All-Versus-Nothing Nonlocality Test of Quantum Mechanics by Two-Photon Hyperentanglement

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We report the experimental realization and the characterization of polarization and momentum hyperentangled two-photon states, generated by a new parametric source of correlated photon pairs. By adoption of these states an "all-versus-nothing" test of quantum mechanics was performed. The two-photon hyperentangled states are expected to find at an increasing rate a widespread application in state engineering and quantum information.

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The violation of Bell's inequalities has been recognized as the first, paradigmatic test of quantum nonlocality [1,2]. As such, in the last decades it has been somewhat successfully realized by many two-particle experiments mostly performed by optical techniques [3,4]. However, in spite of the fact that today bipartite pure two-photon polarization-entangled states can be rather easily produced by spontaneous parametric down-conversion (SPDC) in a nonlinear (NL) crystal, a quite unsatisfactory feature of Bell's inequality method is that its probative effectiveness only applies to statistical measurement procedures. In the framework of this method the EPR local realistic picture can indeed explain perfect correlations implied by predictions to be tested by any definite, single experiment. This shortcoming does not affect the Hardy's "nonlocality test without inequalities," also referred to as "ladder proof of nonlocality," where a contradiction with Einstein's local realism can be demonstrated, at least in principle, by any single experimental test but only for a small fraction of the generated photon pairs [5,6]. The extension to the full set of pairs is important as it provides a complete and startling demonstration of the conflict existing between the laws of quantum mechanics and the expectations of local realism. This test is offered by the "all-versus-nothing" nonlocality proof [7] and is based on the Greenberg-Horne-Zeilinger (GHZ) theorem [8]. It applies for the set of systems that are in the same GHZ state and has been experimentally demonstrated either for three- or four-photon entanglement [9–11]. In addition to their conceptual relevance, tests of quantum mechanics performed by states operating in a large dimension Hilbert space exhibit deviations from local realist expectations which are larger and more robust against noise [12].

Recently, it has been suggested that the realization of GHZ theorem can be extended to the case of two-photon hyperentangled states in a (4×4) Hilbert space [13], i.e., simultaneously maximally entangled in 2 degrees of freedom, as for instance the field's polarization (π) and the spatial momentum $(\mathbf{k})$ [14]. In this way the intrinsic limitation of SPDC where no more than one photon pair is created in any elementary annihilation-creation process can be easily overcome. In most protocols of quantum information (QI) these twofold hyperentangled states associated to a pair of correlated particles act as 4 particles in the usual entanglement configuration. Indeed, the first and foremost application of the $(\pi - \mathbf{k})$ entanglement was the first quantum state teleportation experiment carried out in Rome in 1997 [15]. In that experiment the complete analysis at the Alice's site of the four orthogonal Bell states with 100% efficiency was realized, a result otherwise impossible to achieve with standard entanglement and linear optical techniques [16]. In this Letter we report on the systematic production and characterization of $(\pi - \mathbf{k})$ hyperentangled photon pairs, generated by an "*ad hoc*" flexible parametric source recently developed in our laboratory. This device will be then adopted to realize the experimental "all-versus-nothing" test of quantum nonlocality.

The source adopted in this experiment is schematically shown in Fig. 1(a). It has been extensively described in previous papers [17]. The 2-photon states generated over the emission cone of a 0.5 mm thick β-barium-borate (BBO) type I crystal were simultaneously entangled in polarization and momentum. Polarization (π) entanglement was obtained by quantum superposition of the two overlapping radiation cones generated at the same wavelength $\lambda = 728$ nm by BBO, excited in two opposite directions by a cw Argon laser at $(\lambda_p = \lambda/2)$. The two cones were then carefully overlapped by means of a spherical back mirror (M). The overall radiation is expressed by the maximally entangled state $|\Phi\rangle = 2^{-1/2}(|H_1\rangle|H_2\rangle + e^{i\theta}|V_1\rangle|V_2\rangle)$ in the horizontal (H) and vertical (V) polarization basis, with phase θ easily and reliably controlled by micrometric displacements of the back mirror. It may be locally transformed into the state $|\Psi\rangle = 2^{-1/2}(|H_1\rangle|V_2\rangle + e^{i\theta}|V_1\rangle|H_2\rangle)$ by the zero-order $\lambda/2$ wave plate (HW^*) inserted in one of the two correlated directions: Fig. 2(a). Entangled states, either pure or controllable mixed states, have been created in a flexible way by the same source [18]. Momentum $(\mathbf{k})$ entanglement was realized, under excitation of either one of the two overlapped radiation

