Frequency Hopping in Quantum Interferometry: Efficient Up-Down Conversion for Qubits and Ebits

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(Received 28 January 2002; published 16 January 2003)

A novel Mach-Zehnder interferometer terminated at two different frequencies realizes in a quantum regime the nonlinear frequency conversion of optical quantum superposition states. The *information-preserving* character of the relevant unitary transformation has been experimentally demonstrated for input *qubits* and *ebits*. Besides its own intrinsic fundamental interest, the new scheme is expected to find important applications in modern quantum information technology.

DOI: 10.1103/PhysRevLett.90.027902 PACS numbers: 03.67.–a, 03.65.Ta, 42.50.Dv, 89.70.+c

Interferometry of quantum particles is rooted at the core of modern physics as it provides a unique tool of investigation and a direct demonstration of fundamental properties of nature as complementarity, nonlocality and quantum nonseparability [1]. In modern times all methods and protocols of quantum information (QI) and quantum computation involve interferometry through the very definition of the conceptual cornerstones of this science, viz., the *qubit* and the *ebit* [2]. In this framework it is well known that the first-order interference of particles, e.g., optical photons in the present work, is an utterly fragile property that can be easily spoiled by decoherence. This process is particularly inconvenient if the overall system exhibits a large degree of complexity expressed by a large number of interferometric channels, as in a network of gates in a quantum computer. More fundamentally and very generally, it is a common notion that in order to preserve the quantum features the interfering particles cannot be substantially disturbed by any decohering (*soft* or *hard*) collisions [3].

In this Letter we demonstrate that the last seemingly obvious condition is indeed not a necessary one. In fact, here we deal with the *hardest* possible collisions, the ones implying the *annihilation* or the *creation* of the particles themselves. More generally, we show once more that there exists a class of *information-preserving* unitary transformations of parametric type allowing a nonlinear (NL) frequency conversion of all quantum interferometric dynamical structures via a QED particle annihilation or/and creation process [4]. In this connection the present NL interferometric process may be considered as the dynamical "reversal" of the *quantum injected* NL parametric particle amplification/squeezing process that has been realized recently by several experiments [4]. In addition, the frequency conversion implied by the parametric process may be noiseless and may be easily realized with a "quantum efficiency" close to its maximum value, $QE \approx 1$. These are indeed very useful properties that are expected to be of large technological interest in the domain of quantum information and computation, as we shall see.

Refer to Fig. 1 showing the schematic diagram of a new kind of *single-photon* Mach-Zehnder (MZ) first-order interferometer (IF). The optical structure consists of an input 50/50 beam splitter (BS) coupled to two photon wave vector (wv) modes $\mathbf{k}_j$ $(j = 1, 2)$ at the wavelength (wl) λ, in our case lying in the infrared (IR) spectral region. Before standard mode recombination by the output BS_2, this simple mode structure is interrupted by a device $\mathbf{U}$ providing a unitary NL transformation $\mathbf{U}$ on the input single-photon superposition state, i.e., the *qubit* defined in a two-dimensional Hilbert $\mathbf{k}$ space,

$$|\Phi\rangle = \alpha|1\rangle_1|0\rangle_2 + \beta|0\rangle_1|1\rangle_2, \tag{1}$$

where the state labels 1, 2 refer to the IF modes k_1 and k_2, respectively. Assume now for simplicity and with no lack of generality $\beta = \alpha \exp(i\Phi)$ and $\alpha \equiv 2^{-1/2}$. Let us consider the frequency up-conversion process. The NL frequency-conversion unitary evolution operator,

$$\mathbf{U} \equiv \mathbf{exp}\left[\tilde{g}\sum_{j=1,2}(\hat{a}_j\hat{\bar{a}}_j^{\dagger}) + \text{h.c.}\right], \tag{2}$$

provides the QED *annihilation* of the input qubit (1) defined by the momenta $\hbar\mathbf{k}_j$, phase Φ and wl λ, and the simultaneous QED *creation* of a new qubit $|\Psi\rangle$ defined by

