Amplification of Quantum Entanglement

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The new process of quantum injection into an optical parametric amplifier operating in entangled configuration is adopted to “amplify” into a large dimensionality spin-½ Hilbert space the quantum entanglement and superposition properties of the photon couples generated by parametric down-conversion. The structure of the Wigner function and of the field’s correlation functions shows a decoherence-free multiphoton Schroedinger-cat behavior of the emitted field which is largely detectable against the squeezed-vacuum noise.

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The generation of classically distinguishable quantum states, a major endeavor of modern physics, has long been the object of extensive theoretical studies. In recent times important experimental investigation with atoms has been carried out in this field by various research groups [1–5]. In this context it has been proved that the realization of the Schroedinger-cat program is generally challenged by an extremely rapid decoherence process due to the stochastic interactions of any freely evolving mesoscopic system with the environment [6,7]. Within the framework of quantum computation, the same process has also been recognized to represent a major limitation toward the realization of quantum computation, the same process has also been recognized to represent a major limitation toward the realization of quantum computation, the same process has also been recognized to represent a major limitation toward the realization of quantum computation, the same process has also been recognized to represent a major limitation toward the realization of quantum computation, the same process has also been recognized to represent a major limitation toward the realization of quantum computation, the same process has also been recognized to represent a major limitation toward the realization of quantum computation, the same process has also been recognized to represent a major limitation toward the realization of quantum computation, the same process has also been

In order to prevent any EPR type state reduction affecting the overall superposition process, the photon emitted over the output mode \( k_3 \) is filtered by a polarization analyzer with axis oriented at 45° to the horizontal (t.h.) before being detected by \( D_3 \) [16]. A click at \( D_3 \) opens a gate selecting all registered outcomes, thus providing the conditional character of the overall experiment. The photon emitted over \( k_1 \) provides the quantum injection into the OPA, physically consisting of the other NL crystal. The input state to our system may be expressed in terms of the Fock states associated with the modes \( k_j \) (\( j = 1, 2 \)) cut for type II phase matching are excited by two beams derived from a common UV laser beam at a wavelength (wl) \( \lambda_p = 2\pi |k_p|^{-1} \). Crystal 1 is the SPDC source of \( \pi \)-entangled photon couples emitted, with wl \( \lambda = 2\lambda_p \) over the modes \( k_1, k_3 \) determined by two fixed pinholes.

