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Quantum spin-flipping by the Faraday mirror

V. Secondi, F. Sciarrino, and F. De Martini
Dipartimento di Fisica and Istituto Nazionale per la Fisica della Materia Università di Roma "La Sapienza," Roma, 00185 Italy
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The paper gives a thorough analysis of the spin-flipping action induced by the Faraday mirror on any qubit realized by a polarization-encoded single photon. The results of the theory show that the device acts as a nonuniversal NOT gate, in full agreement with quantum mechanical expectations. Furthermore, the universal optical compensation method devised by Martinelli, of common use in long distance quantum-cryptography, has been analyzed. All theoretical results are substantiated experimentally by the modern methods of quantum state tomography and entanglement-assisted quantum process tomography.

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In the last few years in the quantum information community, a great deal of attention has been attracted by the "optimal" realization of "forbidden" quantum transformations: i.e., the quantum maps that cannot be realized by nature in their exact form since they lack of the required completely positive (CP) character [1]. The most important of such maps are the "quantum cloning" and "flipping" of any unknown qubit $|\Psi\rangle$. Let us consider the latter map, viz., one that transforms any qubit $|\Psi\rangle$ into the orthogonal one $|\psi_{\perp}\rangle$, such as the corresponding representative points on the Poincaré sphere are antipodes one of each other: $\langle\Psi|\Psi_{\perp}\rangle=0$ [2–4]. Even if the basic spin-flipping process is unrealizable in its exact form, being a positive (P) map, it can be approximated by a universal NOT (UNOT) gate implementing optimally the universality condition, i.e., by a fidelity $F<1$. Recently, this optimal gate has been experimentally realized either deterministically by means of a nonlinear parametric amplification [3,4] and probabilistically by a linear symmetrization procedure leading via quantum teleportation to a new protocol: the Tele-UNOT gate [5].

In light of this scenario, recent discussions arose in the literature [2,6,7] about the role attributable in quantum information (QI) to the spin-flipping operation induced by the Faraday mirror (FRM), a rather exotic, passive, and then noiseless magneto-optical device [8]. Because of these properties, the FRM flipping transformation has been deemed to be exact, i.e., implying a full transfer of quantum information from $|\Psi\rangle$ to $|\Psi_{\perp}\rangle$. In addition from a more technological perspective, the FRM spin-flipping process is at the core of the FRM-compensation effect proposed years ago by Martinelli [9]. That very useful process is being currently adopted in modern long-distance quantum cryptography to eliminate efficiently spurious birefringence effects due to polarization fluctuations in optical fibers [6].

In order to clarify the problem and to gain further insight in this interesting issue, the present work reports a detailed quantum mechanical account of the effect of the Faraday mirror on any input qubit realized by the polarization state of a photon. In addition, a comprehensive quantum theory of the FRM-compensation process is given in the paper. In order to avoid any further misinterpretations of the phenomenon, all results are attained in this work through several equivalent but different theoretical paths, and are finally supported by a thorough experimental investigation, including the adoption of the modern methods of the entanglement-assisted quantum process tomography (EAQPT) [10,11].

The FRM is a device consisting of an optical mirror (M) that reflects a monochromatic light beam, associated with a (single Gaussian) mode $\mathbf{k}$, into the mode $-\mathbf{k}$. In front of the mirror is placed a Faraday rotator (FR), a laboratory device generally used to avoid spurious optical backreflection effects, in which a transparent paramagnetic spin-glass rod is acted upon by a static magnetic field $\vec{B}$, parallel to $\mathbf{k}$. By the magneto-optical Faraday effect, the incoming polarization (π) is rotated by keeping the same π character: a linear π is transformed into a linear π', a circular π into a circular π', etc. In particular, the field $\vec{B}$ and the π-rotation angle θ can be tuned as to realize the exact orthogonality between π and π' [8]. Indeed, precisely for this useful property, the FRM device is commercially advertised in optical technology.

