Measurement-induced quantum operations on multiphoton states

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We investigate how multiphoton quantum states obtained through optical parametric amplification can be manipulated by performing a measurement on a small portion of the output light field. We study in detail how the macroqubit features are modified by varying the amount of extracted information and the strategy adopted at the final measurement stage. At last the obtained results are employed to investigate the possibility of performing a microscopic-macroscopic nonlocality test free from auxiliary assumptions.

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I. INTRODUCTION

The possibility of performing quantum operations in order to tailor quantum states of light on demand has been widely investigated in the last few years. Several fields of research have been found to benefit from the capability of generating fields possessing the desired quantum properties. Nonclassical states of light, such as sub-Poissonian light [1], squeezed light [2,3], or the quantum superposition of coherent states [4,5], have been generated in a conditional fashion. In this context, continuous-variable (CV) quantum information represents one of the most promising fields where conditional and measurement-induced non-Gaussian operations can find application. To this end, quantum interactions can be induced by exploiting linear optics, detection processes, and ancillary states [6]. For example, the process of coherent photon subtraction has been exploited to increase the entanglement present in Gaussian states [7,8] and to engineer quantum operations in traveling light beams [9]. Finally, very recently, conditional operations lead to the realization of different schemes for the implementation of the probabilistic noiseless amplifier [10–12], which can find interesting applications within the context of quantum phase estimation [13].

Strictly related to the engineering of quantum states of light for applications to quantum information, there is the problem of beating the decoherence due to losses which affect quantum resources interacting with an external environment. In the last few years a large investigation effort has been devoted to the decoherence process and the robustness of increasing size quantum fields, realized by nonlinear optical methods [14–17]. Recently, quantum phenomena generated in the microscopic world and then transferred to the macroscopic one via parametric amplification have been experimentally investigated. In Ref. [16] it has been reported the realization of a resilient to decoherence multiphoton quantum superposition (MQS) [18] involving a large number of photons and obtained by parametric amplification of a single photon belonging to a microscopic entangled pair: $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$, where A,B refer to spatial modes $\mathbf{k_A},\mathbf{k_B}$ and the kets refer to single-photon polarization states $\vec{\pi}$ ($\pi = H,V$). This process has been realized through a nonlinear crystal pumped by an ultraviolet (UV) high power beam acting as a parametric amplifier on the single entangled injected photon (i.e., the qubit $|\phi\rangle_B$ on spatial mode $\mathbf{k_B}$). In virtue of the unitarity of the optical parametric amplifier (OPA), the generated state was found to keep the same superposition character and the interfering properties of the injected qubit [14,15,19] and, by exploiting the amplification process, the single-photon qubit has been converted into a macroqubit involving a large number of photons.

In this paper we consider several strategies for the realization of measurement-induced quantum operations on these multiphoton states, generated through the process of optical parametric amplification. We investigate theoretically how the measurement strategies, applied on a part of the multiphoton state before the final identification measurement, affect the distinguishability of orthogonal macroqubits. Such measurements based on the discrimination of multiphoton probability distributions combine features of both continuous and discrete variables techniques. The interest in improving the capability of identifying the state generated by the quantum injected optical parametric amplifier (QIOPA) system mainly relies on two motivations: the first one concerns the development of a discrimination method able to increase the transmission fidelity of the state after the propagation over a lossy channel, and hence to overcome the imperfections related to the practical implementation. Such increased discrimination capability in lossy conditions could find applications within the quantum communication context. The second reason concerns the scenario in which an appropriate preselection of the macroqubits could be adopted to demonstrate the microscopic-macroscopic nonlocality, free from the auxiliary assumptions requested if the filtering procedure was applied at the final measurement stage.

In previous papers [15,16] a probabilistic discrimination method, the orthogonality filter (OF), was introduced and successfully applied to an entanglement test in a microscopic-macroscopic bipartite system. The application of the OF strategy, acting at the measurement stage, is indeed not suitable for the demonstration of loophole-free microscopic-macroscopic nonlocality because of the presence of inconclusive results [20]. These correspond to the selection of different subensembles of data, depending on the choice of the measurement

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basis. In Ref. [21] Popescu showed that the preselection of data before the final measurement could encompass the problem of a base-dependent filtering of the detected state. More specifically, he showed that performing a sequence of measurements on each of the two parts of a bipartite state could reveal the presence of *hidden* nonlocality which was not observable with a single measurement. This method corresponds to a selection of a subensemble of data *independently* from the measurement performed in the nonlocality test. This preselection scenario was subsequently extended by Pawlowski *et al.* in Ref. [22] in the more general context of quantum communication.

The capability to generate at the output of the parametric amplifier a quantum state of large size allows one to act on a small portion of the field in order to modify the features of the remaining part by a suitable selection. Conditional manipulation of quantum states of light depending on the measurement carried out on part of the beam could increase the capability of discriminating among the generated multiphoton states, as suggested in [23]. Starting from the original proposal of a preselection apparatus in a macroscopic-macroscopic configuration of Ref. [24], we consider the particular case in which a macrostate generated by the QIOPA is split by an unbalanced beam splitter (UBS) and manipulated by measuring the state on the reflected mode. The conceptual scheme underlying the present investigation is shown in Fig. 1(b): a part of the wave function is measured and the results of the measurement are exploited to conditionally activate an optical shutter placed in the transmitted part. Such a shutter, whose realization has been recently reported in Ref. [23], is used to allow the transmission of the optical beam only in the presence of a trigger event (i.e., in this case the results of the measurement performed in the reflected part of the state). Several detection schemes will be investigated in this paper. In Fig. 2(I) is reported the filtering method based on the intensity preselection. As analyzed in more detail in Sec. II, the signal of the reflected part of the macrostate is analyzed by a threshold detector. If the measured intensity value is above a certain threshold, the shutter on the transmitted mode is activated. This strategy allows one to overcome the experimental imperfection related to the vacuum injection into the amplifier, and hence to distillate the macrostate from the noise belonging to the crystal spontaneous emission. The application of this scheme has been already proposed by Stobinska *et al.* in Ref. [25] to perform a loophole-free nonlocality test in a macroscopic-macroscopic configuration. In Fig. 2(II) is reported the strategy illustrated in Sec. III, based on a probabilistic discrimination of the reflected macroqubit part performed by the OF device. By changing the polarization analysis basis on the reflected mode, we have investigated how the macrostate visibility obtained by a dichotomic measurement of the transmitted state is affected. In Fig. 2(III) is illustrated the measurement procedure described in Sec. IV, in which both the reflected and the transmitted mode are analyzed by a probabilistic OF-based measurement. At last, Sec. V addresses the case in which the reflected macroqubit part is measured in two different polarization bases, as shown in Fig. 2(IV), and the final measurement is purely dichotomic. This measurement strategy is aimed at the realization of a nonlocality test on the microscopic-macroscopic photon state, without any auxiliary assumption. However, we show that such a scheme does not allow one to obtain a violation of a Bell's inequality since the analyzed strategy has the effect of increasing the correlations present in the microscopic-macroscopic system only in a specific polarization basis while suppressing the correlations in the other basis.

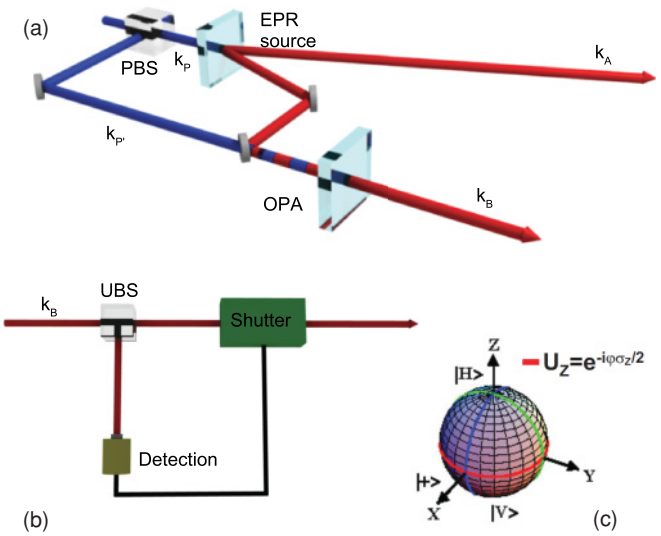

FIG. 1. (Color online) (a) Quantum injected optical parametric amplifier (QIOPA) scheme: an entangled photon pair is generated through spontaneous parametric down-conversion (SPDC). One of the two photons is amplified by a nonlinear crystal, realizing the optimal phase covariant cloning of the injected qubit. (b) Scheme of the measurement-induced quantum operation process. The field is split by an unbalanced beam splitter (UBS), and the reflected portion is measured to conditionally activate the optical shutter placed in the path of the transmitted portion of the field. (c) Schematic view of the single-photon Bloch sphere. The QIOPA device performs the optimal phase covariant process, hence amplifying equally all photons belonging to the equator of the sphere (red (horizontal) line), with polarization $\vec{\pi}_\phi = 2^{-1/2}(\vec{\pi}_H + e^{\imath\phi}\vec{\pi}_V)$.