In the present Letter we present a new approach to the problem based on the amplifying/squeezing operation of the optical parametric amplifier (OPA) operating in a novel entangled configuration and initiated by a process of quantum injection, e.g., provided by the sub-Poissonian character of a single photon in the Fock state \( n = 1 \). This photon may belong to a couple generated by spontaneous parametric down-conversion (SPDC), e.g., in a \( \Phi \) phase tunable entangled state of linear polarization \( \pi \), defined in a Hilbert space of dimensionality \( 2 \times 2 \). The SPDC process has been adopted within recent tests of violation of Bell’s inequalities [12], of quantum-state teleportation [13], and of all processes generally belonging to the chapter of nonlocal entangled interferometry [14,15]. The key idea of the present work relates to the possibility of “amplifying” this quite interesting phenomenology to a higher dimensionality spin-½ Hilbert space, i.e., involving a large number of photon couples. We show that this can be realized by a novel optical device, the quantum-injected, entangled OPA leading to a new entangled Schroedinger-cat (S-cat) configuration which may be decoherence free, in the ideal case. Consider the diagram shown in Fig. 1. Two equal and equally oriented nonlinear (NL) crystals, e.g., beta-barium-borate (BBO)
and with the two $\pi$ components, respectively, parallel and orthogonal (t.h.): $|\Psi_0\rangle = 2^{-1/2} |0\rangle_1 1 \otimes |0\rangle_2 2 \otimes |1\rangle_1 1 \otimes |0\rangle_1 1 + e^{i\Phi} |0\rangle_1 1 \otimes |1\rangle_1 1 | \rangle$. For a type II NL crystal operating in noncollinear configuration, the overall amplification process taking place over $k_F$ is contributed by two equal and independent amplifiers $\text{OPA}_A$ and $\text{OPA}_B$ inducing unitary transformations, respectively, on two couples of time dependent field operators: $[\hat{a}_1(t) = \hat{a}_1 1 , \hat{a}_2(t) = \hat{a}_2 2 ]$ and $[\hat{b}_1(t) = \hat{b}_1 1 , \hat{b}_2(t) = \hat{b}_2 2 ]$ for which, at the initial time of the interaction, $t = 0$, is $[\hat{a}_1(0), \hat{b}_1(0)] = [\hat{b}_1(0), \hat{b}_2(0)] = \delta_{ij}$ and $[\hat{a}_i(0), \hat{b}_j(0)] = 0$ for any $i$ and $j$ and $i,j = 1,2$. A quantum analysis of the dynamics of the system leads to a linear dependence of the field operators on the corresponding input quantities, e.g., for $\text{OPA}_A$, $\hat{a}_1(t) = C \hat{a}_1(0) + S \hat{a}_1(0) 1$, $\hat{a}_2(t) = C \hat{a}_2(0) + S \hat{a}_1(0) 1$, being $C = \cosh g, S = \sinh g, g = \chi t = \text{amplification gain}$ having assumed a classical, undepleted pump field. The interaction time $t$ may be determined in our case by the length $L$ of the NL crystal. The evolution operator for $\text{OPA}_A$ is then expressed in the form of the squeeze operator: $U_A(t) = \exp_g (\hat{A}^\dagger - \hat{A})$ being $A^\dagger = \hat{a}_1(0) 1 \hat{a}_2(0) 1$, $\hat{A} = \hat{a}_1(0) 1 \hat{a}_2(0) 1$. A corresponding $U_B(t)$ for $\text{OPA}_B$ is given by the replacement $\hat{a}_i \to \hat{b}_i$. By use of the overall propagator $U_A(t)U_B(t)$ and of the disentangling theorem [17], the output state is found:

$$\Psi = G \{ |\Psi_A(0)\rangle \otimes |\Psi_A(1)\rangle + e^{i\Phi} |\Psi_B(0)\rangle \otimes |\Psi_B(1)\rangle \},$$

(1)

where $G = (\sqrt{2} C^2)^{-1}$, $|\Psi_A(0)\rangle = \sum_{n=0}^{\infty} \sqrt{P_n} |n\rangle 1 \otimes |n\rangle 2$, $|\Psi_B(0)\rangle = \sum_{n=0}^{\infty} \sqrt{Q_n} |n\rangle 1 \otimes |n\rangle 2$, $\Gamma = S/C$, and $P_n = \frac{n^\Gamma}{(1 + n^\Gamma)} = (\Gamma^{2n}/C)$ is a thermal distribution accounting for the squeezed-vacuum noise with average photon number $\bar{n} = S^2$ [18]. The two states expressed in (1) as $|\Psi_A(1)\rangle = \sum_{n=0}^{\infty} \Gamma^n \sqrt{n^\Gamma + 1} \otimes |n+1\rangle 1 \otimes |n\rangle 2$, and $|\Psi_B(1)\rangle = \sum_{n=0}^{\infty} \Gamma^n \sqrt{n^\Gamma + 1} \otimes |n\rangle 1 \otimes |n+1\rangle 2$, represent the effect of the one-photon quantum injection. Since this sum is extended over the complete set of $n$ states, the appeal to the macroscopic quantum coherence is justified. The output state function, written in the form $|\Psi\rangle = [|\Psi_A\rangle + e^{i\Phi} |\Psi_B\rangle]$ with $|\Psi_A\rangle = |\Psi_A(0)\rangle \otimes |\Psi_A(1)\rangle$, expresses the condition of quantum superposition between two pure, multiparticle states originating, through unitary OPA transformations, from the input single-particle state $|\Psi_0\rangle$, keeping in this process its original phase $\Phi$. Furthermore, most important, since $|\Psi\rangle$ is not factorizable in terms of $\pi$ states, it keeps its original $\pi$-entanglement character, thus transferring into the multiparticle regime the striking quantum nonseparability and Bell-type nonlocality properties of the microscopic (i.e., two-particle) systems [16,19].