We gain insight into the overall process by following the path through FRM of the short light pulse associated with any single photon carrying the qubit. Consider the (real) spatial reference frame (x,y,z) the qubit is referred to before reflection by M: let the axes x,y belong to the plane of M, while the z axis, orthogonal to M, is parallel to and oriented as the input photon momentum $\hbar\mathbf{k}_2$. Assume that the qubit's π states are identified according to the right-hand (r.h.) coordinate convention, henceforth referred to as frame convention (FC). Of course, the preservation of FC upon M or FRM reflection is a necessary requirement for a correct understanding of the reflection dynamics, since the input and output qubits are identified precisely on the basis of the real-space reference frame. This implies that, after reflection the π states of any photon are identified on the basis of a new coordinate frame $(x'=x, y'=y, z'=-z)$, obtained by rotating the old frame around the $\vec{x}$ axis by an angle of 180°.

In order to visualize the process with the help of the Poincaré sphere (P sphere) in the spin space with orthogonal axes σ_i, $i=1,2,3$, let us consider the M reflection first. Let the point P to represents the input qubit when the input photon hits M, $|\Psi\rangle=\psi_1|H\rangle+\psi_2|V\rangle$, $|\psi_1|^2+|\psi_2|^2=1$, being ψ_i the complex components of the spinor $\left|\begin{smallmatrix}\psi_1\\ \psi_2\end{smallmatrix}\right|$. Classical optics show that the reversal of the particle motion and the preservation of FC upon M reflection implies a rotation of $\vec{p}$ around the spin reference axis σ_3 by an angle 180°, i.e., by the unitary map: $\mathbb{U}_3=\exp(i\frac{1}{2}\pi\sigma_3)=i\sigma_3$. Note that, after the corresponding representative point P being drawn into P' by the M reflection transformation, the same P sphere can still be adopted for the future qubit dynamics. In summary, the M reflection process transforms the input qubit $|\Psi\rangle$ above as follows: $\mathbb{U}_3\left|\begin{smallmatrix}\psi_1\\ \psi_2\end{smallmatrix}\right|=i\left|\begin{smallmatrix}\psi_1\\ -\psi_2\end{smallmatrix}\right|$ [6].

Let us now turn our attention to the FRM reflection. If P now represents the qubit before entering the device, the first passage through FR draws P into O by a positive rotation by 90° around σ_2, i.e., corresponding to the evolution operator: $\mathbb{U}_{2+}=\exp(i\frac{1}{4}\pi\sigma_2)$. The M reflection contributes with the map $\mathbb{U}_3$, leading from the point O to O', as just seen. The second passage through FR after M reflection is represented by $\mathbb{U}_{2-}=\exp(-\frac{1}{4}\pi\sigma_2)$ since the change of the spatial reference frame leaves unchanged the (axial) magnetic-polarization pseudovector $\vec{\mathcal{M}}\propto\vec{B}$ while the sign of the phase $\pi/2$ is inverted because of the particle's propagation reversal [12]. In summary, the overall evolution of the input qubit $|\Psi\rangle$ through the FRM leaves it in the final state: $|\Psi'\rangle=\mathbb{U}_{FRM}|\Psi\rangle=\mathbb{U}_{2-}\mathbb{U}_3\mathbb{U}_{2+}|\Psi\rangle=i\sigma_1|\Psi\rangle$. Correspondingly, the initial point P on the sphere is drawn into the successive positions O, O' and P'. Hence, the overall FRM process results in a overall 180° rotation around the axis σ_1 of the initial representative vector $\vec{p}$, i.e., the input qubit is acted upon by the unitary: $\mathbb{U}_{FRM}=\exp(i\frac{1}{2}\pi\sigma_1)=i\sigma_1$, as seen. The final representative point P' is not the antipode of any possible P on the sphere and then FRM does not realize the universal quantum-NOT gate, in agreement with the principles of quantum mechanics (QM). Indeed the spin-flip action is realized exactly only for all representative points P belonging to the equatorial plane orthogonal to the σ_1 axis, such as for the qubits $|H\rangle$, $|V\rangle$, and $|C_\pm\rangle$. Note that a similar analysis could be applied to any other passive π-rotator classical device. For instance, a λ/n optical waveplate realizes a spin-flip map that cannot be universal.