II. FILTERING OF THE MACROQUBIT

In this section we discuss the specific preselection scheme, sketched in Fig. 2(I), proposed in Ref. [25] in a macroscopic-macroscopic scenario. One of the main experimental challenges for the realization of the microscopic-macroscopic system of Fig. 1 is the achievement of spectral, spatial, and temporal matching between the optical mode of the injected single-photon state and the optical mode of the amplifier. In ideal conditions, as shown in [15,16], the microscopic-macroscopic system is realized by the amplification process performed over an entangled couple $|\psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$ generated by the EPR source in Fig. 1(a). In realistic conditions, the injected microscopic-microscopic system is given by $\hat{\rho}_{\psi^-} = p|\psi^-\rangle_{AB}\langle\psi^-| + (1-p)/2\hat{I}_A \otimes |0\rangle_B\langle 0|$, where, as said, $|\psi^-\rangle_{AB}$ is the entangled singlet state connecting the spatial modes A and B, and the parameter p expresses the amount of mode matching between the seed and the amplifier. The microscopic-macroscopic amplified state $\hat{\rho}_{\Psi^-}$ is obtained after the amplification of the $\hat{\rho}_{\psi^-}$ injected state: $\hat{\rho}_{\Psi^-} = (\hat{I}_A \otimes \hat{U}_B)\hat{\rho}_{\psi^-}(\hat{I}_A \otimes \hat{U}_B^\dagger)$, where $\hat{U} = e^{-\imath\hat{H}_I t/\hbar}$ is

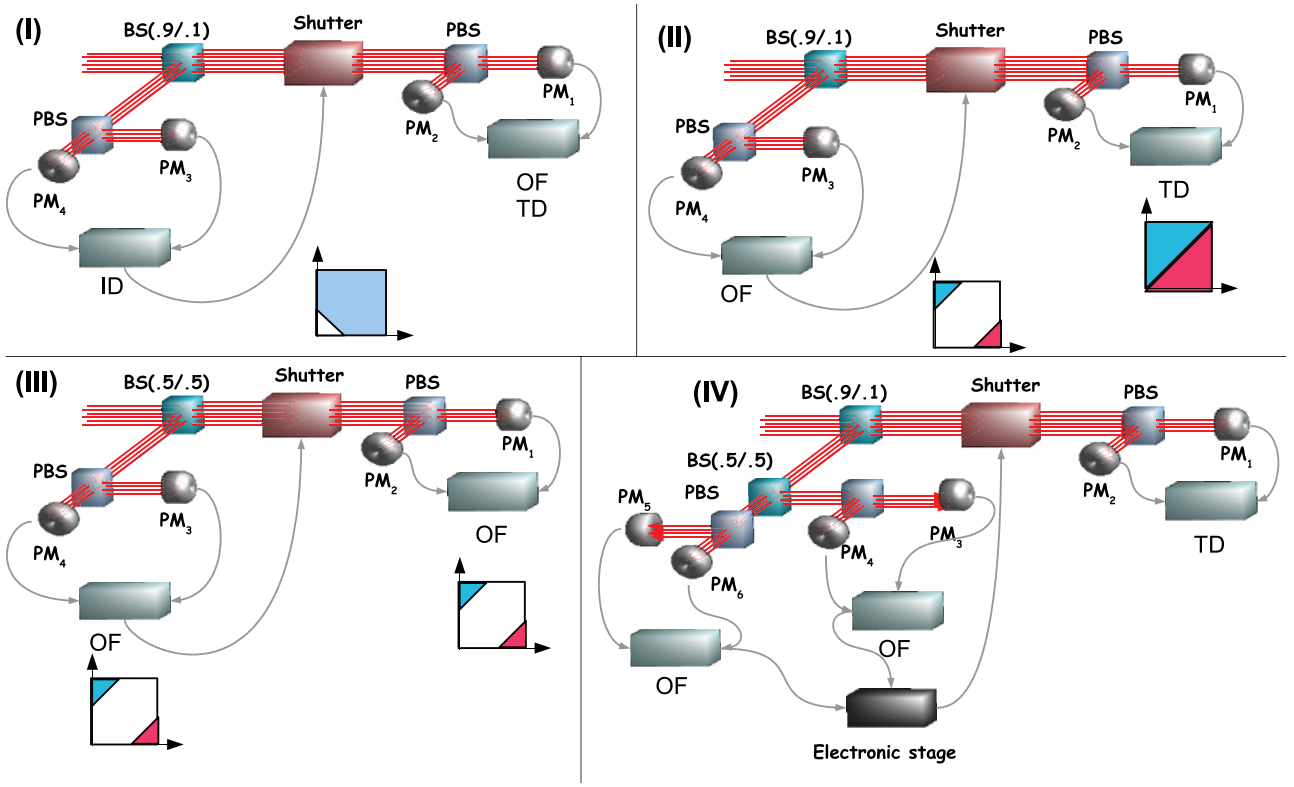

FIG. 2. (Color online) Measurement strategies devoted to the manipulation of the macroqubit state. (I) The shutter activation is conditioned to an intensity measurement on the reflected portion of the macrostate; (II) the small reflected part of the macrostate is analyzed in polarization and detected through an OF-based measurement strategy; (III) the macrostate is split in two equal parts, and both the reflected and the transmitted components are detected through an OF device; (IV) a double basis measurement is performed on the small reflected portion of the macroqubit.

the time-evolution operator associated with the amplifier, defined by the interaction Hamiltonian $\hat{H}_I = \imath\hbar\chi\hat{a}_H^\dagger\hat{a}_V^\dagger + \text{H.c.}$. Then in the expression of the number of photons $N_{\pi_\pm}(\varphi)$ generated by the amplifier when a single photon with the equatorial [Fig. 1(c)] polarization state $|\phi\rangle$ is injected, the spontaneous emission has to be taken into account: $N_{\pi_\pm}(\varphi) = p\{\bar{m} + \frac{1}{2}(2\bar{m}+1)[1 \pm \cos(\varphi)]\} + (1-p)\bar{m}$. When the single photon is injected correctly in the OPA, a pulse with a higher photon number is generated since stimulated emission processes occur in the amplifier. The filtering method here presented exploits this feature to reduce the noise introduced by the spontaneous emission of the amplifier.

Let us now discuss the propagation of the multiphoton field produced by the amplifier and the preselection procedure obtained through an intensity threshold detector (ID) and the shutter device. As shown in Fig. 2(I), the amplified state is split by an unbalanced beam splitter (UBS) 0.90–0.10 in two parts: the smaller portion on mode $\mathbf{k_D}$ is analyzed by the ID, and the larger one on mode $\mathbf{k_C}$ is conditionally pushed through a polarization-preserving shutter [23], and measured in polarization by a dichotomic measurement. The ID-based filtering strategy allows one then to obtain a better discrimination between the orthogonal macrostates, by minimizing the noise related to the vacuum injection into the amplifier. It is worth nothing that, at variance with the techniques which will be introduced in the following sections, the ID action is invariant for rotation on the Fock space. It indeed selects the same region of the macroqubit either in the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis or in the $\{\vec{\pi}_R,\vec{\pi}_L\}$ one. The action of the ID on mode $\mathbf{k}_D$ and of the shutter on mode $\mathbf{k_C}$ allows one to distill the macroqubit from the noise generated by the amplifier and related to the spontaneous emission of the crystal. In the ideal case, this measurement corresponds to the projection of the impinging field onto the subspace: $\hat{\Pi}_k = \sum_{m+n>h} |n\pi,m\pi_\perp\rangle\langle n\pi,m\pi_\perp|$, where $|n\pi,m\pi_\perp\rangle$ represents a quantum state with n photons with polarization π and m photons with polarization $\pi_\perp$. The measurement method is hence based on a threshold detection scheme, in which the ID clicks only if $n_\pi + m_{\pi_\perp} > h$, where h is a threshold conveniently selected. This click activates the shutter on the transmitted UBS mode, ensuring the presence of the higher (i.e., correctly injected) pulses. This scheme has the peculiar property of selecting an invariant region of the Fock space with respect to rotations of the polarization basis. As said, the action of the ID device allows one to decrease the noise due to the vacuum injection into the amplifier since it preserves only the higher pulses, and hence the ones that, with a higher probability, belong to the amplification process. These considerations can be quantified in the following way. The parameter of interest is the conditional injection probability (i.e., the injection probability conditioned to the activation of the shutter given by the threshold condition of the ID). We then evaluated numerically this quantity for several values of the unconditioned injection probability p. It turns out that the