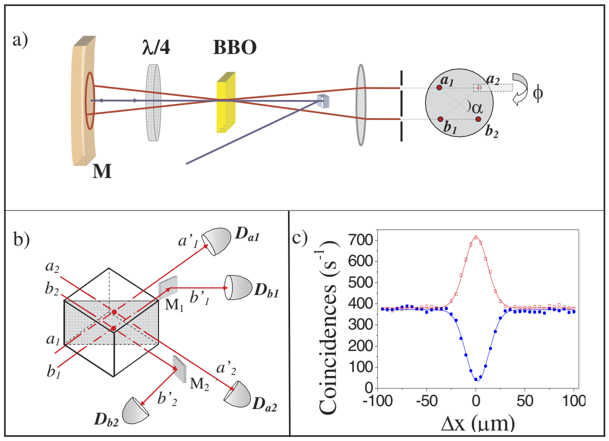

FIG. 1 (color online). (a) Parametric source of polarization-momentum hyperentangled two-photon states. Phase setting $\theta = 0, \pi$ and $\varphi = 0, \pi$ are obtained by micrometric translation of the spherical back mirror and by tilting of the glass plate on mode a_1; (b) spatial coupling of input modes $a_1 - b_1$, $a_2 - b_2$ on the BS plane. The BS output modes, $a_1' - b_1'$, $a_2' - b_2'$, are also shown. M_1 and M_2 are mirrors inserted to separate upper and lower modes; (c) coincidence rate $C(a_1', b_1')$ vs Δx: $\theta = 0$, $\phi = 0$ (upper curve), $\theta = 0$, $\phi = \pi$ (lower curve).

cones, by a four hole screen which allowed to select two pairs of correlated $\mathbf{k}$ modes, $a_1 - b_2$ and $a_2 - b_1$ within the conical emission of the crystal: Fig. 1(a) [19]. The straight lines connecting on the screen the holes leaving through the correlated pairs intercross at an angle $\alpha = 18°$. The "phase-preserving" character of the SPDC process allowed to keep the phase difference ϕ between the two pair emission to the value $\phi = 0$, regardless the value of α.

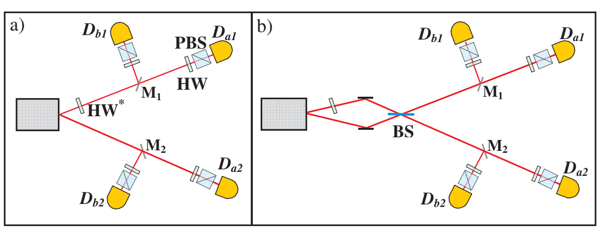

FIG. 2 (color online). Experimental apparatus to measure the expectation values of the nine operators comparing in $\mathcal{O}$. (a) Modes a_1, b_1, a_2, and b_2, separated by the pickoff mirrors M_1 and M_2, are directly coupled to detectors; polarization analysis is performed in the orthonormal bases, either $H - V$ or $D - \bar{D}$, by rotating the half-wave plates HW before the detectors. The half-wave plate HW^* performing the $|\Phi\rangle \rightarrow |\Psi\rangle$ transformation is also shown. By this configuration one can evaluate the values of $z_1 \cdot z_2$, $x_1 \cdot x_2$, $z_1' \cdot z_2'$, $z_1 z_1' \cdot z_2 \cdot z_2'$, and $x_1 \cdot z_1' \cdot x_2 z_2'$. (b) The two mode sets $a_1 - b_1$ and $a_2 - b_2$ are spatially combined onto the BS before being coupled to detectors. In this way one can perform the transformation on the momentum basis, and measure the values of $x_1' \cdot x_2'$, $x_1 x_1' \cdot x_2 \cdot x_2'$, and $z_1 \cdot x_1' \cdot z_2 x_2'$. The same apparatus is used for performing the Bell state analysis by removing HW^* plate, in order to evaluate the value of $z_1 z_1' \cdot x_1 x_1' \cdot z_2 x_2' \cdot x_2 z_2'$.

The phase ϕ could be set by means of a tilted thin glass plate intercepting mode a_2. Hence, for each SPDC generated cone, the $\mathbf{k}$-entangled states $|\psi\rangle = 2^{-1/2}(|a_1\rangle|b_2\rangle + e^{i\phi}|b_1\rangle|a_2\rangle)$ could be generated.