In order to inspect at a deeper level these results, consider the Wigner function of the output field. Evaluate first the symmetrically ordered characteristic function of the set of complex variables $(\eta_j, \eta_j^*, \xi_j, \xi_j^*) = (\eta, \xi), (j = 1, 2)$:

$$\langle \eta, \xi \rangle = \langle \Psi_0 | D[\eta_1(t)D[\eta_2(t)D[\xi_1(t)]D[\xi_2(t)] | \Psi_0 \rangle, \langle \eta, \xi \rangle = \langle \Psi_0 | D[\eta_1(t)D[\eta_2(t)D[\xi_1(t)]D[\xi_2(t)] | \Psi_0 \rangle$$

expressed in terms of the displacement operators: $D[\eta_1(t)] = \exp[i \eta_j^* \hat{a}_j(0)]$, $D[\eta_2(t)] = \exp[i \eta_j^* \hat{a}_j(0)]$, $D[\xi_1(t)] = \exp[i \xi_j^* \hat{a}_j(0)]$, $D[\xi_2(t)] = \exp[i \eta_j^* \hat{a}_j(0)]$, where $\eta_j(t) = (\eta_j - \eta_j^*)$, $\eta_j = (\eta_j C - \eta_j^* S)$, $\eta_j = (\eta_j C - \eta_j S)$, $(\eta_j C - \eta_j S)$, $\eta_j = (\eta_j C - \eta_j S)$, and $\eta_j = (\eta_j C - \eta_j S)$. The Wigner function, expressed in terms of the corresponding complex phase-space variables $(\alpha_j, \beta_j, \beta_j^*) = (\alpha, \beta)$ is the eight-dimensional Fourier transform of $\chi_S(\eta, \xi)$. By a lengthy application of operator algebra and integral calculus, we could evaluate analytically in closed form both functions $\chi_S(\eta, \xi)$ and $W[\alpha, \beta]$. The exact result is

$$W[\alpha, \beta] = -\bar{W}_A[\alpha] \bar{W}_B[\beta][1 - |e^{i\Phi} \Delta_A(\alpha) + \Delta_B(\beta)|^2],$$

(2)

where $\Delta_A(\alpha) = 2^{-1/2}(\gamma_A + i \gamma_A^*)$ and $\Delta_B(\beta) = 2^{-1/2}(\gamma_B - i \gamma_B^*)$ are expressed in terms of the squeezed variables: $\gamma_A = (a_1 + a_2^*) e^{i\epsilon}$, $\gamma_A^* = (a_1 - a_2^*) e^{i\epsilon}$, $\gamma_B = (b_1 + b_2^*) e^{i\epsilon}$, and $\gamma_B^* = (b_1 - b_2^*) e^{i\epsilon}$. The Wigner functions $\bar{W}_A[\alpha] = (2/\pi)^2 \exp(-[|\gamma_A|^2 + |\gamma_A^*|^2])$ and $\bar{W}_B[\beta] = (2/\pi)^2 \exp(-[|\gamma_B|^2 + |\gamma_B^*|^2])$, definite positive over the four-dimensional spaces $\{\alpha\}$ and $\{\beta\}$, represent the effect of squeezed vacuum, i.e., emitted, respectively, by $\text{OPA}_A$ and $\text{OPA}_B$ in the absence of any injection. Inspection of Eq. (1) shows that precisely the quantum superposition character of the injected state $|\Psi_0\rangle$ determines the dynamical quantum superposition of the devices $\text{OPA}_A$ and $\text{OPA}_B$, the ones that otherwise act as uncoupled and independent objects. From another perspective, since the quasiprobability functions $\bar{W}_A[\alpha]$ and $\bar{W}_B[\beta]$ corresponding to the macrostates in the absence of quantum superposition are defined in two totally separated and independent spaces, their respective “distance” in the overall phase space of the system $\{\alpha, \beta\}$ can be thought of as “macroscopic,” as generally required by any standard $S$-cat dynamics in a two-dimensional phase space [2]. The link between the spaces $\{\alpha\}$ and $\{\beta\}$ is provided by the quantum superposition term in Eq. (2): $2 \text{Re}[e^{i\Phi} \Delta_A(\alpha) \Delta_B(\beta)]$. Note also in Eq. (2) and in Fig. 2 the absence of definite positivity of $W[\alpha, \beta]$ over its definition space, which assures the overall quantum character of our multiparticle, quantum-injected amplification scheme [18,20].