The FRM action on single photon states has been characterized experimentally by the adoption, as said, of the powerful tools of standard quantum state tomography (QST) [13] and of the EAQPT [10,11]. A Spontaneous Parametric Down Conversion (SPDC) process was excited in a 1.5 mm thick BBO (β-barium borate) nonlinear (NL) crystal slab, by a mode-locked pulsed UV beam with wavelength (wl) λ_p (Fig. 1). By this setup entangled photon pairs in a singlet state of polarization (π) were created with common wl's, $\lambda=2\lambda_p=795$ nm. Each excitation laser pulse had a time duration $\tau\approx 140$ fs. In virtue of the nonlocal correlation existing between the photons of each pair, the qubit $|\Psi\rangle$ injected over the mode $\mathbf{k}_2$ into the FRM device was deterministically selected by the measurement apparatus coupled to mode $\mathbf{k}_1$. This one consisted of a π analyzer, i.e., a polarizing beam splitter (PBS), a pair of $(\lambda/2+\lambda/4)$ optical waveplates (wpl) and of a tunable Babinet compensator (BC). The detector D_1, and the end detector D_2, were equal SPCM-AQR14 Si-avalanche single photon units with quantum efficiencies (QEs)$\cong 0.55$. One interference filter with bandwidth $\Delta\lambda=3$ nm was placed in front of each D. The Faraday rotator (FR), a Linos-FR-660/1100-8 device accurately $\vec{B}$ tuned for the wl λ, could be optionally removed from the optical network. It was terminated by a 100% reflectivity mirror M. In

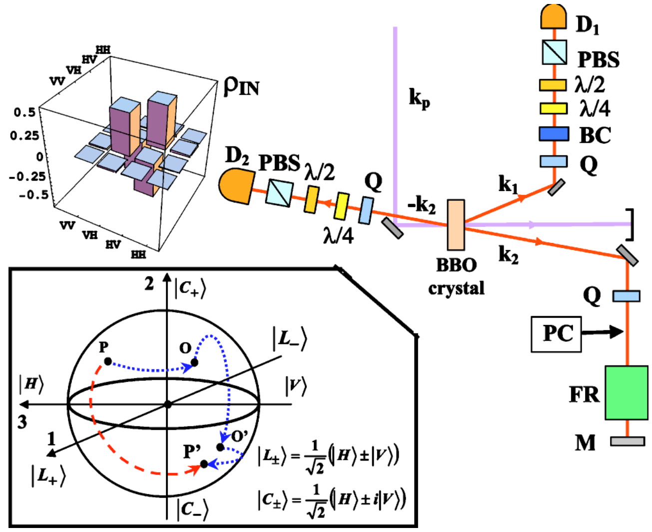

FIG. 1. Experimental setup adopted for the characterization of the FRM reflection process. Inset: Representation of the FRM action on the Poincaré sphere. The tomographic reconstruction of the input singlet state $\rho^{in}_{k1,-k2}$ is reported. It was obtained by direct measurements taken on the mode $\mathbf{k}_2$, i.e., by replacing FRM with the whole measurement apparatus terminated by D_2.

order to avoid the use of other mirrors, the output qubit $|\Psi'\rangle$ emitted by FRM over the mode $-\mathbf{k}_2$ was transmitted through the transparent BBO NL slab and finally detected by the end measurement apparatus, made of a PBS, a pair of $(\lambda/2+\lambda/4)$ wpl sets, and a detector D_2. Three quartz slabs (Q) inserted on the modes $\mathbf{k}_1$, $\mathbf{k}_2$, $-\mathbf{k}_2$ and the Babinet compensator on mode $\mathbf{k}_1$ secured the accurate compensation of all residual birefringence effects, e.g., of the BBO slab.