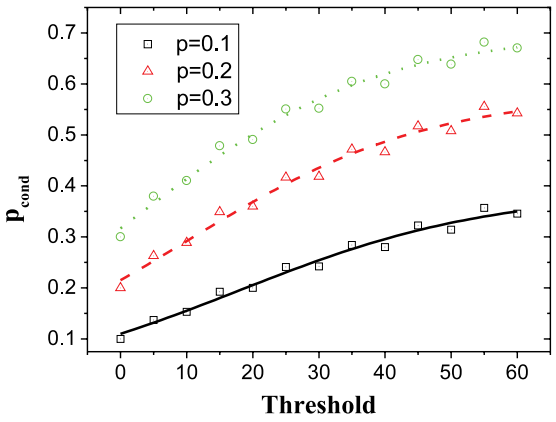

FIG. 3. (Color online) Trend of the injection probability as a function of the ID threshold, for different initial values of p. The nonlinear gain of the amplifier is set at $g = 1.5$.

value of p_{cond} is increased as shown in Fig. 3, in which we report the trend of the conditional injection probability p_{cond} as a function of the ID threshold h.

III. DETERMINISTIC TRANSMITTED STATE IDENTIFICATION

In this section we are interested in exploiting the action of a different preselection strategy, no more based on the intensity filtering but on a comparison between orthogonally polarized signals. This configuration is illustrated in Fig. 2(II) and is based on a peculiar feature of the equatorial macrostates. Indeed, any macroqubit belonging to the injection of an equatorial qubit, due to the phase covariance of the amplifier, can be discriminated through a measurement based on a comparison. Precisely, we can measure the intensity signals belonging to orthogonal polarization components of the same macrostate and subsequently compare them above a certain threshold k. If analyzed in the same polarization basis of the injected qubit, the two signals will be unbalanced with a high probability. This can be explained by analyzing the probability distribution of the amplified states, reported in Fig. 4(a) in the mutually unbiased equatorial polarization basis with respect to the injected state and Fig. 4(b) in the same basis as the injected qubit one.

We will address two cases in which the state generated by the amplifier is either $|\Phi^+\rangle$ or $|\Phi^R\rangle$, obtained by the amplification of a single photon polarized $\vec{\pi}_+ = \frac{\vec{\pi}_H+\vec{\pi}_V}{\sqrt{2}}$ and $\vec{\pi}_R = \frac{\vec{\pi}_H+\imath\vec{\pi}_V}{\sqrt{2}}$, respectively. In both cases the analysis basis corresponding to the UBS-reflected mode is fixed to $\{\vec{\pi}_+,\vec{\pi}_-\}$, while the transmitted mode is analyzed in the same basis in which the injected qubit has been encoded. Let us discuss the experimental setup shown in Fig. 2(II). The macrostate $|\Phi^+\rangle$ (or $|\Phi^R\rangle$) generated by the QIOPA impinges on the UBS. A small portion of the field is reflected on mode $\mathbf{k_d}$ and measured on the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis. The two signals belonging to orthogonal polarizations are then compared by an orthogonality filter. When the two signals are unbalanced (i.e., $|p-q| > k$), being p,q the number of photons $\vec{\pi}_+$,$\vec{\pi}_-$ polarized and k an appropriate threshold, the shutter on mode $\mathbf{k_c}$ is activated and

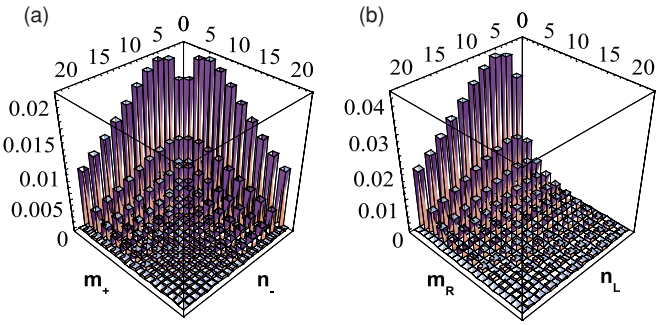

FIG. 4. (Color online) (a) Probability distribution of the state $|\Phi^R\rangle$ as a function of the number of photons $\{\vec{\pi}_+,\vec{\pi}_-\}$. (b) Probability distribution of the state $|\Phi^R\rangle$ as a function of the number of photons $\{\vec{\pi}_R,\vec{\pi}_L\}$. In both distributions $g = 1.5$.

the field on that mode is conditionally transmitted (see Fig. 5). The macrostate $|\Phi^+\rangle$ ($|\Phi^R\rangle$) is then analyzed in the $\{\vec{\pi}_+,\vec{\pi}_-\}$ (or $\{\vec{\pi}_R,\vec{\pi}_L\}$) basis. In the following sections we will address the problem of discriminating the final macrostate, given the acquired information on the small portion of the reflected field.

A. Probability of shutter activation

Let us first evaluate the probability P of activating the shutter when the impinging state is detected on the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis, depending on the value of k, with an OF technique. As shown in Fig. 6, the probability of activating the shutter is the same for the two output fields $|\Phi^+\rangle$ and $|\Phi^R\rangle$. This result can be explained by considering the probability distributions of the state $|\Phi^R\rangle$ in the two mutually unbiased equatorial bases shown in Fig. 4. Due to the linearity of the quantum mechanics, the state $|\Phi^R\rangle$ can be written as $|\Phi^R\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + \imath|\Phi^-\rangle)$. Hence, due to the peculiar features of the two macrostates $|\Phi^\pm\rangle$, that have nonzero contributions for terms with different parity, the probability distribution of the macrostate $|\Phi^R\rangle$ in

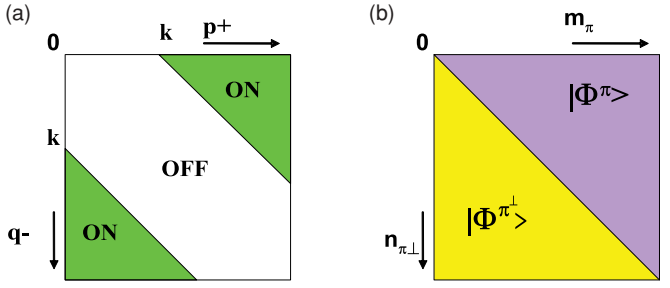

FIG. 5. (Color online) (a) Measurement scheme adopted for the conditional activation of the shutter. If the OF, on the reflected mode, measures the state on the green (shaded corner) regions, the shutter, on the transmitted mode, is conditionally activated. The green (shaded corner) regions correspond to the state for which the signals belonging to orthogonal polarizations are unbalanced over a certain threshold k (i.e., $|p-q| \geqslant k$). (b) Scheme for the final detection of the output state. Conditioned on a measurement result in the ON region on the reflected mode, the transmitted mode is identified by a dichotomic measurement in the $\{\pi,\pi_\perp\}$ basis. The diagonal contribution to the quantum state is assigned randomly to the state $|\Phi^\pi\rangle$ or $|\Phi^{\pi\perp}\rangle$.

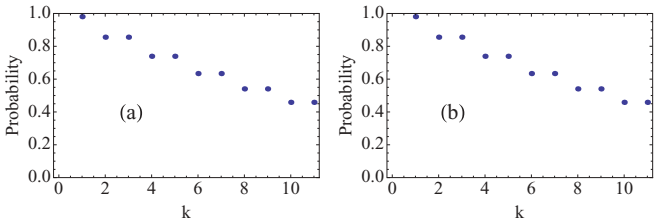

FIG. 6. (Color online) (a) Probability of activating the shutter when the state $|\Phi^R\rangle$ is analyzed in the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis versus the threshold k of the OF. (b) Probability of activating the shutter when the state $|\Phi^+\rangle$ is analyzed on the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis. The nonlinear gain of the amplifier is set at $g = 1.5$.

the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis is given as the sum of the two probability distributions of the states $|\Phi^+\rangle$ and $|\Phi^-\rangle$ in the same basis. Since shot by shot the OF identifies the state $|\Phi^+\rangle$ or $|\Phi^-\rangle$ with the same probability, the activation of the shutter has the same probability of occurrence for any linear combination of $|\Phi^-\rangle$ and $|\Phi^+\rangle$ impinging on the BS.

