Amplification of Quantum Entanglement

Francesco De Martini
Dipartimento di Fisica and Istituto Nazionale di Fisica della Materia, Università "La Sapienza," Roma 00185, Italy
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The new process of *quantum injection* into an optical parametric amplifier operating in *entangled* configuration is adopted to "amplify" into a large dimensionality spin-$\frac{1}{2}$ Hilbert space the quantum entanglement and superposition properties of the photon couples generated by parametric down-conversion. The structure of the Wigner function and of the field's correlation functions shows a decoherence-free multiphoton *Schroedinger-cat* behavior of the emitted field which is largely detectable against the squeezed-vacuum noise. [S0031-9007(98)07189-0]

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The generation of classically distinguishable quantum states, a major endeavor of modern physics, has long been the object of extensive theoretical studies. In recent times important experimental investigation with atoms has been carried out in this field by various research groups [1–5]. In this context it has been proved that the realization of the Schroedinger-cat program is generally challenged by an extremely rapid decoherence process due to the stochastic interactions of any freely evolving mesoscopic system with the environment [6,7]. Within the framework of quantum computation, the same process has also been recognized to represent a major limitation toward the coherent superposition of the qubits carrying the quantum information [8]. In the domain of quantum optics, several strategies have been proposed to overcome the problem, e.g., the back-action evasion [9] and cavity control by optical feedback [10,11].

In the present Letter we present a new approach to the problem based on the amplifying/squeezing operation of the optical parametric amplifier (OPA) operating in a novel entangled configuration and initiated by a process of *quantum injection,* e.g., provided by the sub-Poissonian character of a single photon in the Fock state $n = 1$. This photon may belong to a couple generated by spontaneous parametric down-conversion (SPDC), e.g., in a Φ-*phase tunable* entangled state of linear polarization π, defined in a Hilbert space of dimensionality 2×2. The SPDC process has been adopted within recent tests of violation of Bell's inequalities [12], of quantum-state teleportation [13], and of all processes generally belonging to the chapter of *nonlocal entangled interferometry* [14,15]. The key idea of the present work relates to the possibility of "amplifying" this quite interesting phenomenology to a higher dimensionality spin-$\frac{1}{2}$ Hilbert space, i.e., involving a large number of photon couples. We show that this can be realized by a novel optical device, the quantum-injected, *entangled OPA* leading to a new *entangled* Schroedinger-cat (S-cat) configuration which may be *decoherence free,* in the ideal case. Consider the diagram shown in Fig. 1. Two equal and equally oriented nonlinear (NL) crystals, e.g., beta-barium-borate (BBO) cut for type II phase matching are excited by two beams derived from a common UV laser beam at a wavelength (wl) $\lambda_p = 2\pi|\mathbf{k}_p|^{-1}$. Crystal 1 is the SPDC source of π-entangled photon couples emitted, with wl $\lambda = 2\lambda_p$ over the modes $\mathbf{k}_1$, $\mathbf{k}_3$ determined by two fixed pinholes. In order to prevent any EPR type state reduction affecting the overall superposition process, the photon emitted over the output mode $\mathbf{k}_3$ is filtered by a polarization analyzer with axis oriented at 45° to the horizontal (t.h.) before being detected by D_3 [16]. A click at D_3 opens a gate selecting all registered outcomes, thus providing the *conditional* character of the overall experiment. The photon emitted over $\mathbf{k}_1$ provides the quantum injection into the OPA, physically consisting of the other NL crystal. The input state to our system may be expressed in terms of the Fock states associated with the modes $\mathbf{k}_j$ ($j = 1, 2$)