The mode sets $a_1 - b_1$ and $a_2 - b_2$ were split along the vertical direction by a prismlike two-mirror system and then recombined onto a symmetric beam splitter (BS): Fig. 2(b). A trombone mirror assembly mounted on a motorized translation stage (not shown in the figure) allowed fine adjustments of the path offset delay Δx between the input mode pairs $a_1 - b_1$ and $a_2 - b_2$. Figure 1(b) shows the spatial recombination of modes $a_1 - a_2$ and $b_1 - b_2$ in two different points of the BS plane. The photons associated with the output BS modes, a_1', b_1' and a_2', b_2', were independently detected by four avalanche single photon detectors (D), mod. SPCM-AQR14 D_{a1}, D_{b1}, D_{a2}, and D_{b2} in Fig. 1(b). This is obtained by inserting a mirror on each output arm [M_1 and M_2 in Fig. 1(b)], intercepting the modes b_1' and b_2', respectively. This configuration could be further improved by coupling the four spatial modes to single mode optical fibers. Equal interference filters, with bandwidth $\Delta\lambda = 6$ nm, placed in front of each D, determined the coherence time of the detected pulses: $\tau_{\rm coh} \approx 150$ f sec. Two-photon coincidences were registered for either one of the following mode combinations: $a_1' - b_1'$, $a_1' - b_2'$, $a_2' - b_2'$, $a_2' - b_1'$, while no coincidence was detected for modes $a_1' - a_2'$ and $b_1' - b_2'$. The $(\pi - \mathbf{k})$ hyperentangled states realized in the present experiment could be then expressed as:

$$|\Xi^{\pm\pm}\rangle = |\Psi^{\pm}\rangle \otimes |\psi^{\pm}\rangle = \tfrac{1}{2}(|H\rangle|V\rangle \pm |V\rangle|H\rangle) \otimes (|a_1\rangle|b_2\rangle \pm |b_1\rangle|a_2\rangle), \quad (1)$$

where, in $|\Psi^{\pm}\rangle$, $\theta = 0, \pi$ and in $|\psi^{\pm}\rangle$, $\varphi = 0, \pi$.

Figure 1(c) shows the characteristic quantum resonance effect arising in the BS linear superposition of the two components of any bipartite entangled state [20]. It is expressed here for $\mathbf{k}$ entanglement by the coincidence rate $C(a_1', b_1')$ as a function of Δx. The transition from the symmetric ($|\Xi^{++}\rangle$) to the antisymmetric ($|\Xi^{+-}\rangle$) state condition upon change of the phase θ is shown with a resonance "visibility" ≈ 0.90. Similar results are obtained for the other photon pair coincidences $C(a_1', b_2')$, $C(a_2', b_1')$, $C(a_2', b_2')$, measured, by varying either θ or φ.

Equation (1) expresses a two-photon hyperentangled state spanning a (4×4) Hilbert space which in the ideal case of a perfect pure state allows the generalization of the GHZ theorem. This argument is purely logical and does not involve inequalities. However, inequalities are necessary as a quantitative test in a real experiment in order to avoid the conceptual problems associated to the realization of a null experiment. This can be given by an all versus nothing violation of local realism [13].

In our experiment the hyperentangled state $|\Xi^{--}\rangle = |\Psi^-\rangle \otimes |\psi^-\rangle$ has been adopted to perform the nonlocality test. It is based on the estimation of the operator $\mathcal{O} = -z_1 \cdot z_2 - z'_1 \cdot z'_2 - x_1 \cdot x_2 - x'_1 \cdot x'_2 + z_1 z'_1 \cdot z_2 \cdot z'_2 + x_1 x'_1 \cdot x_2 \cdot x'_2 + z_1 \cdot x'_1 \cdot z_2 x'_2 + x_1 \cdot z'_1 \cdot x_2 z'_2 - z_1 z'_1 \cdot x_1 x'_1 \cdot z_2 x'_2 \cdot x_2 z'_2$, and the condition for violation of local realism is $\mathcal{O} > 7$, while the expected value of $\mathcal{O}$ according to quantum mechanics is $\mathcal{O} = 9$ [13]. It is worth noting that the local boundary $\mathcal{O} \leq 7$ is obtained by employing a hidden variable model which allows correlations between the $(\pi - \mathbf{k})$ degrees of freedom of the same photon. This point has been emphasized by Cabello, who also stressed in his theory that the present nonlocality test involves only two observers [11,13]. In the above expression the Pauli operators:

$$
\begin{aligned}
z_i &= \sigma_{z_i} = |H\rangle\langle H| - |V\rangle\langle V|, \\
x_i &= \sigma_{x_i} = |H\rangle\langle V| + |V\rangle\langle H| \qquad (i = 1, 2) \\
z'_1 &= \sigma'_{z_1} = |a_1\rangle\langle a_1| - |a_2\rangle\langle a_2|, \\
x'_1 &= \sigma'_{x_1} = |a_1\rangle\langle a_2| + |a_2\rangle\langle a_1| \\
z'_2 &= \sigma'_{z_2} = |b_1\rangle\langle b_1| - |b_2\rangle\langle b_2|, \\
x'_2 &= \sigma'_{x_2} = |b_1\rangle\langle b_2| + |b_2\rangle\langle b_1|.
\end{aligned} \tag{2}
$$

allow the state transformations:

$$
\begin{aligned}
z_1 \cdot z_2 |\Xi^{--}\rangle &= -|\Xi^{--}\rangle \\
z'_1 \cdot z'_2 |\Xi^{--}\rangle &= -|\Xi^{--}\rangle \\
x_1 \cdot x_2 |\Xi^{--}\rangle &= -|\Xi^{--}\rangle \\
x'_1 \cdot x'_2 |\Xi^{--}\rangle &= -|\Xi^{--}\rangle \\
z_1 z'_1 \cdot z_2 \cdot z'_2 |\Xi^{--}\rangle &= |\Xi^{--}\rangle \\
x_1 x'_1 \cdot x_2 \cdot x'_2 |\Xi^{--}\rangle &= |\Xi^{--}\rangle \\
z_1 \cdot x'_1 \cdot z_2 x'_2 |\Xi^{--}\rangle &= |\Xi^{--}\rangle \\
x_1 \cdot z'_1 \cdot x_2 z'_2 |\Xi^{--}\rangle &= |\Xi^{--}\rangle \\
z_1 z'_1 \cdot x_1 x'_1 \cdot z_2 x'_2 \cdot x_2 z'_2 |\Xi^{--}\rangle &= -|\Xi^{--}\rangle.
\end{aligned} \tag{3}
$$

The nine operators which contribute to determine the expected value of the $\mathcal{O}$ must be evaluated in order to measure a violation of local realism. It can be observed if the minimum value of entanglement visibility is 7/9 [13]. The measured visibilities of polarization and momentum entanglement obtained by our system are suitable on this purpose.

The experimental apparatuses to observe violation are sketched in Figs. 2(a) and 2(b). In both cases the polarization analyzers on each detection arm, a'_1, a'_2, b'_1, b'_2, allow to perform the polarization measurement in either the $H - V$ or the $D - \bar{D}$ basis, with $D = 2^{-1/2}(H + V)$ and $\bar{D} = 2^{-1/2}(H - V)$. The two photons generated by the source are sent to the Alice and Bob sites which can be indifferently chosen because of the conical emission symmetry of the parametric radiation. In the present experiment, Alice and Bob perform the measurements by the upper $(D_{a1} - D_{a2})$ and lower $(D_{b1} - D_{b2})$ detectors, respectively. By referring to the optical setup of Fig. 2(a), the modes $a_1 - a_2$ and $b_1 - b_2$ are sent directly to the Alice and Bob sites and the corresponding signals are analyzed by a half-wave plate (HW) and a polarizing beam splitter (PBS) in each arm. By this apparatus we could measure the four terms $x_1 \cdot x_2$, $z_1 \cdot z_2$, $z'_1 \cdot z'_2$, $z_1 z'_1 \cdot z_2 \cdot z'_2$, and $x_1 \cdot z'_1 \cdot x_2 z'_2$.

Figure 2(b) shows the optical setup for measuring $x'_1 \cdot x'_2$, $x_1 x'_1 \cdot x_2 \cdot x'_2$, $z_1 \cdot x'_1 \cdot z_2 x'_2$. Before being analyzed in each arm and coupled to detectors, the two mode sets $a_1 - b_1$ and $a_2 - b_2$ are spatially combined onto the BS which performs the transformation from the $a_1 - a_2$ to the $d - \bar{d}$ basis, where $d = 2^{-1/2}(a_1 + a_2)$ and $\bar{d} = 2^{-1/2}(a_1 - a_2)$ and, similarly, for modes b_1 and b_2. The experimental apparatus of Fig. 2(b) realizes a double interferometer operating with a single BS, avoiding the need for any active phase stabilization.

The last term of the operator $\mathcal{O}$, $z_1 z'_1 \cdot x_1 x'_1 \cdot z_2 x'_2 \cdot x_2 z'_2$, is measured by the same setup of Fig. 2(b), simply by removing the half-wave plate (HW^*) which performs the $|\Phi^-\rangle \rightarrow |\Psi^-\rangle$ transformation. This operation realizes the Bell state analysis performed in the teleportation experiments of Ref. [15] and allows to evaluate the expectation values of the operator $z_1 z'_1 \cdot x_1 x'_1 \cdot z_2 x'_2 \cdot x_2 z'_2$ [13]. In this measurement the Alice's and Bob's detectors, $(D_{a1} - D_{a2})$ and $(D_{b1} - D_{b2})$, perform the measurements in the $H - V$ basis.