B. Analysis of the macrostate $|\Phi^+\rangle$

Let us analyze the evolution of the state $|\Phi^+\rangle$ passing through an unbalanced beam-splitter. We start with the expression of the macroqubit [16]:

$$|\Phi^+\rangle = \frac{1}{C^2}\sum_{ij}^{\infty}\left(\frac{-\Gamma}{2}\right)^j\left(\frac{\Gamma}{2}\right)^i\frac{\sqrt{(2j)!(2i+1)!}}{j!i!} \times |(2i+1)+,2j-\rangle_b = \frac{1}{C^2}\sum_{ij}^{\infty}\left(\frac{-\Gamma}{2}\right)^j\left(\frac{\Gamma}{2}\right)^i\frac{(\mathbf{b}_+^\dagger)^{\mathbf{2i+1}}}{j!}\frac{(\mathbf{b}_-^\dagger)^{\mathbf{2j}}}{i!}|0\rangle. \quad (1)$$

The UBS transformation equations for the creation operators on spatial mode $\mathbf{b}$ read

$$\mathbf{b}_\pm = \sqrt{\tau}\mathbf{c}_\pm + \imath\sqrt{1-\tau}\,\mathbf{d}_\pm, \quad (2)$$

where the subscript $\pm$ refers to the polarization modes $\vec{\pi}_\pm = \frac{\vec{\pi}_H \pm \vec{\pi}_V}{\sqrt{2}}$, and $\mathbf{c}^\dagger$ and $\mathbf{d}^\dagger$ refer to the creation operators on the spatial modes transmitted and reflected by the UBS. Hence, after the UBS the output state becomes

$$|\Phi^+\rangle^{\text{out}} = \frac{1}{C^2}\sum_{ij}^{\infty}\sum_{k}^{2i+1}\sum_{l}^{2j}\binom{2i+1}{k}\binom{2j}{l}\left(\frac{-\Gamma}{2}\right)^j\left(\frac{\Gamma}{2}\right)^i\frac{1}{j!}\frac{1}{i!}(\sqrt{\tau}\mathbf{c}_+^\dagger)^k(\imath\sqrt{1-\tau}\mathbf{d}_+^\dagger)^{2i+1-k} (\sqrt{\tau}\mathbf{c}_-^\dagger)^l(\imath\sqrt{1-\tau}\mathbf{d}_-^\dagger)^{2j-l}|0\rangle. \quad (3)$$

By applying the creation operators to the vacuum state the output state reads

$$|\Phi^+\rangle^{\text{out}} = \frac{1}{C^2}\sum_{ij}^{\infty}\sum_{k}^{2i+1}\sum_{l}^{2j}\left(\frac{-\Gamma}{2}\right)^j\left(\frac{\Gamma}{2}\right)^i\frac{1}{j!}\frac{1}{i!}\frac{\sqrt{\tau}^{k+l}}{\sqrt{k!}} \frac{(2i+1)!(2j)!}{\sqrt{(2i+1-k)!(2j-l)!}}\frac{(\imath\sqrt{1-\tau})^{2i+1+2j-k-l}}{\sqrt{l!}} |k+,l-\rangle_d|(2i+1-k)+,(2j-l)-\rangle_c. \quad (4)$$

Let us now consider the case in which the reflected mode by the UBS is measured on the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis. The state $|p+,q-\rangle_d$ is detected on the reflected mode, the transmitted state then reads

$$|\Phi^+\rangle^{\text{meas}} = \frac{1}{C^2}\sum_{ij}^{\infty}\left(\frac{-\Gamma}{2}\right)^j\left(\frac{\Gamma}{2}\right)^i\frac{\sqrt{\tau}^{2i+1+2j-p-q}}{j!} \frac{(\imath\sqrt{1-\tau})^{p+q}}{i!}\frac{(2i+1)!(2j)!}{\sqrt{p!q!(2i+1-p)!(2j-q)}} |(2i+1-p)+,(2j-q)-\rangle_c. \quad (5)$$

We are interested in investigating the distinguishability between orthogonal macrostates by varying the preselection performed over the multiphoton state itself. It is worth noting that our investigation addresses the coherence between two different polarization modes which is quantified by the visibility of the intensity curve obtained by rotating a polarizer. Our first scope is to investigate the visibility of the transmitted mode as a function of the unbalancement between $\vec{\pi}_+$ and $\vec{\pi}_-$ photons, detected on the reflected mode. Namely, if $|p-q| > k$ on mode $\mathbf{k_d}$, what is the visibility of the state $|\Phi^+\rangle^{\text{meas}}$ on mode $\mathbf{k_c}$?

This quantity can be quantified in the following way. Due to the peculiar shape of the photon number probability distribution [Fig. 4(b)], the identification of the state $|\Phi^+\rangle$ can be performed by discriminating between the number of photons $\vec{\pi}_+$ and $\vec{\pi}_-$ polarized. Let us define the following quantities: $P^+(m,n|p,q)$ is the probability that, if the state $|p+,q-\rangle_d$ is detected on spatial mode $\mathbf{k_d}$, $m > n$ is obtained on spatial mode $\mathbf{k_c}$, and hence the macrostate $|\Phi^+\rangle$ is identified (m,n being the number of photons $\vec{\pi}_+$ and $\vec{\pi}_-$ polarized). On the contrary $P^-(m,n|p,q)$ is the probability that, given the detection of the state $|p+,q-\rangle_d$ on spatial mode $\mathbf{k_d}$, $n > m$ is obtained on spatial mode $\mathbf{k_c}$, and hence the macrostate $|\Phi^-\rangle$ is identified, even if the initial state impinging on the UBS was $|\Phi^+\rangle$. We can than derive the visibility as a function of the threshold k such that $|p-q| > k$:

$$V(k) = \frac{\sum_{m,n}\sum_{p,q}\left(P_{m,n}^{p,q\,+}(k) - P_{m,n}^{p,q\,-}(k)\right)}{\sum_{m,n}\sum_{p,q}\left(P_{m,n}^{p,q\,+}(k) + P_{m,n}^{p,q\,-}(k)\right)}, \quad (6)$$

where $P_{m,n}^{p,q\,\pm}(k) = P^\pm(m,n||p-q| > k)$. The trend of visibility as a function of k is reported in Fig. 7(a). We observe that, increasing the value of k, hence detecting a higher unbalancement between $\vec{\pi}_+$ and $\vec{\pi}_-$ photons on mode $\mathbf{k_d}$, we obtain a higher visibility of the state on mode $\mathbf{k_c}$.

C. Analysis of the macrostate $|\Phi^R\rangle$

Let us consider the case in which the state $|\Phi^R\rangle$ impinges on the UBS:

$$|\Phi^R\rangle = \frac{1}{C^2}\sum_{ij}^{\infty}\left(\frac{\imath\Gamma}{2}\right)^j\left(\frac{\imath\Gamma}{2}\right)^i\frac{\sqrt{(2j)!}}{j!}\frac{\sqrt{(2i+1)!}}{i!} \times |(2i+1)R,(2j)L\rangle_b = \frac{1}{C^2}\sum_{ij}^{\infty}\left(\frac{\imath\Gamma}{2}\right)^j\left(\frac{\imath\Gamma}{2}\right)^i\frac{(\mathbf{b_R}^\dagger)^{\mathbf{2i+1}}}{j!}\frac{(\mathbf{b_L}^\dagger)^{\mathbf{2j}}}{i!}|0\rangle. \quad (7)$$

After the UBS, the state can be written as

$$|\Phi^R\rangle^{\text{out}} = \frac{1}{C^2}\sum_{ij}^{\infty}\sum_{k}^{2i+1}\sum_{l}^{2j}\left(\frac{\iota\Gamma}{2}\right)^j\left(\frac{\iota\Gamma}{2}\right)^i\frac{1}{j!}\frac{1}{i!}\frac{\sqrt{\tau}^{k+l}}{\sqrt{k!}} \frac{(2i+1)!(2j)!}{\sqrt{(2i+1-k)!(2j-l)!}}\frac{(\iota\sqrt{1-\tau})^{2i+1+2j-k-l}}{\sqrt{l!}} |kR,lL\rangle_c|(2i+1-k)R,(2j-l)L\rangle_d. \tag{8}$$