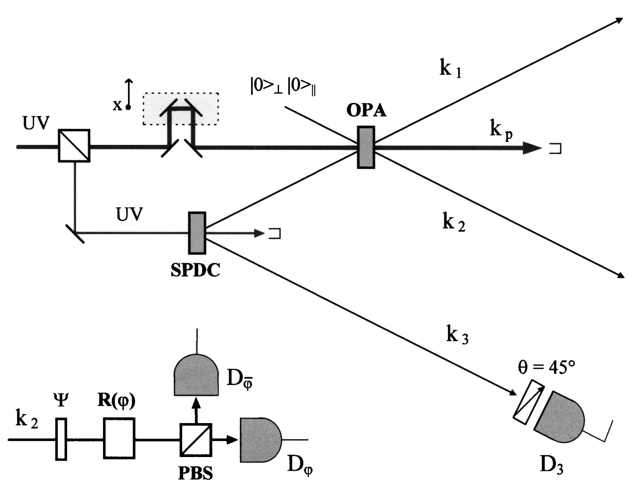

FIG. 1. Optical configuration of the *quantum-injected,* entangled optical parametric amplifier realizing the process of multiphoton quantum superposition. The SPDC quantum injector is provided by a type II Φ-phase tunable generator of linear polarization (π)-entangled photon couples. The crystal realizing the OPA action is cut for type II, noncollinear phase matching and is equal to the one realizing SPDC. The detection system consists of a birefringent plate Ψ, a π rotator $R(\varphi)$, a polarizing beam splitter PBS and two cooled photomultipliers. In the experiment a similar system is inserted on mode $\mathbf{k}_2$.

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and with the two π components, respectively, parallel and orthogonal (t.h.): $|\Psi_0\rangle = 2^{-1/2}|0\rangle_{2\perp} \otimes |0\rangle_{2\|} \otimes [|1\rangle_{1\perp} \otimes |0\rangle_{1\|} + e^{i\Phi}|0\rangle_{1\perp} \otimes |1\rangle_{1\|}]$. For a type II NL crystal operating in noncollinear configuration, the overall amplification process taking place over $\mathbf{k}_j$ is contributed by two equal and independent amplifiers OPA$_A$ and OPA$_B$ inducing unitary transformations, respectively, on two couples of time dependent field operators: $[\hat{a}_1(t) \equiv \hat{a}_{1\perp}, \hat{a}_2(t) \equiv \hat{a}_{2\|}]$ and $[b_1(t) \equiv \hat{a}_{1\|}, \hat{b}_2(t) \equiv \hat{a}_{2\perp}]$ for which, at the initial time of the interaction, $t = 0$, is $[\hat{a}_i(0), \hat{a}_j(0)^\dagger] = [\hat{b}_i(0), \hat{b}_j(0)^\dagger] = \delta_{ij}$ and $[\hat{a}_i(0), \hat{b}_j(0)^\dagger] = 0$ for any i and j and $i, j = 1, 2$. A quantum analysis of the dynamics of the system leads to a linear dependence of the field operators on the corresponding input quantities, e.g., for OPA$_A$, $\hat{a}_1(t) = C\hat{a}_1(0) + S\hat{a}_2(0)^\dagger$, $\hat{a}_2(t) = C\hat{a}_2(0) + S\hat{a}_1(0)^\dagger$, being $C \equiv \cosh g$, $S \equiv \sinh g, g \equiv \chi t =$ amplification gain having assumed a classical, undepleted pump field. The interaction time t may be determined in our case by the length L of the NL crystal. The evolution operator for OPA$_A$ is then expressed in the form of the *squeeze operator*: $U_A(t) = \exp[g(\hat{A}^\dagger - \hat{A})]$ being $\hat{A}^\dagger \equiv \hat{a}_1(t)^\dagger \hat{a}_2(t)^\dagger$, $\hat{A} \equiv \hat{a}_1(t)\hat{a}_2(t)$. A corresponding $U_B(t)$ for OPA$_B$ is given by the replacement $\hat{a}_i \rightarrow \hat{b}_i$. By use of the overall propagator $U_A(t)U_B(t)$ and of the disentangling theorem [17], the output state is found:

$$|\Psi\rangle \equiv G\{|\Psi_A(0)\rangle \otimes |\Psi_A(1)\rangle + e^{i\Phi}|\Psi_B(0)\rangle \otimes |\Psi_B(1)\rangle\}, \tag{1}$$

where $G \equiv (\sqrt{2}\,C^2)^{-1}$, $\Psi_A(0)\rangle \equiv \sum_{n=0}^{\infty} \sqrt{P_n}\, |n\rangle_{1\|} \otimes |n\rangle_{2\perp}$, $|\Psi_B(0)\rangle \equiv \sum_{n=0}^{\infty} \sqrt{P_n}\, |n\rangle_{1\perp} \otimes |n\rangle_{2\|}$, $\Gamma \equiv S/C$, and $P_n \equiv \overline{n}^n/(1 + \overline{n})^{(1+n)} = (\Gamma^{2n}/C)$ is a thermal distribution accounting for the squeezed-vacuum noise with average photon number $\overline{n} = S^2$ [18]. The two states expressed in (1) as $|\Psi_A(1)\rangle \equiv \sum_{n=0}^{\infty} \Gamma^n \sqrt{n+1}\, |n + 1\rangle_{1\perp} \otimes |n\rangle_{2\|}$ and $|\Psi_B(1)\rangle \equiv \sum_{n=0}^{\infty} \Gamma^n \sqrt{n+1}\, |n + 1\rangle_{1\|} \otimes |n\rangle_{2\perp}$ represent the effect of the one-photon quantum injection. Since this sum is extended over the complete set of n states, the appeal to the *macroscopic* quantum coherence is justified. The output state function, written in the form $|\Psi\rangle = [|\Psi_A\rangle + e^{+i\Phi}|\Psi_B\rangle]$ with $|\Psi_A\rangle \equiv |\Psi_A(0)\rangle \otimes |\Psi_A(1)\rangle$, expresses the condition of quantum superposition between two *pure,* multiparticle states originating, through unitary OPA transformations, from the input single-particle state $|\Psi_0\rangle$, keeping in this process its original phase Φ. Furthermore, most important, since $|\Psi\rangle$ is not factorizable in terms of π *states,* it keeps his original π-entanglement character, thus transferring into the multiparticle regime the striking quantum nonseparability and Bell-type nonlocality properties of the microscopic (i.e., two-particle) systems [16,19].

In order to inspect at a deeper lever these results, consider the Wigner function of the output field. Evaluate first the symmetrically ordered characteristic function of the set of complex variables $(\eta_j, \eta_j^*, \xi_j, \xi_j^*) \equiv \{\eta, \xi\}, (j = 1, 2)$: $\chi_s\{\eta, \xi\} \equiv \langle\Psi_0|D[\eta_1(t)]D[\eta_2(t)]D[\xi_1(t)]D[\xi_2(t)]|\Psi_0\rangle$ expressed in terms of the *displacement* operators: $D[\eta_j(t)] \equiv \exp[\eta_j(t)\hat{a}_j(0)^\dagger - \eta_j^*(t)\hat{a}_j(0)]$, $D[\xi_j(t)] \equiv \exp[\xi_j(t)\hat{b}_j(0)^\dagger - \xi_j^*(t)\hat{b}_j(0)]$ where $\eta_1(t) \equiv (\eta_1 C - \eta_2^* S)$, $\eta_2(t) \equiv (\eta_2 C - \eta_1^* S)$, $\xi_1(t) \equiv (\xi_1 C - \xi_2^* S)$, and $\xi_2(t) \equiv (\xi_2 C - \xi_1^* S)$. The Wigner function, expressed in terms of the corresponding complex phase-space variables $(\alpha_j, \alpha_j^*, \beta_j, \beta_j^*) \equiv \{\alpha, \beta\}$ is the eight-dimensional Fourier transform of $\chi_S\{\eta, \xi\}$. By a lengthy application of operator algebra and integral calculus, we could evaluate analytically in closed form both functions $\chi_S\{\eta, \xi\}$ and $W\{\alpha, \beta\}$. The *exact* result is