In a first experiment we wanted to fully characterize by standard QST the input-entangled state generated by SPDC, in absence of FRM. The tomographically reconstructed density matrix $\rho^{in}_{k1,k2}$ of the photon pair is reported in Fig. 1. There the realization of the singlet input pure state condition $|\Phi\rangle=2^{-1/2}(|H\rangle_1|V\rangle_2-|V\rangle_1|H\rangle_2)$ is clearly shown, where $|H\rangle_i$ and $|V\rangle_i$ stands for linear-π horizontal and vertical qubits, associated respectively to modes $\mathbf{k}_i$, $i=1,2$. We then analyzed the transformation induced by the mirror reflection only **(**a condition henceforth referred to as M reflection and by the complete FRM device (FRM reflection)**)**. By the knowledge of $|\Phi\rangle$ we prepared the input qubit $|\Psi\rangle$ injected over $\mathbf{k}_2$ by exploiting the nonlocal π correlation existing between the two photons of each pair. Precisely, we limited ourselves to prepare six input qubits $|\Psi\rangle:|H\rangle,|V\rangle$, $|L_\pm\rangle\equiv 2^{-1/2}(|H\rangle\pm|V\rangle)$, $|C_\pm\rangle\equiv 2^{-1/2}(|H\rangle\pm i|V\rangle)$. The results of two experiments carried out respectively in absence and in presence of FR, were the following:

M reflection: $|H\rangle\to|H\rangle$; $|V\rangle\to|V\rangle$; $|L_\pm\rangle\to|L_\mp\rangle$; $|C_\pm\rangle\to|C_\mp\rangle$, with fidelities $\mathcal{F}=0.98,0.98,0.90,0.90$, respectively.

FRM reflection: $|H\rangle\to|V\rangle$; $|V\rangle\to|H\rangle$; $|L_\pm\rangle\to|L_\pm\rangle$; $|C_\pm\rangle\to|C_\mp\rangle$ with $\mathcal{F}=0.99,0.99,0.92,0.92$, respectively.

Here the arrows indicate the mapping from the input to the output states, associated with the input–output modes $\mathbf{k}_2$ and $-\mathbf{k}_2$, correspondingly, and the $\mathcal{F}$ values were given by the statistical fractions of the right outcomes.

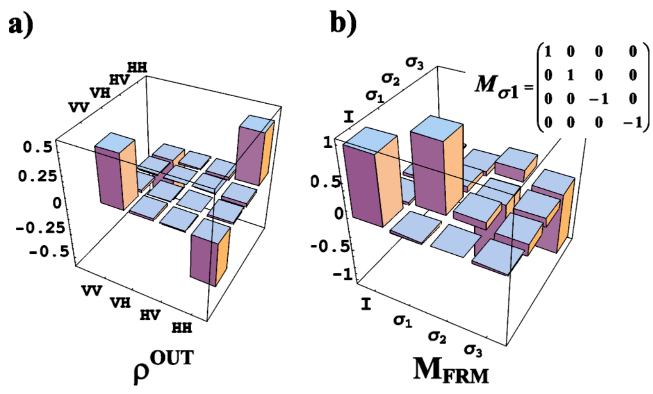

FIG. 2. Tomographic representation of the state $\rho^{out}_{k1,-k2}$ at the FRM output. Only the real parts are reported for the input and output states. The maximum size of the experimentally determined imaginary parts was, respectively, 0.02 and 0.05 in the corresponding scales. The QPT reconstructed matrix $\mathbf{M}_{FRM}$ can be compared with the structure of $\mathbf{M}_{\sigma_1}$.

