Teleportation Scheme Implementing the Universal Optimal Quantum Cloning Machine and the Universal NOT Gate

M. Ricci, F. Sciarrino, C. Sias, and F. De Martini

Dipartimento di Fisica and Istituto Nazionale per la Fisica della Materia, Università di Roma "La Sapienza," Roma, 00185 Italy

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By a significant modification of the standard protocol of quantum state teleportation, two processes "forbidden" by quantum mechanics in their exact form, the universal NOT gate and the universal optimal quantum cloning machine, have been implemented contextually and optimally by a fully linear method. In particular, the first experimental demonstration of the tele-UNOT gate, a novel quantum information protocol, has been reported. The experimental results are found in full agreement with theory.

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Classical information is encoded in bits, viz., dichotomic variables that can assume the values 0 or 1. For such variables there are no theoretical limitations as far as the "cloning" and/or "spin-flipping" processes are concerned. However, manipulations on the quantum analogue of a bit, a qubit, have strong limitations due to fundamental requirements by quantum mechanics. For instance, it has been shown that an arbitrary unknown qubit cannot be perfectly cloned: $|\Psi\rangle \rightarrow |\Psi\rangle|\Psi\rangle$, a consequence of the so-called "no cloning theorem" [1]. Another "impossible" device is the quantum NOT gate, the transformation that maps any qubit into the orthogonal one $|\Psi\rangle \rightarrow |\Psi^{\perp}\rangle$ [2]. In past years a great deal of theoretical investigation has been devoted to finding the best approximation allowed by quantum mechanics for these processes and to establishing the corresponding "optimal" values of the *fidelity* $F < 1$. This problem has been solved in the general case [3,4]. In particular, it was found that a one-to-two universal optimal quantum cloning machine (UOQCM), i.e., able to clone one qubit into two qubits ($1 \rightarrow 2$), can be realized with a fidelity $F_{\mathrm{CLON}} = \frac{5}{6}$. The UOQCM has been experimentally realized by a quantum optical *amplification* method, i.e., by associating the cloning effect with a QED "stimulated emission" process [5,6]. Very recently it has been argued [6] that when the cloning process is realized in a subspace H of a larger nonseparable Hilbert space $H \otimes K$, which is acted upon by a physical apparatus, the same apparatus performs *contextually* in the space K the "flipping" of the input injected qubit, then realizing a ($1 \rightarrow 1$) universal-NOT gate (UNOT) with a fidelity $F_{\mathrm{NOT}} = \frac{2}{3}$ [4]. As an example, a UOQCM can be realized on one output mode of a nondegenerate *quantum-injected* optical parametric amplifier (QIOPA), while the UNOT transformation is realized on the other mode [7].

In the present work this relevant, somewhat intriguing result is investigated under a new perspective implying a *modified* quantum state teleportation (QST) protocol, according to the following scheme. The QST protocol implies that an unknown input qubit $|\phi\rangle_S = \alpha|0\rangle_S + \beta|1\rangle_S$ is destroyed at a sending place (Alice: $\mathcal{A}$) while its perfect replica appears at a remote place (Bob: $\mathcal{B}$) via dual *quantum* and *classical* channels [8]. Let us assume that Alice and Bob share the entangled *singlet* state $|\Psi^-\rangle_{AB} = 2^{-1/2}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)$ and that we want to teleport the generic qubit $|\phi\rangle_S \equiv |\phi\rangle$. The singlet state is adopted hereafter because its well known invariance under SU(2) transformations will ensure the "universality" of the cloning and UNOT processes, as we shall see [5–7]. The overall state of the system is then $|\Omega\rangle_{SAB} = |\phi\rangle_S|\Psi^-\rangle_{AB}$. Alice performs a Bell measurement by projecting the joint state of the qubits S and A into the four Bell states, $\{|\Psi^-\rangle_{SA}, |\Psi^+\rangle_{SA}, |\Phi^-\rangle_{SA}, |\Phi^+\rangle_{SA}\}$, spanning the four-dimensional Hilbert space $H \equiv H_A \otimes H_S$, and then sends the result to Bob by means of two bits of classical information. In order to obtain $|\phi\rangle_B$, Bob applies to the received state the appropriate unitary transformation U_B according to the following protocol: $|\Psi^-\rangle_{SA} \rightarrow U_B = \mathbb{I}$, $|\Psi^+\rangle_{SA} \rightarrow U_B = \sigma_Z$, $|\Phi^-\rangle_{SA} \rightarrow U_B = \sigma_X$, $|\Phi^+\rangle_{SA} \rightarrow U_B = \sigma_Y$, where the kets express the received corresponding information, I, σ_Z, and σ_X are, respectively, the identity, phase flip, and spin flip operators, and $\sigma_Y = -i\sigma_Z\sigma_X$. At last, the QST channel acts on the input state $\rho_S \equiv |\phi\rangle\langle\phi|$ as the identity operator: $E_{\mathrm{QST}}(\rho_S) = \rho_B$. In absence of the classical channel, i.e., of the appropriate U_B transformation, the apparatus realizes the map $E_B(\rho_S) = \frac{1}{2}\mathbb{I}_B$, corresponding to the *depolarization* channel E_{DEP}. This is the worst possible case because any information about the initial state ρ_S is lost.

