Experimental Test of the No-Signaling Theorem

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In 1981 N. Herbert proposed a gedanken experiment in order to achieve by the "First Laser-Amplified Superluminal Hookup" (FLASH) a faster-than-light (FTL) communication by quantum nonlocality. The present work reports the first experimental realization of that proposal by the optical parametric amplification of a single photon belonging to an entangled EPR pair into an output field involving $N = 5 \times 10^3$ photons. A theoretical and experimental analysis explains in general and conclusive terms the precise reasons for the failure of the FLASH program as well as of any similar FTL proposals.

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The theory of special relativity lies on a very basic hypothesis: anything carrying information cannot travel faster than the light in vacuum. On the other side, quantum physics possesses marked nonlocal features implied by Bell's theorem. Nevertheless, it has been shown theoretically, on the basis of several properties of the Hilbert space, that a "no-signaling theorem" holds: one cannot exploit the quantum entanglement between two spacelike separated particles in order to realize any "faster-than-light" (FTL) communication [1–3]. Indeed the quantum theory of communications teaches us that a single particle cannot carry information about its coding basis at the receiving station. Then, in an attempt to break the "peaceful coexistence" between quantum mechanics and relativity, Nick Herbert proposed in 1981 a feasible experimental scheme, i.e., a First Laser-Amplified Superluminal Hookup (FLASH) machine [4]. The proposal, based on the amplification by stimulated emission of a photon in an entangled state, elicited a debate among theorists leading at last to the formulation of the quantum "no-cloning theorem" [5,6]. In the last two decades this theorem was investigated by focusing the attention on only one relevant aspect of the process: the reduction of the cloning "fidelity" $F < 1$ due to an increase of noise affecting the generated clones [7–9]. Indeed it is found that the noise increase implied by the realizable "optimal" fidelity is not large enough to justify the failure of the FLASH program. This is demonstrated in the present experiment by the fairly large signal-to-noise ratio affecting the well resolved fringe patterns of Fig. 2, the ones that according to the FLASH proposal should allow for the FTL communication. A more subtle aspect of the general no-cloning theorem should indeed take place. On the other hand, the complex dynamics underlying any amplification process did not allow anyone to single out a clear and unambiguous theoretical argument justifying the FLASH failure. After so many years a recourse to the experiment was held necessary by imposing greater attention on the effect of the high-order correlations established among the generated clones, as we shall see. The present endeavor contributes to a deeper, more complete understanding of the quantum cloning process and of the distribution of quantum information among the generated clones.

The setup proposed by Herbert is reported in Fig. 1. Two spacelike distant observers, say Alice and Bob, share two polarization entangled photons generated by a common EPR source (crystal 1). Alice detects by the phototubes D^A_φ and $D^A_{\varphi\perp}$ the polarization $\vec{\pi}_A$ of her photon in two mutually unbiased measurement bases: linear *polarizations* $\{\vec{\pi}_\pm = 2^{-1/2}(\vec{\pi}_V \pm \vec{\pi}_H)\}_A$ and *circular* $\vec{\pi}$'s polarizations $\{\vec{\pi}_R = 2^{-1/2}(\vec{\pi}_H + i\vec{\pi}_V), \vec{\pi}_L = 2^{-1/2}(\vec{\pi}_V - i\vec{\pi}_H)\}_A$, where $\vec{\pi}_H$ and $\vec{\pi}_V$ are horizontal and vertical $\vec{\pi}$'s polarizations, respectively. Consider that the choice of the basis is the only coding method accessible to Alice in order to establish a meaningful communication with Bob. If Bob could guess the coding basis chosen by Alice, then a FTL signaling process would be established. Can Bob obtain any information about Alice's settings by a local manipulation of his quantum system? Since the detection of an unknown single particle cannot carry any information on Alice's coding basis [3], Herbert proposed that Bob could make a "new kind" of measurement on the photon through the amplification by a "nonselective laser gain tube," namely, a polarization independent amplifier, i.e., a "universal cloner." The amplified beam is then split by a symmetric beam splitter (BS) so that Bob can perform a measurement in the $\{\vec{\pi}_\pm\}_B$ basis on half of the N amplified