The striking quantum mechanical features of the system are also revealed by the 1st- and 2nd-order correlation functions of the OPA output fields. Before detection over the modes $k_j (j = 1,2)$ the fields are phase shifted by $\psi_j = (\psi_{j+} - \psi_{j-})$ by birefringent plates and filtered by $\pi$ analyzers with axes oriented at the angles: $45^\circ \pm \varphi_j$ (t.h.). Each $\pi$ analyzer may consist of the combination of a Fresnel-rhomb $\pi$ rotator, $R(\varphi)$, and of a polarizing beam splitter, PBS: cf. Fig. 1, inset. The field associated with the mode $k_j$ is detected at the space-time positions $x_j$ by two linear detectors.
injected OPA as a function of the squeezed variables: \(X = (\alpha + B^*)e^{-s}; Y = i(\beta - \alpha^*)e^{+s}\), for a parametric gain \(g = 2.5\) and \(\Delta \Phi = 0\).

\[D_{jk} = D_{jk}(\varphi)\text{, with } \varphi = \varphi + 90^\circ\text{. The 1st-order correlation functions } G_{jk}^{(1)}(x_j, x_j) = \langle \Psi_0 | \hat{N}_j(t) | \Psi_0 \rangle \text{ are ensemble averages of the number operators } \hat{N}_j(t) = \hat{c}_j^\dagger(t) \hat{c}_j(t) \text{ written in terms of the detected fields: } \hat{c}_j(t) = [\hat{c}_j, \hat{a}_j(t) + \hat{\beta}_j(t)], [\hat{c}_j, \hat{e}_j(t)] = \delta_{ij}, \hat{e}_j(t) = 2^{-1/2}(\cos \varphi_j + \sin \varphi_j) \exp(i \varphi_{ij}), \text{ and } \hat{e}_j(t) = 2^{-1/2} \times (\cos \varphi_j - \sin \varphi_j) \exp(i \varphi_{ij}).\]

By comparison with the corresponding (\(\varphi_j\) and \(\Delta \Psi\)-independent) averages over the input vacuum state: \(G_{1, vac}^{(1)} = G_{2, vac}^{(1)} = \bar{n}\), we obtain the signal-to-noise ratio to the S-cat detection: \(s/n = 2\), for \(\Delta \Phi = \varphi = 0\).

The above result immediately suggests a 1st-order \(\pi\)-interferometric method for S-cat detection on a single \(k_2\) beam, with visibility: \(V = (G_{max}^{(1)} - G_{min}^{(1)})/(G_{max}^{(1)} + G_{min}^{(1)}) \approx \frac{1}{3}\).

The Wigner function plotted in Fig. 2 refers to the output field detected by this method on the mode \(k_2\).

The 2nd-order functions \(G_{jk}^{(2)}(x_j, x_j; x_j, x_j) = \langle \Psi_0 | \hat{N}_j(t) \hat{N}_k(t) | \Psi_0 \rangle\) are also found: \(G_{11}^{(2)} = 2\bar{n}^2 + (\bar{n} + 1) \times [1 + \cos(2\varphi_j)cos(\Delta_1 \Phi)], \quad G_{22}^{(2)} = 2\bar{n}^2[1 + [1 + \cos(2\varphi_1) \times \cos(\Delta_2 \Phi)], \quad G_{22}^{(2)} = 2\bar{n}^2[1 + [1 + \cos(2\varphi_2) \times \cos(\Delta_2 \Phi)], \quad G_{12}^{(2)} = 2\bar{n}^2 + \bar{n} + 1/2[1 + \cos(2\varphi_1) \times \cos(\Delta_2 \Phi) + \bar{n} + 1/2(1 + \cos(2\varphi_2) \times \cos(\Delta_2 \Phi)] + \bar{n}(\bar{n} + 1)[1 + \cos(\Delta \Psi) \cos^2 \Delta \varphi + [1 + \cos(\Delta \Psi)] \sin^2 \Delta \varphi].\]