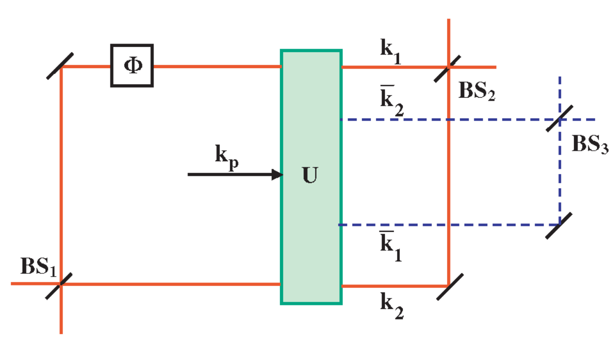

FIG. 1 (color online). Schematic diagram of the nonlinear Mach-Zehnder interferometer (MZ-IF) terminated at two different correlated frequencies.

the momenta $\hbar\overline{\mathbf{k}}_j$ $(j = 1, 2)$, phase Ψ and wl $\overline{\lambda} = (\lambda^{-1} + \lambda_p^{-1})^{-1}$ in our case lying in the ultraviolet (UV) spectral region. The second-order tensor parameter $\tilde{\mathbf{g}}$ is proportional to the interaction time t , to the second-order susceptibiliy $d^{(2)}$ of the NL medium, and to the "pump" field $\mathbf{E}_p$, with wl λ_p, wv $\mathbf{k}_p$, phase Θ. The pump field is assumed to be a single plane-wave coherent "classical field" undepleted by the interaction. The wl's and wv's $\overline{\mathbf{k}}_j$, $\mathbf{k}_j$, $\mathbf{k}_p$ are mutally connected by the energy conservation $\overline{\lambda}^{-1} = (\lambda^{-1} + \lambda_p^{-1})$ and by the *phase-matching condition* (PMC) for the three-wave interaction $\mathbf{k}_j + \overline{\mathbf{k}}_j + \mathbf{k}_p = 0$, leading in our simple plane case to an equation with two solutions: $j = 1, 2$. Similarly, the qubit phases at different wl's are also connected by $\Phi - \Psi + \Theta = 0$. Assume $\Theta = 0$ for convenience. We see that by introduction of the device U and of an additional output BS_3 the standard MZ-IF is transformed into a new kind of interferometer terminated at two *different* wavelengths λ and $\overline{\lambda}$. Correspondingly, two sets of *interference fringes* can be retrieved upon changes $\Delta\Phi = 2^{3/2}\pi X/\lambda$ of the mutual phase of the modes $\mathbf{k}_j$ via displacements X of an optical mirror M activated by a piezotransducer. Note that since in the present experiment PMC couples *deterministically* each mode $\mathbf{k}_j$ to a corresponding $\overline{\mathbf{k}}_j$, no additional quantum interference effects arise in the overall NL coupling process [5].

Let us now venture into a more detailed account of the experiment shown in Fig. 2. The input field was generated by a cw linearly polarized, single transverse mode diode laser (RLT8810MG), operating at the IR wl $\lambda = 876.1$ nm with a FWHM linewidth $\Delta\lambda \approx 1$ nm. The laser beam was highly attenuated by a set of neutral density (ND) filters to the approximate *single photon* level [6]. The achievement of this important condition was tested in agreement with the following considerations. Assuming a single-mode coherent field with mean photon number m, the probability of n photon excitation is given by the Poisson statistics $P(n, m) = \frac{m^n}{n!} e^{-m}$. Accordingly, the ratio of the probability of the two-photon vs one-photon mode excitation is $\sigma \equiv \frac{P(2,m)}{P(1,m)} = \frac{m}{2}$, and the "single photon" condition implies $\sigma \ll 1$. Because of the strong beam attenuation, in our experiment a value as small as $\sigma = 4.1 \times 10^{-7}$ at wl $\lambda = 876.1$ nm has been evaluated on the basis of the number of coincidence pulses detected simultaneously by the photodiodes D_A and D_B coupled with the two output modes of the MZ interferometer set with the IF phase $\Phi = \pi/2$. In the context of our experiment, this condition also implies that any up-converted single photon on mode $\overline{\mathbf{k}}_j$ was associated with the vacuum field on $\mathbf{k}_j$ after the NL interaction, and vice versa.