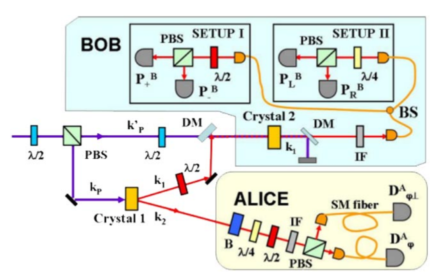

FIG. 1 (color online). Configuration of the quantum injected optical parametric amplifier. The polarization entangled photon couple is generated by crystal 1, while crystal 2 realizes the optical parametric amplifier (OPA) action.

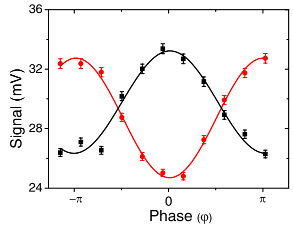

FIG. 2 (color online). Average signal versus the phase of the input qubit: I^B_+ (square marks), I^B_- (circle marks).

particles, and in the $\{\vec{\pi}_R, \vec{\pi}_L\}_B$ basis on the other half: Fig. 1. The electronic signals registered simultaneously by the couples of photodetectors acting in the two corresponding "measurement boxes" are $\{I^B_+, I^B_-\}$ and $\{I^B_L, I^B_R\}$. Such a double analyzer is able to identify the polarization of a pulse of N photons, either linearly or circularly polarized. Furthermore, the identification could also be achieved for polarization randomly distributed between $\{\vec{\pi}_\pm\}$ and $\{\vec{\pi}_R, \vec{\pi}_L\}$. As shown by the fringe patterns of Fig. 2, the measurement carried out by Bob in the "correct" polarization basis, i.e., coincident with Alice's basis, say $\{\vec{\pi}_\pm\}$, should exhibit a neat difference between the signals I^B_+ and I^B_-. This difference disappears if Bob measures in the "wrong" basis $\{\vec{\pi}_R, \vec{\pi}_L\}$, since there the average values of I^B_R and I^B_L are equal. According to the FLASH proposal precisely this difference should allow for FTL communication. In order to challenge this proposal, theorists resorted to the no-cloning theorem which forbids the perfect duplication of quantum states. Actually Herbert, aware of the presence of spontaneous emission noise, proposed to extract an appropriate "signature" by a discrimination between the two measured quantities $\langle |I^B_+ - I^B_-|\rangle$ and $\langle |I^B_R - I^B_L|\rangle$, the averages being taken over many events measured for the same Alice basis. For instance, taking $\{\vec{\pi}_\pm\}$ as the Alice basis, Herbert was expecting $\langle |I^B_+ - I^B_-|\rangle > \langle |I^B_R - I^B_L|\rangle$. But this was found incorrect, as we shall see.

Let us now account for the experiment. We first selected a convenient amplifier model by which the test should be carried out: Fig. 1. A laser beam at wavelength (wl) $\lambda_P = 397.5$ nm was split into two beams through a $\lambda/2$ wave plate and a polarizing beam splitter (PBS) and excited two nonlinear (NL) crystals (β-barium borate) cut for type II phase matching [7]. Crystal 1, excited by the beam $\mathbf{k}_P$, is the spontaneous parametric down-conversion source of entangled photon couples of wl $\lambda = 2\lambda_P$, emitted over the two output modes $\mathbf{k}_i$ ($i = 1, 2$) in the *singlet* state $|\Psi^-\rangle_{k1,k2} = 2^{-1/2}(|H\rangle_{k1}|V\rangle_{k2} - |V\rangle_{k1}|H\rangle_{k2})$. The single photon state generated over the mode $\mathbf{k}_1$ was directed toward Bob's station, simultaneously with a pump beam (mode $\mathbf{k}'_P$). In virtue of the nonlocal correlation acting on modes $\mathbf{k}_1$ and $\mathbf{k}_2$, the input qubit on mode $\mathbf{k}_1$ was codified in the polarization state $\vec{\pi}$ by measuring the photon on mode $\mathbf{k}_2$ in the appropriate polarization basis. The injected photon and the UV pump beam $\mathbf{k}'_P$ were superposed by a dichroic mirror (DM) with high reflectivity at λ and high transmittivity at λ_P.