In order to implement the UOQCM and UNOT at Alice's and Bob's sites, in the present work we modify the QST protocol by performing a different measurement on the system $S + A$. This leads to a different content of information to be transferred by the classical channel from $\mathcal{A}$ to $\mathcal{B}$. Precisely, the *Bell measurement,* able to discriminate between the four Bell states, is replaced here by a dichotomic *projective measurement* able to identify $|\Psi^-\rangle_{SA}$, i.e., the *antisymmetric* subspace of $H \equiv H_A \otimes H_S$, and its complementary *symmetric* subspace. Let us analyze the outcomes of such strategy, schematically

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represented by Fig. 1. With a probability $p = \frac{1}{4}$ the $|\Psi^-\rangle_{SA}$ is detected by $\mathcal{A}$. In this case the correct QST channel E_{QST} is realized. However, if this is not the case, with probability $p = \frac{3}{4}$, Bob cannot apply any unitary transformation to the set of the nonidentified Bell states, $\{|\Psi^+\rangle_{SA}, |\Phi^-\rangle_{SA}, |\Phi^+\rangle_{SA}\}$, and then the QST channel implements the statistical map $E(\rho) = \frac{1}{3}[\sigma_Z\rho\sigma_Z + \sigma_X\rho\sigma_X + \sigma_Y\rho\sigma_Y]$. As we shall see, this map coincides with the map $E_{UNOT}(\rho_S)$ which realizes the universal optimal-NOT gate, i.e., the one that approximates *optimally* the flipping of one qubit $|\phi\rangle$ into the orthogonal qubit $|\phi^\perp\rangle$, i.e., ρ_S into $\rho_S^\perp \equiv |\phi^\perp\rangle\langle\phi^\perp|$. Bob identifies the two different maps realized at his site by reading the information (1 bit) received by Alice on the classical channel. For example, such *bit* can assume the value 0 if Alice identifies the Bell state $|\Psi^-\rangle_{SA}$ and 1 if she does not. We name this process tele-UNOT since it consists in the teleportation of an optimal antiunitary map acting on any input qubit, the UNOT gate [7,9].

As already stated, it has been shown that in a bipartite entangled system the optimal UNOT gate is generally realized contextually together with an optimal quantum cloning process [6,7]. Therefore, it is worth analyzing what happens when the overall state $|\Omega\rangle_{SAB}$ is projected onto the subspace orthogonal to $|\Psi^-\rangle_{SA}\langle\Psi^-|_{SA} \otimes H_B$ by the projector:

$$P_{SAB} = (\mathbb{I}_{SA} - |\Psi^-\rangle_{SA}\langle\Psi^-|_{SA}) \otimes \mathbb{I}_B. \tag{1}$$