The input approximate single-photon field with wl λ was injected into the input 50/50 beam splitter BS_1 of the double MZ-IF with output modes $\mathbf{k}_j$, $j = 1, 2$. These modes were mutually Φ dephased by a piezoelectrically driven mirror (M) and the associated fields were brought by a short focal length lens within a type I $LiIO_3$, $l =$

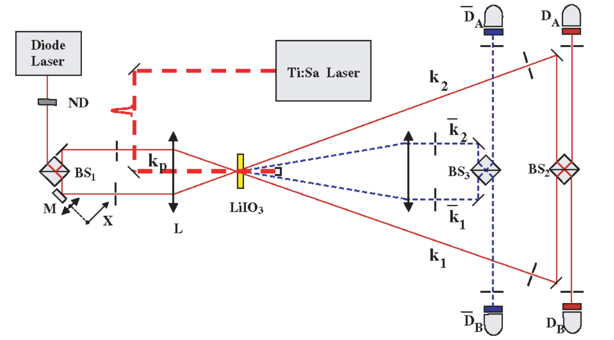

FIG. 2 (color online). Layout of the single-particle MZ-IF experiment.

1 mm thick crystal. Here a strong NL three-wave interaction took place between the input field associated with $\mathbf{k}_j$, the up-converted field associated with $\overline{\mathbf{k}}_j$, and the pump field associated with the pulses emitted at wl $\lambda_p = 795$ nm by a 76 MHz Ti-Sa laser ($\Delta t = 170f$ sec).

Finally, the output beams $\mathbf{k}_j$ emerging from the NL crystal were again superimposed by a 50/50 beam splitter BS_2 thus completing the usual MZ-IF scheme at the input wl λ. In a similar way the two up-converted output beams at the UV wl $\overline{\lambda} = 416.8$ nm were superimposed on an independent 50/50 BS_3, thus completing the MZ-IF scheme at the up-converted wl $\overline{\lambda}$. The difficult task of filtering the very weak beam at wl $\overline{\lambda}$ against the very strong UV beam at wl $\lambda_p/2$ was overcome by spatial discrimination after the NL crystal and by the adoption of two interference filters at 416.8 nm with bandwidth 10 nm.

All detectors operating at the IR wl λ were equal Si avalanche single-photon SPCM-200PQ diodes with *quantum efficiency* QE $\approx 30\%$, while the two detectors operating at the UV wl $\overline{\lambda}$ were photomultipliers: Philips-56DUVP (QE $\approx 23\%$) and Hamamatsu-R943-02 (QE $\approx 21\%$). A computer interfaced Stanford Research 400 counter was adopted for counting and averaging the detected signals.

Note that the up-conversion unitary transformation of the quantum superposition state, a *qubit*, at the IR wl λ into the "UV qubit" with wl $\overline{\lambda}$ is a *noise free* process since energy conservation does not allow any amplification of the input vacuum state. Of course, the inverse transformation process is also possible as an input qubit with wl $\overline{\lambda} > \lambda_p$ can be frequency "down-converted" into a corresponding one with wl $\lambda = \overline{\lambda}\lambda_p(\overline{\lambda} - \lambda_p)^{-1} > \overline{\lambda}$. This last down-conversion, "optical parametric amplifying," process is, of course, affected by a *squeezed vacuum* noise due to the amplification of the input vacuum state at wl λ [4].