Let us now describe more precisely how the amplifier, i.e., the *cloning machine*, was realized [7]. We adopted a quantum injected–optical parametric amplifier (QI-OPA), exploiting the process of stimulated emission in a NL crystal (crystal 2) in the highly efficient collinear regime [10]. This process, often referred to as *optimal phase-covariant quantum cloning*, realizes the constant amplification of qubits belonging to the equatorial plane orthogonal to the z axis of the corresponding Bloch sphere [11–13]. Since they are identified only by a phase φ, the qubits belonging to this plane will also be referred to as $|\varphi\rangle = 2^{-1/2}(|H\rangle + e^{i\varphi}|V\rangle)$.

The interaction Hamiltonian accounting for the parametric amplification of crystal 2 reads $\hat{H} = i\chi\hbar(\hat{a}^\dagger_H \hat{a}^\dagger_V) +$ H.c. and acts on the single spatial mode $\mathbf{k}_1$ where $\hat{a}^\dagger_{\vec{\pi}}$ is the one photon creation operator associated to mode $\mathbf{k}_1$ with polarization $\vec{\pi}$ [10]. Its form implies the phase covariance and the optimality of the cloning process [13]. The multiphoton output state $|\Phi^\varphi\rangle$ generated by the OPA amplification injected by a single photon qubit $|\varphi\rangle$ is found $|\Phi^\varphi\rangle = \hat{\mathbf{U}}|\varphi\rangle$, where $\hat{\mathbf{U}} = \exp(-i\hat{H}\hbar^{-1}t)$ is the unitary transformation implied by the QI-OPA process [10]. Consider any two generic qubits $|\varphi\rangle$ and $|\psi\rangle$ on Alice's Bloch sphere. They are connected by the general linear transformation: $|\psi\rangle = (\alpha|\varphi\rangle + \beta|\varphi\perp\rangle)$ with $|\alpha|^2 + |\beta|^2 = 1$. The orthogonal qubits are obtained by the *antiunitary* time-reversal map: $|\varphi^\perp\rangle = \mathbf{T}|\varphi\rangle$ and $|\psi^\perp\rangle = \mathbf{T}|\psi\rangle = (-\beta^*|\varphi\rangle + \alpha^*|\varphi\perp\rangle)$. In addition, the *unitarity* of $\hat{\mathbf{U}}$ leads to $\langle \Phi^\varphi \mid \Phi^\varphi\rangle = 1$, $\langle \Phi^{\varphi\perp} \mid \Phi^\varphi\rangle = 0$, and to

$$\begin{aligned} |\Phi^\psi\rangle &= (\alpha|\Phi^\varphi\rangle + \beta|\Phi^{\varphi\perp}\rangle); \\ |\Phi^{\psi\perp}\rangle &= \mathbf{T}|\Phi^\psi\rangle = (-\beta^*|\Phi^\varphi\rangle + \alpha^*|\Phi^{\varphi\perp}\rangle). \end{aligned} \tag{1}$$

The average photon number $M^B_\pm$ over $\mathbf{k}_1$ with polarization $\vec{\pi}_\pm$ is found to depend on the phase φ of the input qubit as follows: $M^B_\pm(\varphi) = \bar{m} + \frac{1}{2}(2\bar{m} + 1)(1 \pm \cos\varphi)$ with $\bar{m} = \sinh^2 g$, g being the NL gain of the OPA.