$$W\{\alpha, \beta\} = -\overline{W}_A\{\alpha\}\overline{W}_B\{\beta\}[1 - |e^{i\Phi}\Delta_A\{\alpha\} + \Delta_B\{\beta\}|^2], \tag{2}$$

where $\Delta_A\{\alpha\} \equiv 2^{-1/2}(\gamma_{A+} - i\gamma_{A-})$ and $\Delta_B\{\beta\} \equiv 2^{-1/2}(\gamma_{B+} - i\gamma_{B-})$ are expressed in terms of the squeezed variables: $\gamma_{A+} \equiv (\alpha_1 + \alpha_2^*)e^{-g}$, $\gamma_{A-} \equiv i(\alpha_1 - \alpha_2^*)e^{+g}$, $\gamma_{B+} \equiv (\beta_1 + \beta_2^*)e^{-g}$, and $\gamma_{B-} \equiv i(\beta_1 - \beta_2^*)e^{+g}$. The Wigner functions $\overline{W}_A\{\alpha\} \equiv (2/\pi)^2 \times \exp(-[|\gamma_{A+}|^2 + |\gamma_{A-}|^2])$ and $\overline{W}_B\{\beta\} \equiv (2/\pi)^2 \times \exp(-[|\gamma_{B+}|^2 + |\gamma_{B-}|^2])$, definite positive over the four-dimensional spaces $\{\alpha\}$ and $\{\beta\}$, represent the effect of squeezed vacuum, i.e., emitted, respectively, by OPA$_A$ and OPA$_B$ in the absence of any injection. Inspection of Eq. (1) shows that precisely the quantum superposition character of the injected state $|\Psi_0\rangle$ determines the dynamical quantum superposition of the devices OPA$_A$ and OPA$_B$, the ones that otherwise act as *uncoupled* and *independent* objects. From another perspective, since the quasiprobability functions $\overline{W}_A\{\alpha\}$ and $\overline{W}_B\{\beta\}$ corresponding to the two macrostates in the absence of quantum superposition are defined in two totally separated and independent spaces, their respective "distance" in the overall phase space of the system $\{\alpha, \beta\}$ can be thought of as "macroscopic," as generally required by any standard S-cat dynamics in a two-dimensional phase space [2]. The link between the spaces $\{\alpha\}$ and $\{\beta\}$ is provided by the quantum superposition term in Eq. (2): $2\,\mathrm{Re}[e^{i\Phi}\Delta_A\{\alpha\}\Delta_B^*\{\beta\}]$. Note also in Eq. (2) and in Fig. 2 the absence of definite positivity of $W\{\alpha, \beta\}$ over its definition space, which assures the overall quantum character of our multiparticle, quantum-injected amplification scheme [18,20].

The striking quantum mechanical features of the system are also revealed by the 1st- and 2nd-order correlation functions of the OPA output fields. Before detection over the modes $\mathbf{k}_j$ $(j = 1, 2)$ the fields are phase shifted by $\Psi_j = (\psi_{j\perp} - \psi_{j\|})$ by birefringent plates and filtered by π analyzers with axes oriented at the angles: $45° + \varphi_j$ (t.h.). Each π analyzer may consist of the combination of a Fresnel-rhomb π rotator, $R(\varphi)$, and of a polarizing beam splitter, PBS: cf. Fig. 1, inset. The field associated with the mode $\mathbf{k}_j$ is detected at the space-time positions x_j by two linear detectors