The state on mode $\mathbf{k_d}$ is then measured in the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis. The state $|(2i+1-k)R,(2j-l)L\rangle_d$ can then be rewritten as

$$|(2i+1-k)R,(2j-l)L\rangle = \sum_{r}^{2i+1-k}\sum_{s}^{2j-l}\frac{1}{\sqrt{2}^{2i+1+2j-k-l}} \frac{1}{\sqrt{(2i+1-k)!}}\frac{1}{\sqrt{(2j-l)}}\binom{2i+1-k}{s}\binom{2j-l}{s} \times\sqrt{(s+r)!(2i+1+2j-k-l-s-r)!} \times\iota^{2i+1-k-r}\iota^{2j-l-s} |(r+s)+,(2i+1+2j-k-l-r-s)-\rangle, \tag{9}$$

and the overall state reads

$$|\Phi^R\rangle^{\text{out}} = \frac{1}{C^2}\sum_{ij}^{\infty}\sum_{k}^{2i+1}\sum_{l}^{2j}\sum_{r}^{2i+1-k}\sum_{s}^{2j-l}\left(\frac{\iota\Gamma}{2}\right)^j\left(\frac{\iota\Gamma}{2}\right)^i\frac{1}{i!}\frac{1}{j!} \times\frac{(2i+1)!(2j)!\iota^{2i+1-k-r}(-\iota)^{2j-l-s}}{\sqrt{(2i+1-k)!(2j-l)!k!l!}} \times\frac{\sqrt{\tau}^{k+l}\sqrt{\iota(1-\tau)}^{2i+1+2j-k-l}}{\sqrt{2}^{2i+1+2j-k-l}} \frac{\sqrt{(r+s)!}\sqrt{(2i+1+2j-k-l-r-s)!}}{\sqrt{(2i+1-k)!(2j-l)!}} \times\binom{2i+1-k}{r}\binom{2j-l}{s} |(r+s)+,(2i+1+2j-k-l-r-s)-\rangle_d|kR,lL\rangle_c. \tag{10}$$

If the state on mode $\mathbf{k_d}$ is detected, $|(r+s)+,(2i+1+2j-k-l-r-s)-\rangle_d = |p+,q-\rangle_d$, the state on mode $\mathbf{k_c}$ is

$$|\Phi^R\rangle^{\text{meas}} = \frac{1}{C^2}\sum_{ij}^{\infty}\sum_{l}^{2j}\sum_{s}^{2j-l}\left(\frac{\iota\Gamma}{2}\right)^j\left(\frac{\iota\Gamma}{2}\right)^i\frac{1}{i!}\frac{1}{j!} \times\frac{(2i+1)!(2j)!}{\sqrt{(l+p+q-2j)!(2j-l)!}} \times\frac{\sqrt{\tau}^{2i+2j+1-p-q}}{\sqrt{(2i+1+2j-l-p-q)!l!}} \frac{\sqrt{p!q!}}{\sqrt{2}^{p+q}}\frac{\sqrt{\iota(1-\tau)}^{p+q}}{\sqrt{(l+p+q-2j)!(2j-l)!}}\binom{l+p+q-2j}{p-s} \times\binom{2j-l}{s}\iota^{l+q-2j}(-\iota)^{2j-l-s} \times|(2i+1+2j-l-p-q)R,lL\rangle_c, \tag{11}$$

where the following conditions have to be satisfied:

$$\begin{gathered} p>s, \\ 2i+1+2j>l+p+q, \\ 2j<l+p+q. \end{gathered} \tag{12}$$

If the state (11) is measured in the polarization basis $\{\vec{\pi}_R,\vec{\pi}_L\}$ obtaining a state $|mR,nL\rangle_c$, the corresponding probability amplitude is

$$\frac{1}{C^2}\sum_{j}^{\infty}\sum_{s}^{2j-n}\left(\frac{\iota\Gamma}{2}\right)^j\left(\frac{\iota\Gamma}{2}\right)^{(-2j+m+n+p+q-1)/2}\frac{(-\iota)^{2j-n-s}}{\sqrt{m!n!}} \frac{1}{j!}\frac{1}{\left(\frac{-2j+m+n+p+q-1}{2}\right)!}\frac{(-2j+p+q+m+n)!(2j)!}{\sqrt{(n+q-2j+s)!(p-s)!}} \frac{\sqrt{p!q!}}{(2j-n-s)!s!}\sqrt{\tau}^{m+n}(\iota\sqrt{1-\tau})^{p+q}\frac{1}{\sqrt{2}^{p+q}}\iota^{n+p+q-2j-r} \tag{13}$$

and the probability of measuring the state $|mR,nL\rangle_c$ is given by

$$P(m,n|p,q) = \frac{1}{C^4}\sum_{j=0}^{\infty}\sum_{s}^{2j-n}\sum_{i}^{\infty}\sum_{r}^{2i-n}\frac{1}{C^4}\left(\frac{\Gamma}{2}\right)^{m+n+p+q-1} \frac{p!q!}{m!n!}\frac{(-1)^{r-s}}{2^{p+q}}\frac{\tau^{m+n}(1-\tau)^{p+q}}{\left(\frac{p+q+m+n-2j-1}{2}\right)!j!}\frac{1}{\left(\frac{p+q+m+n-2i-1}{2}\right)!i!} \frac{(p+q+m+n-2j)!(2j)!}{(n+q+s-2j)!s!(2j-n-s)!(p-s)!} \times\frac{(p+q+m+n-2i)!(2i)!}{(q+n+r-2i)!r!(2i-n-r)!(p-r)!}. \tag{14}$$

The visibility of the macrostate reads

$$V(k) = \frac{\sum_{m,n}\sum_{p,q}\left(P_{m,n}^{p,q\ R}(k)-P_{m,n}^{p,q\ L}(k)\right)}{\sum_{m,n}\sum_{p,q}\left(P_{m,n}^{p,q\ R}(k)+P_{m,n}^{p,q\ L}(k)\right)}, \tag{15}$$

where $P_{m,n}^{p,q\ R,L}(k) = P^{R,L}(m,n||p-q|>k)$. Here, $P^R(m,n||p-q|>k)$ is the probability that, given the detection of the state $|p+,q-\rangle_d$ on mode $\mathbf{k_c}$, $m>n$ is obtained on mode $\mathbf{k_d}$ [$m(n)$, number of photons polarized $\vec{\pi}_R(\vec{\pi}_L)$]. In this case the state $|\Phi^R\rangle$ is identified; conversely, the state $|\Phi^L\rangle$ is detected even if the state $|\Phi^R\rangle$ impinged on the UBS.

We observe that the visibility of the state $|\Phi^R\rangle$ in the case in which a small portion of the overall state is measured on the $\{\vec{\pi}_+,\vec{\pi}_-\}$ polarization basis, is a decreasing function of the threshold k ($|p-q|>k$). This trend is shown in Fig. 7(c). The decreasing trend of visibility can be explained by considering that the measurements in the two polarization bases correspond to two noncommuting operators acting on the same initial state. Indeed, for asymptotically high values of the threshold $k\to\infty$, the measurement of the $\hat{\Pi}_i$ operators that describe the OF tends to the measurement of the pseudospin operators $\hat{\Sigma}_i$: that is, $\hat{\Sigma}_1 = |\Phi^+\rangle\langle\Phi^+| - |\Phi^-\rangle\langle\Phi^-|$ or $\hat{\Sigma}_2 = |\Phi^R\rangle\langle\Phi^R| - |\Phi^L\rangle\langle\Phi^L|$. More details on the relationship between the OF device and the pseudo-Pauli operator can be found in Ref. [26]. In view of this consideration, the measurement on the $\mathbf{k}_C$ mode corresponds to the measurement of the $\hat{\Sigma}_i$ operators. The information gained on this mode about one of the two

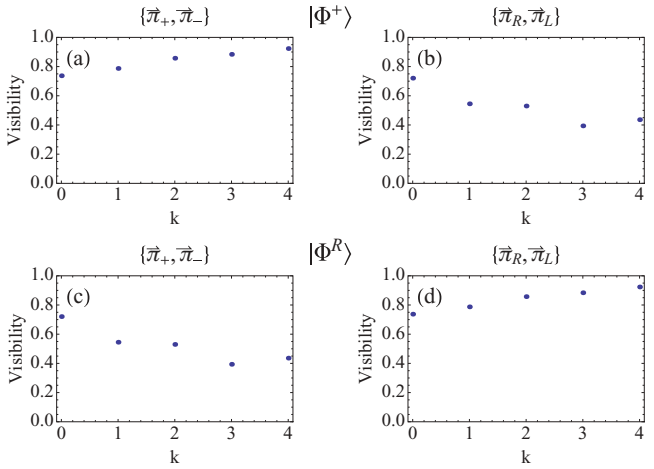

FIG. 7. (Color online) (a)–(b) Trend of the visibility of the state $|\Phi^+\rangle$ measured in the basis $\{\vec{\pi}_+,\vec{\pi}_-\}$ and $\{\vec{\pi}_R,\vec{\pi}_L\}$, respectively, as a function of the threshold k. (c) and (d) Trend of the visibility of the state $|\Phi^R\rangle$ measured in the basis $\{\vec{\pi}_+,\vec{\pi}_-\}$ and $\{\vec{\pi}_R,\vec{\pi}_L\}$, respectively, as a function of the threshold k. The numerical results have been obtained for the value of the gain parameter $g = 1.1$.

pseudospin operators acting on the macroqubit does not allow one to gain information about the orthogonal pseudospin operator. As a further remark, let us stress that this feature of the OF measurement is related to the filtering of different regions of the Fock space depending on the analyzed basis. The portion of the state that survives the action of the OF is indeed different if measured on the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis or in the $\{\vec{\pi}_R,\vec{\pi}_L\}$ one and is shown in Fig. 8.