As an additional deeper-level demonstration, a very general characterization of the FRM device was attained experimentally by the modern EAQPT methods [10,11]. Let us outline the basics of this approach, by first representing the state of the input qubit by the density operator $\rho_{k2}=\frac{1}{2}(\mathbb{I}+\vec{r}\cdot\vec{\sigma})$ and then by the vector $(1,\vec{r})$ in the four-dimensional space $\{\sigma_0\equiv\mathbf{1},\sigma_i\}$, $i=1,2,3$. The single map $\mathcal{E}(\rho)$, expressing the FRM reflection, is completely described by a 4×4 real matrix $\mathbf{M}_{\mathrm{FRM}}$, which maps ρ_{k_2} into the density matrix: $\rho'_{-k2}=\mathbf{M}_{\mathrm{FRM}}\rho_{k_2}$ [13,14]. A most important, exclusive property of the EAQPT method is the exploitation of the quantum parallelism associated with the entanglement condition. Indeed, in our experiment the matrix $\mathbf{M}_{\mathrm{FRM}}$ was experimentally reconstructed by the use of the input pure entangled state $\rho^{in}_{k1,k2}$ (Fig. 1). Then according to EAQPT the qubit associated with mode $\mathbf{k}_1$—call it qubit 1—was left unchanged while the unknown $\mathcal{E}(\rho)$ map, i.e., of FRM reflection, was applied to qubit 2, associated with the mode $\mathbf{k}_2$. Note that by this procedure $\mathcal{E}(\rho)$ was investigated by a complete span over the Hilbert space $\mathcal{H}_2$ of qubit 2 because of its mixed-state condition. The EAQPT result of the output state $\rho^{out}_{k1,-k2}=\mathbb{I}_{k1}\otimes\mathcal{E}_{k_2}(\rho^{in}_{k1,k2})$ is shown in Fig. 2. Finally, the matrix $\mathbf{M}_{FRM}$ corresponding to $\mathcal{E}(\rho)$ was estimated by means of the experimentally determined density matrices $\rho^{in}_{k1,k2}$ and $\rho^{out}_{k1,-k2}$, by adoption of an inversion software. Remember the theoretical result $\mathbb{U}_{FRM}=i\sigma_1$ obtained above. We may check in Fig. 2 the very good correspondence of the EAQPT reconstructed $\mathbf{M}_{FRM}$ with the corresponding matrix $\mathbf{M}_{\sigma_1}$ expressing the σ_1 Pauli operator. We may express quantitatively the overlap of the corresponding maps by the fidelity $\mathcal{F}(\sigma_1,\mathcal{E}(\rho))=\int d\Psi\,\mathcal{F}(\mathcal{E}(|\Psi\rangle\langle\Psi|),\sigma_1(|\Psi\rangle\langle\Psi|))=\frac{1}{2}+\frac{1}{3}(M_{22}-M_{33}-M_{44})=(99.7\pm0.3)\%$ [14].

In summary, our experimental results fully support the previous theoretical analysis. The FRM device does not realize the universal spin-flip action but rather a CP map, i.e., a unitary transformation expressible by a definite rotation of the unit vector $\vec{p}$ around the center of the P sphere (Fig. 1, inset). Correspondingly, the point P, the vertex of $\vec{p}$ expressing the input $|\Psi\rangle$ on the surface of the same sphere is drawn into the point P' the vertex of $\vec{p}'$ expressing the output $|\Psi'\rangle$.