This procedure generates the normalized state $|\tilde{\Omega}\rangle \equiv P_{SAB}|\Omega\rangle_{SAB} = \sqrt{\frac{2}{3}}[|\xi_1\rangle_{SA} \otimes |1\rangle_B - |\xi_0\rangle_{SA} \otimes |0\rangle_B]$, where $|\xi_1\rangle_{SA} = \alpha|0\rangle_S|0\rangle_A + \frac{1}{2}\beta(|1\rangle_S|0\rangle_A + |0\rangle_S|1\rangle_A)$ and $|\xi_0\rangle_{SA} = \beta|1\rangle_S|1\rangle_A + \frac{1}{2}\alpha(|1\rangle_S|0\rangle_A + |0\rangle_S|1\rangle_A)$. By tracing this state over the SA and B manifolds we get $\rho_{SA} \equiv \mathrm{Tr}_B|\tilde{\Omega}\rangle\langle\tilde{\Omega}| = \frac{2}{3}|\phi\rangle|\phi\rangle_{SA}\langle\phi|\langle\phi|_{SA} + \frac{1}{3}|\{\phi, \phi^\perp\}\rangle_{SA}\langle\{\phi, \phi^\perp\}|_{SA}$ and $\rho_B^{\mathrm{out}} \equiv \mathrm{Tr}_{SA}|\tilde{\Omega}\rangle\langle\tilde{\Omega}| = \frac{1}{3}(2\rho_S^\perp + \rho_S)$, where $|\{\phi, \phi^\perp\}\rangle_{SA} = 2^{-1/2}(|\phi^\perp\rangle_S|\phi\rangle_A + |\phi\rangle_S|\phi^\perp\rangle_A)$. We further project on the spaces A and B and obtain the output states, which are found mutually equal: $\rho_S^{\mathrm{out}} \equiv \mathrm{Tr}_A\rho_{SA} = \frac{1}{6}(5\rho_S + \rho_S^\perp) = \rho_A^{\mathrm{out}} \equiv \mathrm{Tr}_S\rho_{SA}$. At last, by these results the expected optimal values for the fidelities of the two forbidden processes are obtained: $F_{\mathrm{CLON}} = \mathrm{Tr}[\rho_S^{\mathrm{out}}\rho_S] = \mathrm{Tr}[\rho_A^{\mathrm{out}}\rho_S] = \frac{5}{6}$ and $F_{\mathrm{UNOT}} = \mathrm{Tr}[\rho_B^{\mathrm{out}}\rho_S^\perp] = \frac{2}{3}$ [6,7].

In recent years many experimental realizations of quantum state teleportation have been achieved [10]. Recently the first *active,* i.e., *complete,* version of the QST protocol was realized by our laboratory by physically implementing Bob's unitary operation [11]. In the present experiment the input qubit was codified as the *polarization* state of a single photon belonging to the input mode k_S: $|\phi\rangle_S = \alpha|H\rangle_S + \beta|V\rangle_S$, $|\alpha|^2 + |\beta|^2 = 1$; see Fig. 2. Here $|H\rangle$ and $|V\rangle$ correspond to the horizontal and vertical polarizations, respectively. In addition, an entangled pair of photons, A and B, was generated on the modes k_A and k_B by spontaneous parametric down-conversion (SPDC) in the singlet state: $|\Psi^-\rangle_{AB} = 2^{-1/2}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$. The projective measurement in the space $H = H_A \otimes H_S$ was realized by linear superposition of the modes k_S and k_A on a 50:50 beam splitter, BS_A. Consider the overall output state realized on the two output modes k_1 and k_2 of BS_A and expressed by a linear superposition of the states: $\{|\Psi^-\rangle_{SA}, |\Psi^+\rangle_{SA}, |\Phi^-\rangle_{SA}, |\Phi^+\rangle_{SA}\}$. It is well known that the realization of the singlet $|\Psi^-_{SA}\rangle$ is identified by the emission of one photon on each output mode of BS_A, while the realization of the set of the other three Bell states implies the emission of two photons either on mode k_1 or on mode k_2. The realization of the last process, sometimes dubbed as Bose mode coalescence (BMC)

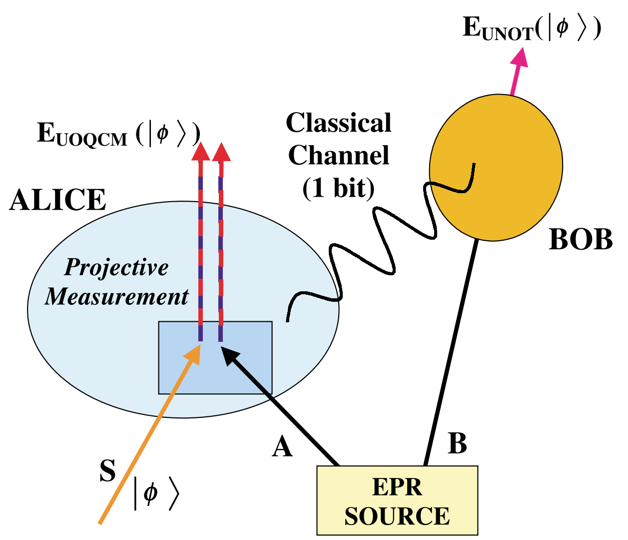

FIG. 1 (color online). General scheme for the simultaneous realization of the tele-UNOT gate and of the *probabilistic* universal quantum cloning machine (UOQCM).