We may observe, e.g., for all \(\Delta \varphi = \varphi = 0\), that our system realizes the maximum quantum mechanical violation of the Cauchy-Schwarz inequality which generally holds in semiclassical field theory: \(\left[g_{12}^{(2)}(0)\right]^2 \leq g_{11}^{(2)}(0)g_{22}^{(2)}(0)\) being \(g_{ij}^{(2)}(0) = G_{ij}^{(2)}(0)[G_{i}^{(1)}(0)G_{j}^{(1)}(0)]^{-1}\) [18]. Furthermore, the given expression of \(G_{12}^{(2)}\) shows the effects of the multiparticle quantum nonseparability and Bell-type nonlocality, contributed by the terms proportional to \(\cos(2\varphi), \cos(\Delta \varphi)\).

The absence of decoherence within our ideal, nondissipative multiparticle system is due to its nature of nonlinearly driven excitation. As such, it is coupled with a continuously rephasing environment here provided by the parametric NL polarization. Similar situations are encountered in physics of the nonlinear dynamical systems, e.g., in nonlinear surface plasmon or surface exciton-polarization generation in solid state NL spectroscopy [21]. Of course, any single photon loss event, mainly contributed in our case by stray reflections, implies an elementary decoherence process. In our laboratory experiment two equal 1 mm thick, BBO crystals are excited by 0.8 ps pulses at \(\lambda_p = 400\) nm second-harmonic generated by a mode-locked Ti:Sa laser at a 76 MHz repetition rate with an average power = 0.3 W. The detection system, consisting of two linear photodetectors connected to an electronic correlator, is equal to the one shown by Fig. 1, inset, but for the absence of the birefringent plate. The initial phase is \(\Phi = 0\). All surfaces are treated by special antireflection coatings resonant at the working \(\lambda = 800\) nm with an overall transmittivity \(T = 99.60\%\). This figure implies the loss of a single photon every \(\approx 20\) pulses with the generation of \(\pi \approx 10\) per pulse. This would make our \(S\)-cat experiment quite feasible.

Within the domain of quantum computation our results suggest the nonlinear interaction among the information carrying particles as an efficient solution toward a large scale implementation of the new methods [22]. In summary, we have given the theory of a novel multiparticle system showing both quantum superposition and quantum entanglement features. This results from a smart interplay of the fundamental paradigms of modern quantum optics, i.e., quadrature squeezing, multiparticle-state entanglement, and quantum-nonseparability in parametric correlations. From a foundational perspective our method allows the first realization of several fundamental nonlocality and noncontextuality tests of quantum mechanics requiring a number of entangled particles larger than two [23].

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Note added.—A very interesting 3-crystal variant of the scheme of Fig. 1 consists of two independent SPDC devices feeding in a symmetrical fashion the OPA on both input modes \(k_j \ (j = 1, 2)\), by two distinct single-photon quantum-injection processes and within a double-conditional experiment. In this case, as we shall analyze.
in a forthcoming paper, the squeezed-vacuum noise can be greatly reduced and the S-cat visibility increased to $V > 1/2$.

[14] C. Monroe, D. Meekhof, B. King, and D. Wineland, Science 272, 1131 (1996); W. L. Munro and M. D. Reid, Quantum Opt. 6, 1 (1994). Owing to space limitations we postpone to another paper the analysis of a conceivable Bell-type multiparticle nonlocality test by our system.
[18] The realization of nonlinear universal logic gates is now being considered as a sensible solution toward experimental quantum computation: D. Di Vincenzo (private communication).