The output state of crystal 2 with wl λ was spatially separated by the fundamental UV beam through a dichroic mirror, spectrally filtered by an interferential filter (IF) with bandwidth 1.5 nm, coupled to a single mode (SM) fiber and split over two output modes by a BS. Then each output field was polarization analyzed and detected by a couple of photomultiplier (PM) tubes ($P^B_{+/R}$ and $P^B_{-/L}$). The adopted PM's were Burle C31034-A02 with GaAs cathode and quantum efficiency (QE) $\eta_{\mathrm{QE}} = 13\%$. The pulse signals were registered by a digital memory oscilloscope (Tektronix TDS5104B) triggered by $\{D^A_\varphi, D^A_{\varphi\perp}\}$. We estimated the experimental gain value $g = 4.45 \pm 0.04$, which corresponds to a generated mean photon number about equal to 4,000 in the stimulated regime [14]. The coherence property of the multiphoton output field $|\Phi^\varphi\rangle$ implied by the quantum superposition character of the input qubit $|\varphi\rangle$ was measured in the basis $\vec{\pi}_\pm$ over the output "cloning" mode $\mathbf{k}_1$ by signals registered by P^B_+ and P^B_- *conditionally* to a single photon detection event by

$D^A_{\varphi\perp}$. The corresponding signals I^B_+ and I^B_-, averaged over 2,500 trigger pulses, are reported versus the phase φ in Fig. 2. The typical fidelity values of the output photon polarization state have been found to be 0.58–0.60 within the theoretical figures. The discrepancy between experimental and theoretical values can be attributed to the reduced *visibility* of input qubit ($\mathcal{V}_{\text{in}} \approx 85\%$) and to the reduced probability $p \simeq 0.4$ of correct amplifier injection under D^A_{φ}-trigger detection. A theoretical model including these imperfections gives rise to an expected value for the visibility $\tilde{\mathcal{V}} = \mathcal{V}_{\text{in}}[p(2\bar{m}+1)]/[p(2\bar{m}+1)+2\bar{m}]$ which fits the experimental data obtained for different values of g.

As a first step toward the FLASH test, we analyzed the correlations between Alice's and Bob's measurements by a *conditional* experiment. Let us assume that Bob analyzes his field in the basis $\{\vec{\pi}_\pm\}_B$, by the experimental setup I in Fig. 1. By this one the observable $\Delta^B_{+,-} = (I^B_+ - I^B_-)$ was measured over 2,500 equally prepared experiments, for different input qubit states $|\varphi\rangle$. As shown by the interference pattern given in Fig. 3(a) the average difference signal $\langle\Delta^B_{+,-}\rangle$ is equal to zero for an input state belonging to the basis $\{\vec{\pi}_R, \vec{\pi}_L\}$, i.e., for $\varphi = -\frac{\pi}{2}, \frac{\pi}{2}$, while it achieves well-defined maximum and minimum values for input qubits equal to $|+\rangle$ ($\varphi = 0$) or $|-\rangle$ ($\varphi = \pi$), respectively. According to Herbert's proposal, the average of the *moduli* of $\Delta^B_{+,-}$, i.e., the value of $\langle|\Delta^B_{+,-}|\rangle$, has been estimated. More precisely, in order to further reduce the effects of the shot-to-shot amplification fluctuations, we registered the average value $\langle|N_{+,-}|\rangle$ with $N_{i,j} = (I^B_i - I^B_j)/(I^B_i + I^B_j)$ for different values of the phase of the input qubit. As shown by Fig. 3(b), even in a *conditional* experiment any information on the input state $|\varphi\rangle$ is deleted by the averaging process then making all different input $|\varphi\rangle$ states fully indistinguishable. By a similar procedure, in correspondence with the $\{\vec{\pi}_R, \vec{\pi}_L\}_B$ basis, no information on the basis could be drawn by $\langle|N_{R,L}|\rangle$. Hence we conclude that the Herbert's measurement scheme fails when applied to the output field of any amplifier or cloning device.