The whole experiment has been carried out by a sequence of measurements each one lasting an average time of 30 sec. The experimental results corresponding to the measurement of the nine terms of $\mathcal{O}$ are summarized in

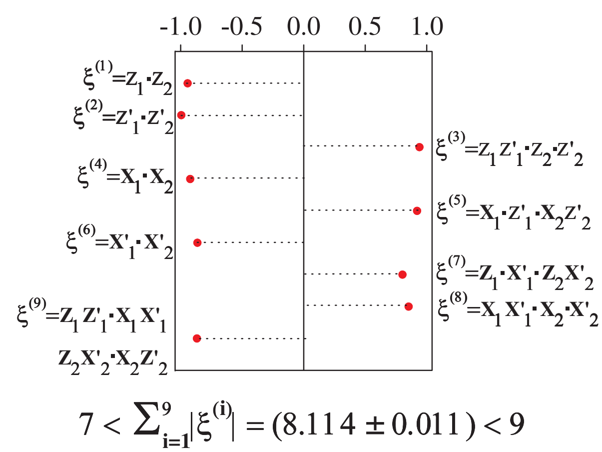

FIG. 3 (color online). Barchart of expectation values for the nine operators involved in the experiment. The following results have been obtained: $z_1 \cdot z_2 = -0.9428 \pm 0.0030$, $z'_1 \cdot z'_2 = -0.9953 \pm 0.0033$, $z_1 z'_1 \cdot z_2 \cdot z'_2 = 0.9424 \pm 0.0030$, $x_1 \cdot x_2 = -0.9215 \pm 0.0033$, $x_1 \cdot z'_1 \cdot x_2 z'_2 = 0.9217 \pm 0.0033$, $x'_1 \cdot x'_2 = -0.8642 \pm 0.0043$ $z_1 \cdot x'_1 \cdot z_2 x'_2 = 0.8039 \pm 0.0040$, $x_1 x'_1 \cdot x_2 \cdot x'_2 = 0.8542 \pm 0.0040$, $z_1 z'_1 \cdot x_1 x'_1 \cdot z_2 x'_2 \cdot x_2 z'_2 = -0.8678 \pm 0.0043$.

the histogram shown in Fig. 3. The experimental value of $\mathcal{O}$, obtained after summation over all the measured values, $\mathcal{O} = 8.114 \pm 0.011$, which, corresponding to a violation of the inequality by $101 - \sigma$ standard deviations, demonstrates a large contradiction with local realism.

In this Letter we have experimentally demonstrated the nonlocal character of two-photon hyperentangled states by an "all-versus-nothing" test of quantum mechanics. In addition to the conceptual relevance of this result concerning one of the most intriguing fundamental properties of Nature, the content of the present work may be viewed as a clear demonstration of the power and the flexibility of the new source here applied for the first time to the generation of hyperentangled states. The simplicity and reliability of the optical scheme together with the conceptual relevance of the underlying quantum process here realized, i.e., the effective doubling to the extension of the Hilbert space spanned by the state of the generated particles, is expected to be appreciated in the near future as a useful and far reaching resource of QI technology. During the preparation of this work, another "all-versus-nothing" test has been performed with a different two photon hyperentanglement source [21].

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Note added in proof.—In the present work, the adoption of the same apparatuses for the measurement of couples of operators as $z_1 \cdot z_1'$ and $z_1 z_1'$ implies the supplementary assumption that the numerical results of the measurement of $z_1 z_1'$ is equal to the product of the results of z_1 and z_1' measured separately. The same argument holds for the measurement of the other operators: $x_1 \cdot x_1'$ and $x_1 x_1'$, $z_2 \cdot x_2$ and $z_2 x_2'$, $x_2 \cdot z_2'$ and $x_2 z_2'$. We are presently adopting a new, more complete measurement method which does not imply that extra assumption. The preliminary results obtained with this new method for the three operators, $z_1' \cdot z_2' = -0.9893 \pm 0.0031$, $z_1 \cdot z_2 = -0.9348 \pm 0.0037$, and $z_1 z_1' \cdot z_2 \cdot z_2' = 0.9218 \pm 0.0037$, are in full agreement with the data presented in this Letter.

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