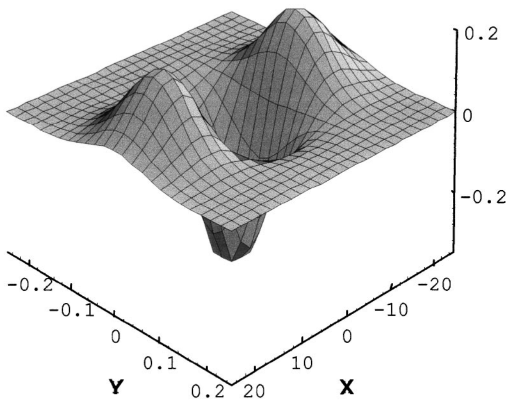

FIG. 2. Tridimensional plot of the Wigner function of the amplified field on mode $\mathbf{k}_2$ at the output of the quantum injected OPA as a function of the squeezed variables: $X = (\alpha + \beta^*)e^{-g}$; $Y = i(\beta - \alpha^*)e^{+g}$, for a parametric gain $g = 2.5$ and $\Delta\Phi = 0$.

$D_{j\varphi}$ and $D_{j\overline{\varphi}}$, with $\overline{\varphi} = \varphi + 90°$. The 1st-order correlation functions $G_j^{(1)}(x_j, x_j) \equiv \langle\Psi_0|\hat{N}_j(t)|\Psi_0\rangle$ are ensemble averages of the *number* operators $\hat{N}_j(t) \equiv \hat{c}_j^\dagger(t)\hat{c}_j(t)$ written in terms of the detected fields: $\hat{c}_j(t) \equiv [\xi_j^- \hat{a}_j(t) + \xi_j^+ \hat{b}_j(t)]$, $[\hat{c}_i(t), \hat{c}_j^\dagger(t)] = \delta_{ij}$, $\xi_j^+ \equiv 2^{-1/2}(\cos\varphi_j + \sin\varphi_j)\exp(i\psi_{j\beta})$, and $\xi_j^- \equiv 2^{-1/2} \times (\cos\varphi_j - \sin\varphi_j)\exp(i\psi_{j\alpha})$, where $\psi_{j\alpha}, \psi_{j\beta}$ are phase shifts induced by the birefringent plate on the fields $\hat{a}_j(t)$ and $\hat{b}_j(t)$. $G_j^{(1)}$ show the expected superposition character of the output field with respect to φ_j and to $\Delta_j^{\mp}\Phi \equiv (\Phi \pm \Psi_j)$: $G_1^{(1)} = \overline{n} + \frac{1}{2}(\overline{n} + 1)[1 + \cos(2\varphi_1)\cos\Delta_1^-\Phi]$ and $G_2^{(1)} = \overline{n} + \frac{1}{2}\overline{n}[1 + \cos(2\varphi_2) \times \cos\Delta_2^+\Phi]$. By comparison with the corresponding (φ_j and $\Delta\Psi$-independent) averages over the input *vacuum state*: $G_{1,\mathrm{vac}}^{(1)} = G_{2,\mathrm{vac}}^{(1)} = \overline{n}$, we obtain the signal-to-noise ratio to the S-cat detection: $s/n = 2$, for $\Delta_j^-\Phi = \varphi_j = 0$. The above result immediately suggests a 1st-order π-interferometric method for S-cat detection on a *single* $\mathbf{k}_j$ beam, with *visibility*: $V = (G_{\max}^{(1)} - G_{\min}^{(1)})/(G_{\max}^{(1)} + G_{\min}^{(1)}) \geqq \frac{1}{3}$. The Wigner function plotted in Fig. 2 refers to the output field detected by this method on the mode $\mathbf{k}_2$.