In order to understand the previous results, we investigate the dynamics of the process by considering the probability distribution $\mathcal{P}^{\varphi}_{\pm}(x)$ of the fluctuating observable $x \equiv \delta^B_\pm = n^B_+ - n^B_-$ for an input qubit with phase φ. There $n^B_\pm$ is the number of photons with polarization $\vec{\pi}_\pm$. Assume the *polarization* basis $\{\vec{\pi}_\pm\}_A$ corresponding to the *phase* basis $\{\varphi = 0, \varphi^\perp = \pi\}_A$. As shown by Fig. 4(a), the probability distribution $\mathcal{P}^0_\pm(x)$ exhibits a peak in correspondence of $\tilde{x}^0 > 0$ while the distribution $\mathcal{P}^{\pi}_{\pm}(x)$ (not reported in the figure) is identical to $\mathcal{P}^0_\pm(x)$ but reversed in respect to the axis $x = 0$ with a peak at $\tilde{x}^\pi = -\tilde{x}^0$. The ensemble average values of the observable x are $\langle x\rangle^0 = 2\bar{m} + 1$ and $\langle x\rangle^\pi = -(2\bar{m}+1)$, respectively. Suppose now that Bob keeps his previous measurement basis $\{\vec{\pi}_\pm\}_B$, but Alice adopts the new coding basis $\{\vec{\pi}_{L/R}\}_A \Rightarrow \{\varphi = \frac{\pi}{2}, \varphi^\perp = -\frac{\pi}{2}\}_A$. The average values now read $\langle x\rangle^{\pi/2} = \langle x\rangle^{-(\pi/2)} = 0$, and the corresponding distributions $\mathcal{P}^{\pm(\pi/2)}_{\pm}(x)$ are equal for both $\pm\pi/2$. Furthermore, most important, Fig. 4(b) shows that the two $\mathcal{P}^{\pm(\pi/2)}_{\pm}(x)$ do not exhibit a single peak centered around the common average value $\tilde{x}^{\pm(\pi/2)} = 0$, as expected for any Gaussian-like distribution, but two symmetrical and balanced peaks that are in exact correspondence with the ones shown for $\tilde{x}^0$ and $\tilde{x}^\pi$, i.e., in correspondence with the former Alice polarization basis $\{\vec{\pi}_\pm\}_A$. Let us now focus our attention on the probability distribution of $|\delta^B_\pm|$ pointing out its implication on FTL communication. From the peculiar behavior of the $\mathcal{P}^{\varphi}_{\pm}(x)$ shown in Fig. 4, it is straightforward to conclude that any input qubit leads to *the same* distribution for $|\delta^B_\pm|$. Indeed this unexpected and somewhat counterintuitive result, confirmed by detailed calculations, lies at the basis of the explanation for the FLASH failure. We consider for simplicity, and without loss of generality, the average of the square function of $\delta^B_\pm$, which is equivalent for our purposes to the average of $|\delta^B_\pm|$. It is found

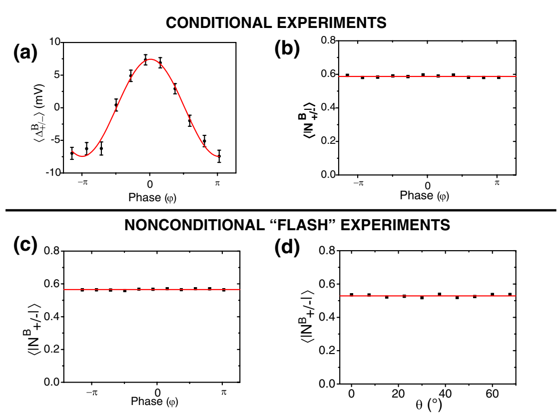

FIG. 3 (color online). (a) Mean value of $\Delta^B_{+,-}$ for different input qubit states. (b) Mean value of $|N_{+,-}|$ versus the phase of the input qubit. Error bars are within the corresponding experimental points. (c) Mean value of $|N_\pm|$ versus the Alice measurement basis $\{\vec{\pi}_\varphi, \vec{\pi}_{\varphi\perp}\}_A$. (d) Mean value of $|N_\pm|$ versus the Alice measurement basis: $\{\vec{\pi}_\theta, \vec{\pi}_{\theta\perp}\}_A$.