IV. PROBABILISTIC TRANSMITTED STATE IDENTIFICATION

In the previous sections we have shown how the visibility of the macroqubit obtained by a pure dichotomic measurement can be modified if a small portion of the beam is identified by a probabilistic measurement strategy.

This section addresses the trend of the macrostate visibility when the field is split in two equal parts by a 0.5-0.5 beam splitter, and both the reflected and the transmitted states are

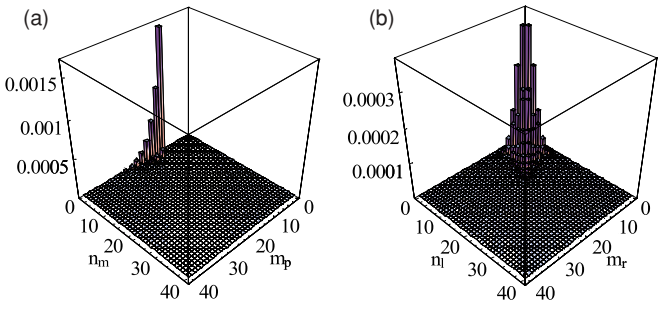

FIG. 8. (Color online) Selected region for the $|\Phi^+\rangle$ state after the measurement with an OF in the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis. (a) Photon number distribution in the $\{\vec{\pi}_+,\vec{\pi}_-\}$ basis. (b) Photon number distribution in the $\{\vec{\pi}_R,\vec{\pi}_L\}$ basis. In both cases $k = 10$ and $g = 1.2$.

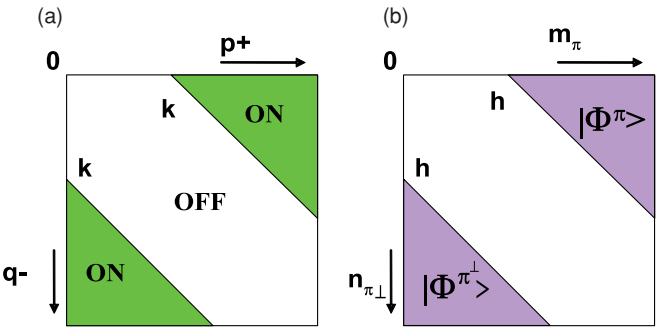

FIG. 9. (Color online) (a) Conditional activation of the shutter. If the OF acting on the reflected mode measures the state on the green (shaded corner) regions, the shutter, on the transmitted mode, is conditionally activated. The green (shaded corner) regions correspond to the state for which the signals belonging to orthogonal polarizations are unbalanced over a certain threshold k (i.e., $|p - q| \geqslant k$). (b) Corresponding to the ON region on the reflected mode, the transmitted mode is identified by a probabilistic measurement in the $\{\pi,\pi_\perp\}$ basis. The identification condition is $|m - n| \geqslant h$.

detected through the OF device. In this case the measurement schemes are shown in Figs. 2(III) and 9: the OF technique is applied in order to extract the maximum information available from the two measured states.

We consider the case in which the portion on the reflected mode is analyzed in the polarization basis orthogonal to the codification one. In Fig. 10 is reported the trend of visibility as a function of the threshold h on the transmitted mode, and k on the reflected one. The two polarization analysis basis are chosen to be mutually unbiased. It can be seen that for equal values of the two thresholds $h = k$ the visibility reaches a value around 0.64, the same obtained through a pure dichotomic measurement, without any preselection procedure on the macrostate. In Fig. 11 is reported the trend of the visibility as a function of the threshold on the reflected mode, keeping fixed the value of the threshold on the transmitted one. We can see that the visibility of the transmitted state decreases when the threshold on the reflected mode increases. If the

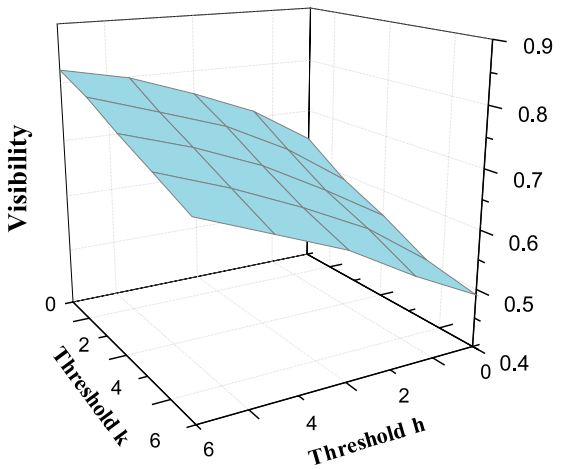

FIG. 10. (Color online) Trend of the visibility of the state $|\Phi^R\rangle$ for different values of the threshold h on the transmitted mode and of the threshold k on the reflected one. The numerical result has been obtained for a value of the parameter $g = 1.2$.

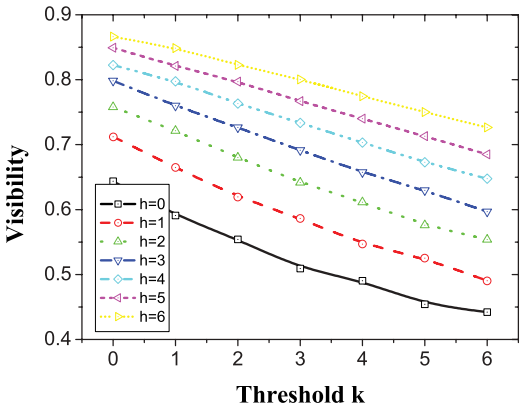

FIG. 11. (Color online) Trend of the macrostate visibility as a function of the threshold k on the reflected mode fixed the threshold h on the transmitted one.

threshold on the transmitted mode is greater than the one on the reflected mode, the visibility results to be higher than 0.64, as expected by the action of the OF, which allows a better discrimination of the macrostate, measured in the codification polarization basis. Otherwise it can be seen how, decreasing the threshold h below the threshold k, the visibility decreases below the "no-filtering value."

From the analysis performed in this paper we can conclude that the macrostates are not suitable for quantum cryptography. The action on a portion of the state can indeed be seen as an eavesdropping attack. If the state is measured in the codification basis, the visibility of the final state results to increase as shown in Figs. 7(a)–7(d). This means that the conclusive results for the eavesdropper would coincide with the conclusive results for the receiver, and the eavesdropper can gain information on the macrostates without introducing noise. Otherwise, if the state is measured by the eavesdropper in the wrong basis, the visibility at the receiver is not affected if the state is measured above a certain filtering threshold. According to these considerations, an eavesdropper could then develop a strategy in which he measures its part of the transmitted state in two bases. With this approach he could gain information on the transmitted signal by considering only the measurement outcome in the right basis, and only a small amount of noise is introduced by keeping the filtering thresholds above a certain value. Related to the security of the macrostates is the possibility of performing a nonlocality test upon them. As a final remark for this section, we remind one that the adoption of the OF device at the measurement stage is not suitable for a nonlocality test, since the filtered portion of the state is *dependent* on the measurement basis [20]. We will then address the nonlocality task in the following section.

V. PRESELECTION FOR ENTANGLEMENT AND NONLOCALITY TESTS

In this section we shall investigate a preselection scheme based on a conditional operation driven by the measurement of a portion of the multiphoton state in two different polarization bases. The setup of this preselection scheme is reported in Fig. 2(IV). A small portion of the generated multiphoton state is reflected by an unbalanced beam splitter of transmittivity $T = 0.9$ and subsequently split by a 50-50 beam splitter in two equal parts. One of the two parts is measured in an equatorial $\{\vec{\pi}_\beta,\vec{\pi}_{\beta_\perp}\}$ basis by two photomultipliers, and the photocurrents $\{I_\beta,I_{\beta_\perp}\}$ are analyzed by an OF device (Fig. 5). The other part undergoes the same measurement process in a different equatorial basis $\{\vec{\pi}_{\beta'},\vec{\pi}_{\beta'_\perp}\}$.