The 2nd-order functions $G_{ij}^{(2)}(x_i, x_j; x_j, x_i) \equiv \langle\Psi_0| : \hat{N}_i(t)\hat{N}_j(t) : |\Psi_0\rangle$ are also found: $G_{11}^{(2)} = 2\overline{n}\{\overline{n} + (\overline{n} + 1) \times [1 + \cos(2\varphi_1)\cos\Delta_1^-\Phi]\}$, $G_{22}^{(2)} = 2\overline{n}^2\{1 + [1 + \cos(2\varphi_2) \times \cos\Delta_2^+\Phi]\}$, and $G_{12}^{(2)} = 2\overline{n}^2 + \overline{n}/2 + \overline{n}[(\overline{n} + 1) \times \cos(2\varphi_1)\cos\Delta_1^-\Phi)] + \overline{n}(\overline{n} + 1/2)[1 + \cos(2\varphi_2) \times \cos\Delta_2^+\Phi] + \overline{n}(\overline{n} + 1)\{[1 + \cos\Delta\Psi]\cos^2\Delta\varphi^- + [1 - \cos\Delta\Psi]\sin^2\Delta\varphi^+\}$ where $\Delta\varphi^\pm \equiv (\varphi_1 \pm \varphi_2)$; $\Delta\Psi \equiv (\Psi_1 + \Psi_2)$. We may prove, e.g., for all $\Delta_j^\pm\Phi = \varphi_j = 0$, that our system realizes the *maximum* quantum mechanical violation of the Cauchy-Schwarz inequality which generally holds in semiclassical field theory: $[g_{12}^{(2)}(0)]^2 \leq g_{11}^{(2)}(0)g_{22}^{(2)}(0)$ being $g_{ij}^{(2)}(0) \equiv G_{ij}^{(2)}(0)[G_i^{(1)}(0)G_j^{(1)}(0)]^{-1}$ [18]. Furthermore, the given expression of $G_{12}^{(2)}$ shows the effects of the *multiparticle* quantum nonseparability and Bell-type nonlocality, contributed by the terms proportional to $\cos(2\varphi_j)$, $\cos\Delta\varphi^\pm$. This is a most relevant manifestation of the nonlocality properties of our quantum injected, entangled parametric system [16,19].

The absence of *decoherence* within our ideal, nondissipative multiparticle system is due to its nature of nonlinearly *driven* excitation. As such, it is coupled with a continuously rephasing environment here provided by the parametric NL polarization. Similar situations are encountered in physics of the nonlinear dynamical systems, e.g., in nonlinear surface plasmon or surface exciton-polariton generation in solid state NL spectrocopy [21]. Of course, any single photon loss event, mainly contributed in our case by stray reflections, implies an elementary decoherence process. In our laboratory experiment two equal 1 mm thick, BBO crystals are excited by 0.8 ps pulses at $\lambda_p = 400$ nm second-harmonic generated by a mode-locked Ti:Sa laser at a 76 MHz repetition rate with an average power $\approx 0.3W$. The detection system, consisting of two linear photodetectors connected to an electronic correlator, is equal to the one shown by Fig. 1, inset, but for the absence of the birefringent plate. The initial phase is $\Phi = 0$. All surfaces are treated by special antireflection coatings resonant at the working $\lambda = 800$ nm with an overall transmittivity $T \approx 99.60\%$. This figure implies the loss of a single photon every $\gtrsim 20$ pulses with the generation of $\overline{n} \approx 10$ per pulse. This would make our S-cat experiment quite feasible.

Within the domain of quantum computation our results suggest the *nonlinear* interaction among the information carrying particles as an efficient solution toward a large scale implementation of the new methods [22]. In summary, we have given the theory of a novel multiparticle system showing both quantum superposition and quantum entanglement features. This results from a smart interplay of the fundamental paradigms of modern quantum optics, i.e., quadrature squeezing, multiparticle-state entanglement, and quantum-nonseparability in parametric correlations. From a foundational perspective our method allows the first realization of several fundamental nonlocality and noncontextuality tests of quantum mechanics requiring a number of entangled particles larger than two [23].

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Note added.—A very interesting 3-crystal variant of the scheme of Fig. 1 consists of two independent SPDC devices feeding in a symmetrical fashion the OPA on *both* input modes $\mathbf{k}_j$ ($j = 1, 2$), by two distinct single-photon quantum-injection processes and within a *double-conditional* experiment. In this case, as we shall analyze in a forthcoming paper, the squeezed-vacuum noise can be greatly reduced and the S-cat *visibility* increased to $V > 1/2$.

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