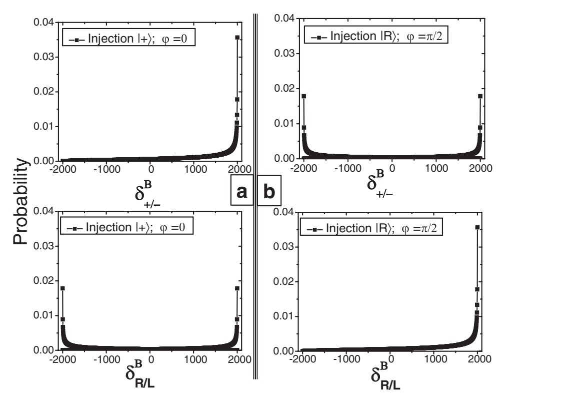

FIG. 4. Theoretical probability distributions of $\delta^B_{+/-}$ and $\delta^B_{R/L}$ and for a fixed number of overall output clones equal to 2,001. (a) Injected state: $|+\rangle$. (b) Injected state: $|R\rangle$.

$\langle(\delta_{\pm}^{B})^2\rangle_{\varphi} = 12\bar{m}^2 + 12\bar{m} + 1$ for any input qubit $|\varphi\rangle$ injected on mode $\mathbf{k}_1$ pointing out the phase independence of the function. Let us note that the previous results are not invalidated by the low detection efficiency η. Indeed the observable $\langle(\delta_{\pm}^{B})^2\rangle_{\varphi}$ behaves as $\bar{m}^2$, corresponding to a super-Poissonian statistics of $\delta_{\pm}^{B}$. At variance with the squeezing phenomenology, its fluctuation properties can be investigated with $\eta \ll 1$.

Let us now account for the set of experiments suggested by the FLASH proposal. In this case, since no classical information channel should connect Alice to Bob, a *nonconditional* configuration is necessary. This test was realized by adopting two different triggering solutions for the registering setups at Bob's site: (1) The trigger was provided by the output of a logic gate XOR fed by the output standard TTL signals registered by $\{D_{\varphi}^{A}$ and $D_{\varphi\perp}^{A}\}$. This device ensured a good discrimination against noise, but any information about Alice's basis was erased. (2) Any channel of information between Alice and Bob was accurately severed. Apart from noise, the two solutions led to identical results. Two experiments were realized. (a) The average value of $|N_{\pm}|$ was measured with Alice measuring her photon polarization in any basis $\{\vec{\pi}_{\varphi}, \vec{\pi}_{\varphi\perp}\}_A$ with $\vec{\pi}_{\varphi} = 2^{-1/2}(\vec{\pi}_H + e^{i\varphi}\vec{\pi}_V)$: Fig. 3(c). (b) The value of $\langle|N_{\pm}|\rangle$ was measured with Alice measuring her photon polarization in any basis $\{\vec{\pi}_{\vartheta}, \vec{\pi}_{\vartheta\perp}\}_A$ belonging to the equatorial plane orthogonal to the y axis. There, $\vec{\pi}_{\theta} = (\cos\theta\vec{\pi}_H + \sin\theta\vec{\pi}_V)$: Fig. 3(d). In both cases the observable $|N_{\pm}|$ was found to be independent from Alice's choice of measurement basis.

All these results may be understood in conclusive terms on the basis of the fundamental *linearity* of quantum mechanics. Suppose that Alice's polarization coding bases consist of any generic couple of qubit sets: $\{\vec{\varphi}, \vec{\varphi}\perp\}_A$ and $\{\vec{\chi}, \vec{\chi}\perp\}_A$. In any *nonconditional* experiment Bob must carry out his measurements on the states he receives, which here are either one of the two mesoscopic states: $\rho_B = \frac{1}{2}(|\Phi^{\varphi}\rangle\langle\Phi^{\varphi}| + |\Phi^{\varphi\perp}\rangle\langle\Phi^{\varphi\perp}|)$ or $\rho_{B'} = \frac{1}{2}(|\Phi^{\chi}\rangle\langle\Phi^{\chi}| + |\Phi^{\chi\perp}\rangle\langle\Phi^{\chi\perp}|)$. On the basis of Eq. (1) these states are equal as they are expressed by the fully mixed state: $\rho_B = \rho_{B'} = \hat{1}/2$. In a different albeit equivalent perspective which refers to previous considerations, the linearity of the Hilbert space requires that the *sum* of the *probability distributions* $\mathcal{P}_{\pm}^{(\varphi,\varphi\perp)}(x) = \mathcal{P}_{\pm}^{\varphi}(x) + \mathcal{P}_{\pm}^{\varphi\perp}(x)$ of the variable x is invariant in respect to the corresponding coding basis $\{\varphi, \varphi\perp\}_A$. Since this result is valid for any measurement setup chosen by Bob, the discrimination of the phase φ is impossible, in spite of the different mean values $\langle x\rangle^{\varphi}$ found for different φ's in the well-resolved fringing patterns of Figs. 2 and 3(a). This result, shown by the experimental data of Figs. 3(c) and 3(d), is general and prevents any FTL communication based on quantum nonlocality.