For the sake of completeness, we also investigate the interesting FRM compensation process discovered by Martinelli that has recently found a widespread application whenever an active compensation of optical path fluctuations is required, as in long a distance quantum cryptography applications with optical fibers [6]. The optical scheme of the process is the following: suppose that Alice (A) and Bob (B), two receiving–sending QI stations are connected by an optical device (PC in Fig. 1) that acts on the transmitted optical signal by (presumably unwanted) unitary transformation $\mathbb{U}$, e.g., corresponding to slowly fluctuating birefringence effects due possibly to temperature changes or to local changes of the curvature of the optical fiber, etc. If the overall communication process implies the transmission of the signal from A to B and then back from B to A, the spurious $\mathbb{U}$ effect can be cancelled if the corresponding device (e.g., the optical fiber) is terminated by a FRM device. For instance, if the FRM device is placed at the site B, the signal sent along the fiber from A to B and the received back is seen by A to be unaffected by the disturbance introduced by the PC device. In summary, looking through the fiber, A can only detect the effect of FRM and not of PC. All this can be easily analyzed with the help of the discussion above. Let the fluctuating effect contributed by PC and affecting the transmission from A to B be represented generally by the unitary $\mathbb{U}=\exp\left[i\frac{1}{2}(\vec{\sigma}\cdot\vec{n})\theta\right]$ where $\vec{n}$ is a unit vector pointing in any direction in the spin space. The propagation back from B to A will be then represented by a corresponding unitary $\mathbb{U}'=\exp\left[-i\frac{1}{2}(\vec{\sigma}\cdot\vec{n}')\theta\right]$ where the phase $(-\theta)$ accounts for the propagation reversal, and the components of the unit vector $\vec{n}'$ in the spin space are connected to $\vec{n}$ as follows: $n'_1=n_1, n'_2=-n_2, n'_3=-n_3$. In other words, by retracing the propagation back through the fiber the rotation-axis involved in the $\mathbb{U}'$ transformation has undergone a rotation by an angle of 180° around the axis σ_1, with respect to the original direction $\vec{n}\to\vec{n}'$. All this can be explained by considering that the fluctuating optical disturbance $\mathbb{U}$, e.g., physically due to birefringence, et., consists of an electrical-polarization $\vec{\mathcal{P}}$ effect. Since $\vec{\mathcal{P}}$ is a true vector, viz., a polar vector, it is not insensitive to any sign change of the spatial coordinates in real space, e.g., the one implied by the FRM reflection [12]. Furthermore, we have already shown that the input/output dynamics due to the FRM reflection implies precisely a rotation by an angle 180° around σ_1 of any unit vector representative of the input–output qubits $\vec{p}\to\vec{p}'$. The overall process can be now formally expressed by a straightforward application of spin algebra leading to the transformation affecting any qubit forwarded by A in a round trip journey through the whole device (PC+FRM): $\mathbb{U}'\mathbb{U}_{FRM}\mathbb{U}=\mathbb{U}_{FRM}=i\sigma_1$, for any vector $\vec{n}$ and angle θ. In other words, the effect of any possible roundtrip unitary process, e.g., introduced by any optical fiber or by any other optical device, is exactly cancelled, i.e., compensated, by this very clever universal FRM technique.

The compensation effect has been experimentally checked by an ergodic procedure, i.e., by insertion of a stochastically time-varying unitary operation $\mathbb{U}(t)$ into the mode $\mathbf{k}_2$ between the NL crystal and the FRM. More precisely this device, represented by PC in Fig. 1, and apt to simulate the most general propagation inside an optical fiber, consisted of a cascade of two equal Pockels cells (Shangai Institute of Ceramics, rise time $\approx$1 ns, $V_{(1/2\lambda)}=600$ V) mutually rotated by 45° and driven by a stochastic time sequence of high-voltage electric pulses. The output density matrices $\widetilde{\rho}_{k1,-k2}^{out}$ and $\rho_{k1,-k2}^{out}$ representing the states of each photon pair after propagation into the PC+FRM device, respectively, upon activation and inactivation of the Pockels cells, were reconstructed by a tomographic technique. The value of the fidelity

$$\mathcal{F}(\rho_{k1,-k2}^{out},\widetilde{\rho}_{k1,-k2}^{out}) = \left(\mathrm{Tr}\sqrt{\sqrt{\widetilde{\rho}_{k1,-k2}^{out}}\rho_{k1,-k2}^{out}\sqrt{\widetilde{\rho}_{k1,-k2}^{out}}}\right)^2$$

expressing the overlapping of the two experimentally determined quantum states has been found: $\mathcal{F}=(99.9\pm0.1)\%$. This figure shows the very high degree of compensation attainable by the method.

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