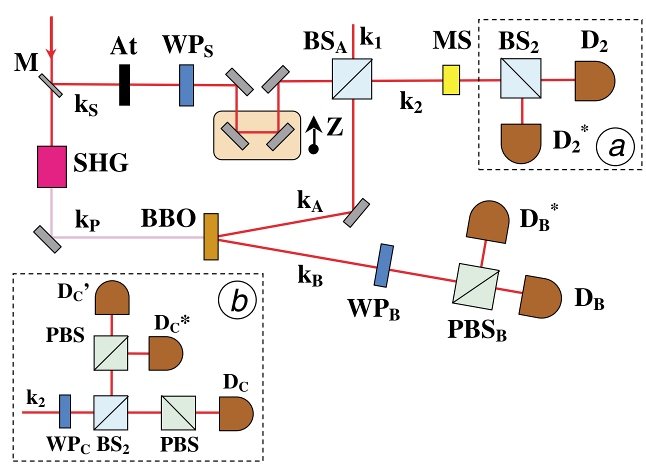

FIG. 2 (color online). Setup for the optical implementation of the tele-UNOT gate and the *probabilistic* UOQCM. The measurement setup used for the verification of the cloning experiment is reported in the inset (b).

[12], was experimentally identified by the simultaneous clicking of the detectors D_2 and D_2^* coupled to the output mode k_2 by the 50:50 beam splitter BS_2. The identical effect expected on mode k_1 was not exploited, for simplicity. As shown by the theoretical analysis above, the last condition implied the simultaneous realization in our experiment of the UNOT and UOQCM processes, here detected by a *postselection* technique. Interestingly enough, the symmetry of the projected subspace of H identified by BMC is implied by the intrinsic Bose symmetry of the two-photon Fock state realized at the output of BS_A.

The source of the SPDC process was a Ti:sapphire mode-locked pulsed laser with wavelength (WL) $\lambda = 795$ nm and repetition rate 76 MHz. A weak beam, deflected from the laser beam by mirror M, strongly attenuated by grey filters (At), delayed by $Z = 2c\Delta t$ via a micrometrically adjustable optical *trombone,* was the source of the quasi-single-photon state injected into BS_A over mode k_s. The average number of injected photons was $\bar{n} \simeq 0.1$, and the probability of a spurious two-photon injection was evaluated to be a factor 0.05 lower than for single-photon injection. Different qubit states $|\phi\rangle_S$ were prepared via the optical wave plate (WP) WP_S, either a $\lambda/2$ or a $\lambda/4$ WP. The main UV laser beam with WL $\lambda_p = 397.5$ nm generated by second harmonic generation, focused into a 1.5 mm thick nonlinear crystal of β-barium borate (BBO) cut for type-II phase matching, excited the SPDC source of the singlet $|\Psi^-\rangle_{AB}$. The photons A and B of each entangled pair were emitted over the modes k_A and k_B with equal WLs $\lambda = 795$ nm. All adopted photodetectors (D) were equal SPCM-AQR14 single-photon counters. One interference filter with bandwidth $\Delta\lambda = 3$ nm was placed in front of each D and determined the coherence time of the optical pulses: $\tau_{\text{coh}} \simeq 350$ fsec.