At last, let us consider again the role of the no-cloning theorem, itself a consequence of the linearity of quantum mechanics. As already emphasized, the limitations implied by a complete quantum cloning theory are not restricted to the bounds on the cloning fidelity, as commonly taken for granted. They also largely affect the high-order correlations existing among the different clones. In fact, noisy but separable, i.e., noncorrelated, copies would lead to a perfect state estimation for $g \rightarrow \infty$ and hence to a real possibility of FTL communication. Indeed the particles produced by a cloning machine are highly interconnected and the high-order correlations established among the clones strongly limit the possibility to exploit them to estimate the output state in Bob's hands. All this is clearly demonstrated by the peculiar distribution functions shown by Fig. 4 and is in agreement with preliminary theoretical investigations by [15,16]. The above interpretation has been confirmed in our laboratory by a side quantum state-tomography experiment based on the technique introduced in [17], showing that the density matrix of a system of two clones $\rho_C^{\mathrm{I,II}}$ clearly differs from the one obtained for two separable particles: $\rho_C^{\mathrm{I}} \otimes \rho_C^{\mathrm{II}}$.

In summary, we have presented a conclusive theoretical and experimental investigation of the physical processes underlying the failure of the FLASH program. Since the relevant quantum-informational aspects of the correlations implied by the cloning process have been little explored in the literature, the present work may elicit an exciting new trend for future research.

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[1] G. C. Ghirardi *et al.*, Nuovo Cimento Soc. Ital. Fis. **78B**, 9 (1983); Phys. Lett. B **166**, 435 (1992); A. Kuzmich *et al.*, Phys. Rev. Lett. **86**, 3925 (2001).
[2] D. Bruss *et al.*, Phys. Rev. A **62**, 062302 (2000).
[3] A. Peres, *Quantum Theory and Measurement* (North-Holland, Amsterdam, 1983).
[4] N. Herbert, Found. Phys. **12**, 1171 (1982).
[5] W. K. Wootters *et al.*, Nature (London) **299**, 802 (1982).
[6] A. Peres, Fortschr. Phys. **51**, 458 (2003).
[7] F. De Martini *et al.*, Prog. Quantum Electron. **29**, 165 (2005).
[8] V. Scarani *et al.*, Rev. Mod. Phys. **77**, 1225 (2005).
[9] N. Cerf *et al.*, Prog. Opt. **49**, 455 (2006).
[10] F. De Martini, Phys. Lett. A **250**, 15 (1998).
[11] D. Bruß *et al.*, **62**, 012302 (2000).
[12] F. Sciarrino and F. De Martini, Phys. Rev. A **72**, 062313 (2005).
[13] F. Sciarrino and F. De Martini, Phys. Rev. A **76**, 012330 (2007).
[14] F. De Martini *et al.*, J. Phys. A **40**, 2977 (2007).
[15] R. Demkowicz-Dobrzanski, Phys. Rev. A **71**, 062321 (2005).
[16] J. Bae and A. Acin, Phys. Rev. Lett. **97**, 030402 (2006).
[17] M. Caminati *et al.*, Phys. Rev. A **74**, 062304 (2006).