When the threshold condition $|I_\pi - I_{\pi_\perp}| > k$ (Fig. 5) is realized in both branches, measured, respectively, in the polarization bases $\{\vec{\pi}_\beta,\vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'},\vec{\pi}_{\beta'_\perp}\}$, a TTL electronic signal is sent to conditionally activate the optical shutter placed in the optical path of the remaining part of the multiphoton state. Then, the field is analyzed at the measurement stage with the dichotomic strategy discussed in the previous paragraphs. For this preselection method, the relevant parameter is the angle ϕ between the two bases $\{\vec{\pi}_\beta,\vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'},\vec{\pi}_{\beta'_\perp}\}$ in which the small portion of the beam is analyzed. The angle ϕ is defined according to the relations between the two polarization bases:

$$\vec{\pi}_{\beta'} = e^{\imath\frac{\phi}{2}}\left[\cos\left(\frac{\phi}{2}\right)\vec{\pi}_\beta - \imath\sin\left(\frac{\phi}{2}\right)\vec{\pi}_{\beta_\perp}\right], \tag{16}$$

$$\vec{\pi}_{\beta'_\perp} = e^{\imath\frac{\phi}{2}}\left[-\imath\sin\left(\frac{\phi}{2}\right)\vec{\pi}_\beta + \cos\left(\frac{\phi}{2}\right)\vec{\pi}_{\beta_\perp}\right]. \tag{17}$$

Let us begin by analyzing the trend of the visibility of the fringe pattern obtained by varying the equatorial polarization $\vec{\pi}_\alpha$ of the injected single-photon state in the amplifier. More specifically, we analyze how the visibility changes as a function of the angle ϕ between the two bases of the preselection branch. In Fig. 12 we show the numerical results obtained by calculating the visibility according to the standard definition $V = \frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}$. In this case, the visibility is evaluated according to the following expression:

$$V(k) = \frac{\sum_{m>n} P_{\overline{\alpha}}[m,n|(|I_\beta - I_{\beta_\perp}| > k)\cap(|I_{\beta'} - I_{\beta'_\perp}| > k)] - \sum_{m<n} P_{\overline{\alpha}}[m,n|(|I_\beta - I_{\beta_\perp}| > k)\cap(|I_{\beta'} - I_{\beta'_\perp}| > k)]}{\sum_{m>n} P_{\overline{\alpha}}[m,n|(|I_\beta - I_{\beta_\perp}| > k)\cap(|I_{\beta'} - I_{\beta'_\perp}| > k)] + \sum_{m<n} P_{\overline{\alpha}}[m,n|(|I_\beta - I_{\beta_\perp}| > k)\cap(|I_{\beta'} - I_{\beta'_\perp}| > k)]}. \tag{18}$$

Here $P_{\overline{\alpha}}[m,n|(|I_\beta - I_{\beta_\perp}| > k)\cap(|I_{\beta'} - I_{\beta'_\perp}| > k)]$ is the photon-number distribution of the state $|\Phi^{\overline{\alpha}}\rangle$ after the preselection stage. More specifically, the value of $\overline{\alpha}$ is chosen in order to maximize the contribution of the $\sum_{m>n}$ term and minimize the contribution of the $\sum_{m<n}$ term:

$$I_{\max} = \sum_{m>n} P_{\overline{\alpha}}[m,n|(|I_\beta - I_{\beta_\perp}| > k)\cap(|I_{\beta'} - I_{\beta'_\perp}| > k)], \tag{19}$$

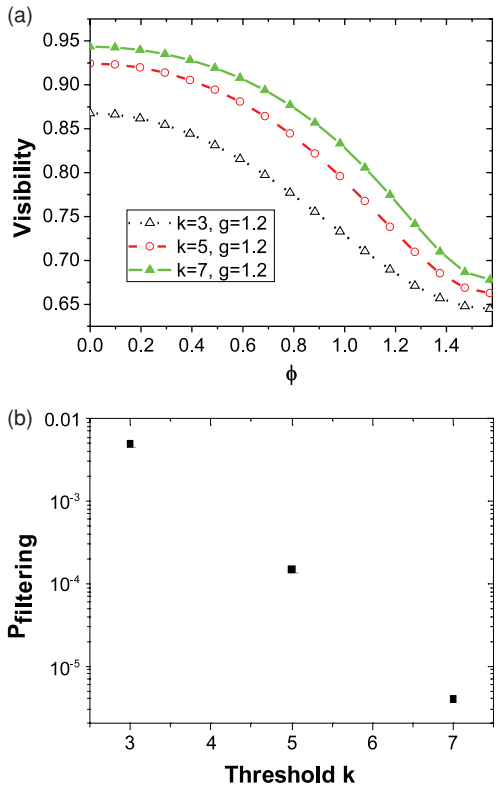

FIG. 12. (Color online) (a) Trend of the visibility for the double-filtering technique as a function of the angle ϕ between the two polarization bases $\{\vec{\pi}_\beta,\vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'},\vec{\pi}_{\beta'_\perp}\}$ of the preselection measurement. Square black points correspond to $k=3$, circular red points to $k=5$, and triangular green points to $k=7$. (b) Filtering probability of the scheme as a function of the threshold k at the preselection measurement stage. All graphs correspond to $g=1.2$.

$$I_{\min}=\sum_{m<n}P_{\overline{\alpha}}[m,n|(|I_\beta-I_{\beta_\perp}|>k)\cap(|I_{\beta'}-I_{\beta'_\perp}|>k)]. \tag{20}$$

Equation (18) then coincides with the usual definition of visibility. We note that the visibility is higher for smaller angles ϕ, since in that case a strong projection of the state is performed in two close bases. This condition is equivalent to the scheme of Fig. 2(II), where the OF measurement performed in one basis allows one to obtain a better discrimination of the detected state only in the polarization basis of the preselection measurement [Figs. 7(a) and 7(b)]. When ϕ is high, a lower visibility can be achieved since the projection of the macrostate occurs in two distant bases. In this case, the increasing effect of the preselection in one basis on the visibility is in contrast with the decreasing effect of the preselection in the other basis, as shown in Sec. III.

We conclude this section by discussing the feasibility of a nonlocality test by exploiting the proposed preselection method. We consider the case of a CHSH inequality [27]. Let us briefly summarize in light of a local hidden variable (LHV) theory the content of Bell's inequalities for a set of dichotomic observables. Consider a quantum state described by the density matrix $\hat{\rho}$ defined in the Hilbert space $\mathcal{H}_1\otimes\mathcal{H}_2$. Define $\hat{O}_a^i$ the positive operator acting on subspace $\mathcal{H}_1$, and the probability of finding the value i after the measurement a as given by $\mathrm{Tr}[\hat{\rho}(\hat{O}_a^i\otimes\hat{I})]$. The same relation holds for the positive operator $\hat{O}_b^j$ acting on subspace $\mathcal{H}_2$.

The existence of an LHV model implies that the expectation values of the a and b observables are predetermined by the value of the parameter λ, $\{X_a(\lambda),X_{a'}(\lambda),X_b(\lambda),X_{b'}(\lambda)\}$, hence the product $a\cdot b$ is equal to $X_a(\lambda)X_b(\lambda)$. For a fixed value of λ the variables X_n with $n=\{a,b,a',b'\}$ take the values $-1,1$ and satisfy the CHSH inequality:

$$X_a(\lambda)X_b(\lambda)+X_a(\lambda)X_{b'}(\lambda)+X_{a'}(\lambda)X_b(\lambda)-X_{a'}(\lambda)X_{b'}(\lambda)\leqslant 2. \tag{21}$$

The same inequality holds by integrating this equation on the space of the hidden variable (λ):

$$\int_\Omega d\mathbb{P}(\lambda)X_a(\lambda)X_b(\lambda)+\int_\Omega d\mathbb{P}(\lambda)X_a(\lambda)X_{b'}(\lambda)+\int_\Omega d\mathbb{P}(\lambda)X_{a'}(\lambda)X_b(\lambda)-\int_\Omega d\mathbb{P}(\lambda)X_{a'}(\lambda)X_{b'}(\lambda)\leqslant 2, \tag{22}$$

where $\mathbb{P}(\lambda)$ is the measure of the λ probability space. If there is a local hidden variable model for quantum measurement taking values $[-1,+1]$, then the following inequality must be satisfied:

$$S_{\mathrm{CHSH}}=E^\rho(a,b)+E^\rho(a,b')+E^\rho(a',b)-E^\rho(a',b')\leqslant 2, \tag{23}$$

where $E^\rho(a,b)=\int_\Omega X_a(\lambda)X_b(\lambda)\,d\mathbb{P}(\lambda)$. The violation of (23) proves that an LHV variable model for the considered experiment is impossible.