In order to realize the tele-UNOT protocol, the BMC process on the output mode k_2 of BS_A was detected by a coincidence technique involving D_2 and D_2^* [Fig. 2, inset (a)]. In order to enhance the *visibility* of the Ou-Mandel interference at the output of BS_A, a π-preserving mode selector (MS) was inserted on mode k_2. On Bob's site, the polarization π state on the mode k_B was analyzed by the combination of the WP_B and the polarization beam splitter PBS_B. For each input π state $|\phi\rangle_B$, WP_B was set in order to make the PBS_B transmit $|\phi\rangle_B$ and reflect $|\phi^\perp\rangle_B$, by then exciting D_B and D_B^* correspondingly. First, consider the teleportation (QST) turned off, by setting the optical delay $|Z| \gg c\tau_{\text{coh}}$, i.e., by spoiling the interference of photons S and A in BS_A. In this case, since the states $|\phi\rangle_B$ and $|\phi^\perp\rangle_B$ were realized with the same probability on mode k_B, the rate of coincidences detected by the D sets $[D_B, D_2, D_2^*]$ and $[D_B^*, D_2, D_2^*]$ were expected to be equal. By turning on the QST, i.e., by setting $|Z| \ll c\tau_{\text{coh}}$, the output state $\rho_B^{\text{out}} = (2\rho_S^\perp + \rho_S)/3$ was realized, implying a factor $R = 2$ *enhancement* of the counting rate $[D_B^*, D_2, D_2^*]$ and *no* enhancement of $[D_B, D_2, D_2^*]$. The actual measurement of R was carried out, and the *universality* of the tele-UNOT process was demonstrated, by the experimental results shown in Fig. 3. These three-coincidence results, involving the sets $[D_B, D_2, D_2^*]$ and $[D_B^*, D_2, D_2^*]$, correspond to the injection of three different input states: $|\phi\rangle_S = |H\rangle$, $|\phi\rangle_S = 2^{-1/2}(|H\rangle + |V\rangle)$, $|\phi\rangle_S = 2^{-1/2}(|H\rangle + i|V\rangle)$. In Fig. 3 the square and triangular markers refer, respectively, to the $[D_B^*, D_2, D_2^*]$ and $[D_B, D_2, D_2^*]$ coincidences versus the delay $Z = 2c\Delta t$. We may check that the tele-UNOT process affects only the $|\phi^\perp\rangle_B$ component, as expected. The signal-to-noise (S/N) ratio R was determined as the ratio between the peak values, i.e., for $Z \simeq 0$, and the no BMC enhancement values, i.e., for $|Z| \gg c\tau_{\text{coh}}$. The experimental values of the UNOT fidelity $F = R(R+1)^{-1}$ are $F_H = 0.641 \pm 0.005$, $F_{H+V} = 0.632 \pm 0.006$, and $F_{H+iV} = 0.619 \pm 0.006$, for the three injection states $|\phi\rangle_S$. These results, to be compared with the optimal value $F_{\text{th}} = 2/3 \approx 0.666$ corresponding to the optimal $R = 2$, have been evaluated by taking into account the reduction, by a factor $\xi = 0.7$, of the coincidence rate due to the spurious

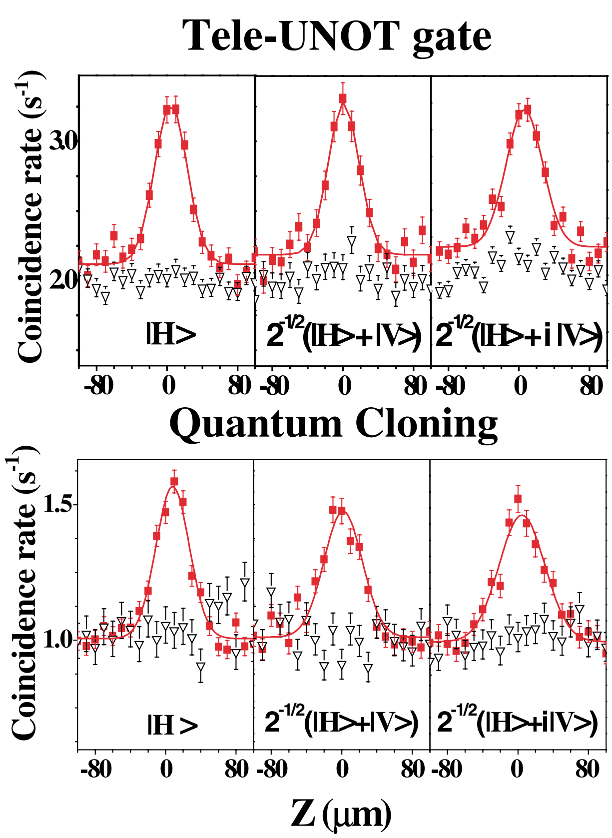

FIG. 3 (color online). Experimental results of the tele-UNOT gate and the UOQCM for three input qubits. Filled squares: plots corresponding to the "correct" polarization. Open triangles: plots corresponding to the "wrong" polarization. The solid line represents the best Gaussian fit expressing the *correct* polarization.