The quantum efficiency (QE) of the up-conversion process, defined as the ratio between the average numbers of scattered and input photons on the modes $\overline{\mathbf{k}}_j$, $\mathbf{k}_j$ as a function of the peak intensity of the pump pulse I_p can be obtained by previous evaluation of the field at the output of the nonlinear interaction: $U^{\dagger}\hat{\mathbf{a}}U$ and $U^{\dagger}\hat{\bar{\mathbf{a}}}U$. An explicit theoretical calculation of QE has been carried out in [7] which refers to the ideal case of a collinear interaction. This last condition can be approximated in our case by improving the spatial superposition of the beams in the NL interaction region. We report in Fig. 3 the theoretical value of QE as a function of the pump intensity I_p, calculated for the 1 mm thick $LiIO_3$ crystal adopted in the experiment. The theoretical value of I_p corresponding to the limit condition $QE = 1$ is found; $I_p = 200\ GW/cm^2$. This figure may be compared with the experimental result $QE \approx 0.4$ that we obtained in a side experiment by focusing a low repetition rate 100 fs, $I_p = 200\ GW/cm^2$ pulse on the same crystal: Fig. 3 [7]. In the present MZ-IF experiment the peak intensity of each Ti-Sa laser pulse was $\approx 1\ GW/cm^2$, and the corresponding measured value of the quantum efficiency was $QE \simeq 3 \times 10^{-3}$.

The persistence of the quantum superposition condition within the IR $\longrightarrow$ UV frequency hopping process is demonstrated in Fig. 4 by the two correlated interference fringe patterns showing an equal periodicity upon changes of the mutal dephasing $\Delta\Phi$ of the IR modes $\mathbf{k}_1$, $\mathbf{k}_2$. As previously emphasized in a different context [4], this is but one aspect of a very general *information preserving* transformation of all unitary NL parametric up- (or down-) conversion transformations. By these any input qubit at wl λ and expressed by Eq. (1) is generally transformed into another at wl $\bar{\lambda} \neq \lambda$ keeping the *same* complex parameters α, β of the original one, i.e., fully reproducing its *quantum information* content. In addition and most importantly, the present work shows that these transformations can be *noise-free* and can be realized with a quantum efficiency close to its maximum value.

Apart from the fundamental relevance of these results due to the peculiar paradigmatic and historical status of quantum interferometry, the present work is expected to have a relevant impact on modern QI technology. This can be illustrated by the following example. Consider a case in which QI is encoded, say, on a single microwave photon with wl λ, e.g., within the cavity of a micromaser. If we want to transfer conveniently this information at a large distance we need to use an optical fiber exhibiting its low loss behavior in the IR spectral region, at wl λ'. This can be done in an *information lossless* manner in a NL waveguide by the up-conversion $\lambda \longrightarrow \lambda'$. If now this information is to be further transferred to a set of trapped atoms in an optical cavity we may need a further lossless up-conversion into the visible: $\lambda' \longrightarrow \lambda''$, etc. This appealing scenario may indeed imply the *only available* solution to sort information out of a nanostructure quantum device and to interconnect it efficiently within a QI network made of heterogeneous components. This may the case of a NMR quantum gate operating at radiofrequency wls or of a SQUID device operating at still larger wls [8]. The latter suggestion is indeed presently being investigated experimentally in our laboratory [9].

So far we have been dealing only with conversion of "qubits." The extension of our NL method to a two photon entangled state, or specifically to elementary

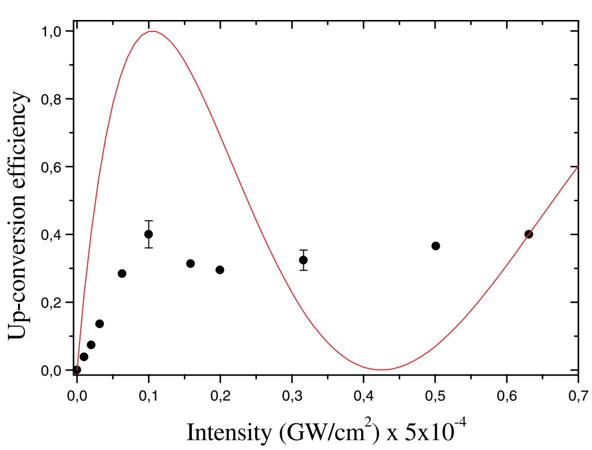

FIG. 3 (color online). Up-conversion quantum efficiency QE as a function of the laser pump intensity I_p: theoretical (continuous line) and experimental results.