We consider the case in which the angle ϕ between the two bases $\{\vec{\pi}_\beta,\vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'},\vec{\pi}_{\beta'_\perp}\}$ is set at $\phi=\pi/4$. This choice is motivated by the following considerations. On one side, low values of ϕ would lead to a microscopic-macroscopic state possessing strong correlations only in one polarization basis, thus not allowing one to violate a Bell's inequality. On the other side, high values of ϕ does not allow one to obtain the necessary enhancement in the correlations of the microscopic-macroscopic system to violate a Bell's inequality. The obtained fringe patterns for the chosen case are reported in Fig. 13 and corresponds to the following conditions. The $(+1)$ outcome of the dichotomic measurement is recorded as a function of the polarization $\vec{\pi}_\alpha$ of the injected single-photon state. In particular, the two chosen equatorial polarization bases $\{\vec{\pi}_\beta,\vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'},\vec{\pi}_{\beta'_\perp}\}$ corresponds to $\beta=0$ and $\beta'=\frac{\pi}{4}$. We then analyzed three different choices for the threshold k at the preselection stage. When the threshold k is set to 0, the fringe pattern corresponding to the two bases $\beta=0$ and $\beta=\frac{\pi}{4}$ are mutually shifted of an angle $\frac{\pi}{4}$, since no filtering and no preselection is performed on the state. When the threshold k is increased, the mutual shift between the fringe pattern is progressively reduced and canceled, since a strong filtering of the state is performed. In particular, the maximum of both the fringe pattern in the $\beta=0$ and $\beta=\frac{\pi}{4}$ bases is obtained for the $|\Phi^{\overline{\alpha}}\rangle$ state with $\overline{\alpha}=\frac{\pi}{8}$. This means that this preselection strategy for a sufficiently high value of k enhances the correlations in the microscopic-macroscopic system in a

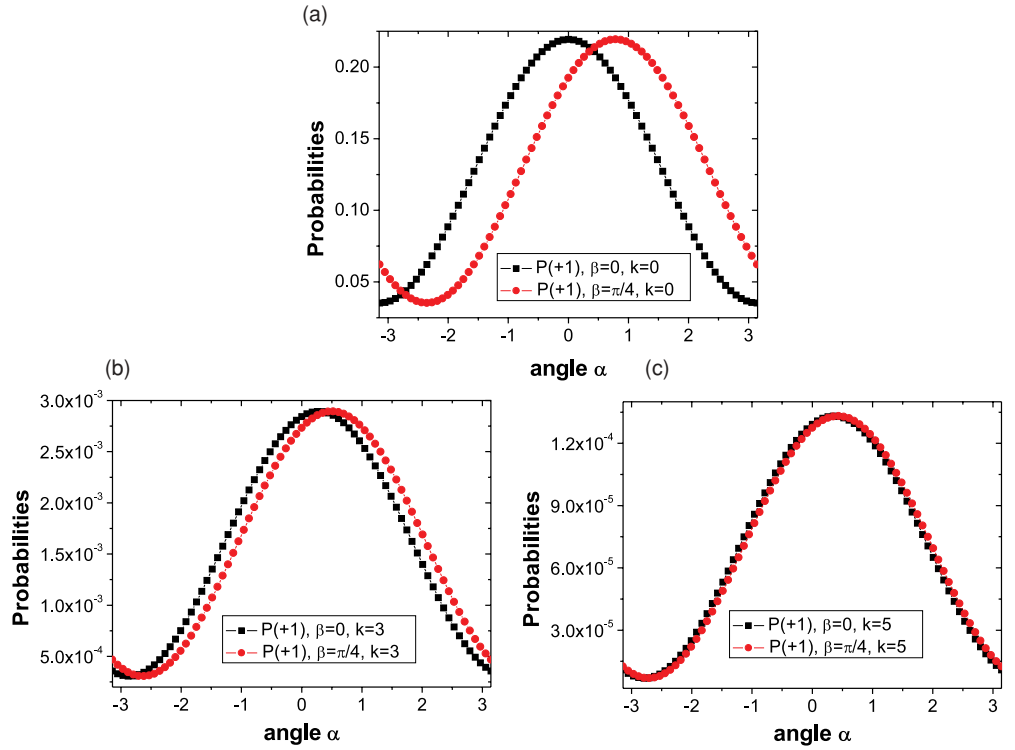

FIG. 13. (Color online) Fringe pattern as a function of the angle α of the polarization basis at the single-photon site. The angle ϕ between the two bases of the preselection stage is set at $\phi = \pi/4$, while $g = 1.2$. (a) Threshold $k = 0$. (b) Threshold $k = 3$. (c) Threshold $k = 5$. Square black points are the fringe patterns obtained by recording the $(+1)$ outcome at the measurement stage, where the measurement basis $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ is set at $\beta = 0$. Circle red points are the fringe patterns obtained by recording the $(+1)$ outcome at the measurement stage, where the measurement basis $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ is set at $\beta = \frac{\pi}{4}$.

specific polarization basis and suppresses the correlations in the other bases. For this reason, the proposed strategy does not allow one to observe the violation of a Bell's inequality in the microscopic-macroscopic system here analyzed. The enhanced value of the visibility could nevertheless be employed in quantum lithography and quantum metrology schemes, in which high-visibility correlation pattern and high-photon number regimes are required. Recently it has indeed been shown how the amplification process of a single-photon probe can beat the detrimental effect of losses which happen in the transmission and detection stages [28]. Such a scheme for noninvasive quantum metrology could benefit from the presented filtering procedures in order to improve the visibility value of the interference fringe pattern.

VI. CONCLUSIONS

In this paper we have analyzed the properties of the macrostates obtained by a quantum injected amplification process, by addressing the behavior of the distinguishability between orthogonal macrostates when a filtering process is applied over a portion of them. More specifically, we analyzed theoretically in detail several schemes for the realization of conditional measurement-induced operations. All these strategies are aimed at the manipulation and filtering of the macrostates for their applications in different contexts, such as the realization of a nonlocality test or quantum communication.

We have identified a strategy, based on the ID device, able to minimize the effects of the noise due to the vacuum injection into the amplifier. The ID-based filtering procedure is independent on the analysis basis and selects the same portion of the state when the measurement is performed in any equatorial polarization basis.

A different filtering procedure, based on the OF device, has been deeply studied: it turned out that when a small portion of the state is analyzed through the OF, the visibility of the overall state, relative to a dichotomic measurement, is affected in a different way depending on the polarization basis in which the small portion has been measured. If the polarization basis is the same of the macroqubit codification, the final visibility increases with the increase of the filtering threshold; otherwise, it decreases. This behavior is related to the impossibility of measuring noncommuting operators on the same quantum state, as explained in Sec. III.

We have further addressed in Sec. IV the trend of the macrostate visibility when an OF discrimination system is used even at the transmitted state detection stage. In this case, the two OF apparatuses in both transmitted and reflected branches play an opposing role in increasing or decreasing the visibility of the fringe pattern obtained in a microscopic-macroscopic configuration. Such analysis shows that the macrostates generated by optical parametric amplification of a single-photon state are not suitable for quantum cryptography, since they are not robust under an eavesdropping attack. Furthermore, we showed that this discrimination method is not suitable for a

microscopic-macroscopic nonlocality test since it performs a base-dependent filtering of the detected state.

Finally, in Sec. V we addressed a preselection scheme for the realization of a Bell's inequality test which does not suffer the same detection loopholes as the one based on postselection strategies [20]. The proposed method, based on the measurement of the reflected part of the wave function in two different bases, does not allow one to violate a Bell's inequality, since it induces the collapse of the correlations present in the macrostates in only a single polarization basis.

Several open points remain to be investigated. The measurement-induced operations analyzed in this paper are all based on dichotomic detection schemes. Other approaches, such as the ones based on continuous variables measurements or on the processes of coherent photon addition and photon subtraction, can lead to a different manipulation of the QIOPA multiphoton states. Systems with different properties from the one analyzed in this paper could be obtained with these methods.

ACKNOWLEDGMENTS

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