simultaneous injection of two photons on the mode k_S and to the simultaneous emission of two SPDC pairs. The factor ξ was carefully evaluated by a side experiment involving the detectors D_2 and D_2^*. Note that the experimental tele-UNOT peaks shown in Fig. 3 indeed demonstrate the simultaneous, contextual realization of the quantum cloning and UNOT processes on Alice's and Bob's sites, respectively.

For the sake of completeness, we wanted to gain insight in the linear *probabilistic* UOQCM process by investigating whether at the output of BS_A the photons S and A were indeed left in the state $\rho_S^{\rm out} = (5\rho_S + \rho_S^{\perp})/6 = \rho_A^{\rm out}$ after the state projection. The cloning analysis was realized on the BS_A output mode k_2 by replacing the measurement set (a) with (b), shown in the inset of Fig. 2. The polarization state on mode k_2 was analyzed by the combination of WP_C and of the polarizer beam splitters PBS. For each input π state $|\phi\rangle_S$, WP_C was set in order to make PBS's transmit $|\phi\rangle_S$ and reflect $|\phi^{\perp}\rangle_S$. The cloned state $|\phi\phi\rangle_S$ was detected on mode k_2 by a coincidence between the detectors D_C and D'_C. The generation of an entangled pair was assured by detecting one photon on the mode k_B; in this case PBS_B was removed and the field of mode k_B was coupled directly to D_B. Any coincidence detected by the sets $[D_C, D'_C, D_B]$ and $[D_C, D^*_C, D_B]$ implied the realization of the states $|\phi\phi\rangle_S$ and $|\phi\phi^{\perp}\rangle_S$, respectively. In analogy with the previous experiment, when $|Z| \gg c\tau_{\rm coh}$ the rate of coincidences detected by $[D_C, D'_C, D_B]$ and $2 \times [D^*_C, D'_C, D_B]$ were expected to be equal. By turning on the cloning machine, $|Z| \ll c\tau_{\rm coh}$, an enhancement by a factor $R = 2$ of the counting rate by $[D_C, D'_C, D_B]$ and no enhancement by $[D_C, D^*_C, D_B]$ were expected. The experimental results of the S/N ratio R, carried out by coincidence measurements involving $[D_C, D'_C, D_B]$ and $[D_C, D^*_C, D_B]$, are reported in the lower plots of Fig. 3, again for the three different input states: $|\phi\rangle_S = |H\rangle$, $|\phi\rangle_S = 2^{-1/2}(|H\rangle + |V\rangle)$, $|\phi\rangle_S = 2^{-1/2}(|H\rangle + i|V\rangle)$. The square and triangular markers there refer, respectively, to the $[D_C, D'_C, D_B]$ and $[D_C, D^*_C, D_B]$ coincidence plots versus the delay Z. The following values of the cloning fidelity $F = (2R + 1)(2R + 2)^{-1}$ were found: $F_H = 0.821 \pm 0.003$, $F_{H+V} = 0.813 \pm 0.003$, and $F_{H+iV} = 0.812 \pm 0.003$, to be compared with the optimal $F_{\rm th} = 5/6 \approx 0.833$ corresponding to the limit S/N value $R = 2$. As for the tele-UNOT gate experiment, the factor $\xi = 0.7$ has been corrected during the evaluation of the fidelities. Finally, note that in the present context the entangled singlet state $|\Psi^-\rangle_{AB}$ was not strictly necessary for the sole implementation of quantum cloning as we could model the *local* effect of the singlet on the input mode k_A by a *fully mixed* state $\rho_A = \frac{1}{2}\mathbb{I}_A$ spanning a two-dimensional space. This has been realized successfully in an earlier version of this paper [13].

In summary, two relevant quantum information processes, forbidden by quantum mechanics in their exact form, are found to be connected contextually by a modified quantum state teleportation scheme and can be optimally realized. At variance with previous experiments, the complete implementation of the new protocol has been successfully performed by a fully *linear* optical setup. The results are found in full agreement with theory.

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