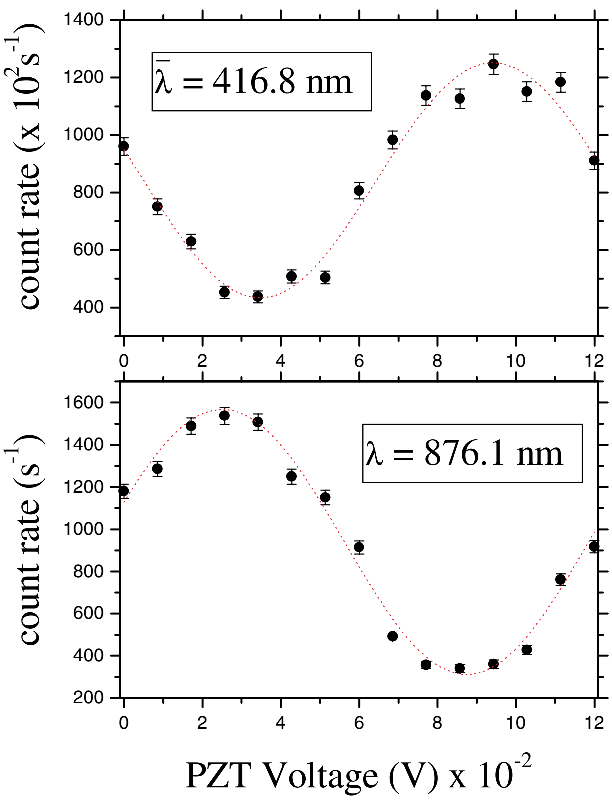

FIG. 4 (color online). Single photon interference fringes obtained within the same experiment at the different correlated wavelengths λ and $\bar{\lambda} = (\lambda^{-1} + \lambda_p^{-1})^{-1}$. The phase period of the fringing patterns is in agreement with the figure of merit (0.7 nm/V) of the piezoelectrical transducer activating the mirror M.

entangled information carriers, i.e., *ebits,* can be easily realized in several ways depending on the nature of the entanglement. Consider, for instance, a linear-polarization ($\boldsymbol{\pi}$) entangled two-photon state emitted over two spatial modes $\mathbf{k}_1$, $\mathbf{k}_2$ by a spontaneous parametric down-conversion process in a NL crystal: $|\Phi\rangle = \alpha|\uparrow\rangle_1|\uparrow\rangle_2 + \beta|\downarrow\rangle_1|\downarrow\rangle_2$ [4,10]. With reference to a technique recently adopted by Kwiat *et al.* [10] the modes $\mathbf{k}_j$ and the strong coherent pump beam with wv $\mathbf{k}_p$ could be focused by the common lens L beam into a combination of two equal thin type I NL plane crystal slabs and placed in mutual contact along their plane orthogonal to $\mathbf{k}_p$. If these slabs are mutually rotated around the axis parallel to $\mathbf{k}_p$ by an angle $\phi = \pi/2$ and the linear polarization $\boldsymbol{\pi}_p$ of the pump beam is also rotated by $\phi = \pi/4$, both nonlocally correlated orthogonal π-state components of the injected entangled state undergo equal NL transformations given by [2], thus realizing an overall, *information preserving* up- (or down-) frequency conversion of $|\Phi\rangle$.

According to a recent conceptual and formal new QI perspective, the optical *field's modes* rather than the photons are taken as the carriers of quantum information and entanglement [11,12]. Furthermore, in that picture any *qubit* is physically implemented by a two-dimensional subspace of Fock states of the optical field, specifically the state spanned by the vacuum state and the one-photon state. According to this perspective the class of information preserving NL transformations of the state given by Eq. (1) investigated in the present work should be more correctly referred to as *entangled states* and may indeed provide a useful new set of unitary transformations for the Hilbert space evolution of these new QI states.

This work has been supported by the FET European Network on Quantum Information and Communication (Contract No. IST-2000-29681: ATESIT) and by PAIS-INFM 2002 (QEUPCO).

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