Witnessing Genuine Multiphoton Indistinguishability

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Bosonic interference is a fundamental physical phenomenon, and it is believed to lie at the heart of quantum computational advantage. It is thus necessary to develop practical tools to witness its presence, both for a reliable assessment of a quantum source and for fundamental investigations. Here we describe how linear interferometers can be used to unambiguously witness genuine n-boson indistinguishability. The amount of violation of the proposed witnesses bounds the degree of multiboson indistinguishability, for which we also provide a novel intuitive model using set theory. We experimentally implement this test to bound the degree of three-photon indistinguishability in states we prepare using parametric down-conversion. Our approach results in a convenient tool for practical photonic applications, and may inspire further fundamental advances based on the operational framework we adopt.

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Introduction.—Various notions of nonclassicality, such as steering and nonlocality, are believed to offer different effective resources for quantum information processing [1]. Similarly, quantum interference promises to enable several applications in quantum information, ranging from quantum communication [2] to quantum computation [3] and simulation [4]. A paradigmatic example of two-particle interference is the Hong-Ou-Mandel (HOM) effect in a balanced beam splitter [5], where output events with a single photon per mode are strictly suppressed. In the many-particle setting, the model known as Boson Sampling [6] results in computational advantage fuelled by quantum interference between n photons evolving in a multimode linear-optical network. This promising model has resulted in a number of experiments implementing these devices [7–19]. Multiphoton interference represents a crucial resource in several tasks such as quantum simulation, enabling the simulation of complex dynamics [20,21], or quantum metrology, being at the heart of quantum enhanced performances in the estimation of one or more physical parameters [22–24]. Following the recognition of the important role that quantum correlations play in quantum information protocols [25], various functionals have been proposed to quantify correlations [1,26] or to testify the nonseparability of specific quantum states [27]. Along the same lines, it is essential to witness and quantify multiparticle quantum interference as a resource to achieve quantum advantage. Such a test is necessary to exclude that observed features are due to interference between only a subset of all particles.

Different techniques to discriminate indistinguishable bosons from distinguishable ones have been proposed [28–35] and experimentally verified [36–43], allowing for the observation of genuine multiparticle quantum interference in specific scenarios. Moreover, it was recently suggested [44,45] that investigating intrinsically multiparticle interference beyond pairwise correlations is essential to obtain more complete knowledge of the full interference landscape. However, up to now no experimental procedure has been proposed to directly witness and quantify genuine n-photon indistinguishability.

In this Letter we propose a novel interferometric scheme capable of witnessing and quantifying genuine n-photon indistinguishability. We test a concrete implementation of the scheme in the form of a linear-optical bulk interferometer, which we use to characterize various indistinguishability regimes of three-photon states, reached by means of spectral filtering and temporal delays. Furthermore, we propose an intuitive set-theoretic model that captures quantitative features of genuine multiphoton indistinguishability. Our results show that it is possible to quantify true n-photon interference with relatively little experimental effort, using an interferometric setup suitable to miniaturization using integrated photonic circuits [13,46–49], and which can be scaled up efficiently using known schemes [39,50].

Genuine multiphoton indistinguishability.—Pure multiphoton states can be described by a set of state distributions $|\phi_i\rangle$, such that two photons i and j are perfectly identical if $\langle\phi_i|\phi_j\rangle = 1$ and distinguishable if $\langle\phi_i|\phi_j\rangle = 0$. These distributions carry information about actual spectra (i.e., frequency distributions), spatial modes, polarization, and in general any photonic degree of freedom that is inaccessible to the detectors.

The states we consider here are convex combinations of the state $\rho^{\parallel}$ of n perfectly indistinguishable photons, together with other states $\rho_i^{\perp}$ having at least two mutually orthogonal photons:

$$\rho = c_1 \rho^{\parallel} + \sum_i c_i \rho_i^{\perp}. \quad (1)$$

We then propose the following criterion: a state ρ [as in Eq. (1)] displays genuine n-photon indistinguishability if $c_1 > 0$ in all of its convex decompositions. The minimum value of c_1 gives a lower bound on the probability that all n photons will behave identically in any possible test.

Demonstrating genuine multiphoton indistinguishability entails more than simply showing some nonzero level of nonclassical two-photon interference for all photon pairs, as there are states (1) with this property for which $c_1 = 0$ (see the Supplemental Material [51] for an example). This phenomenon of nongenuine n-photon interference ($c_1 = 0$) is analogous to the case of multipartite entanglement and nonlocality [56], which may vanish even in states displaying nonclassical correlations across all bipartitions.

Next we describe a family of interferometers that can be used to experimentally demonstrate genuine n-photon indistinguishability. Our approach can be interpreted in an adversarial sense: if a source produces states with no support on the ideal n-photon state; i.e., if $c_1 = 0$ for some decomposition of the state, then it would fail the test no matter what mixtures it creates of states with mutually orthogonal photons.

Interferometers witnessing genuine n-photon indistinguishability.—Consider the family of interferometers depicted in Fig. 1, parametrized by the number of photons n. These interferometers consist of a single $(n-1)$-mode discrete quantum Fourier transform (QFT; in layer A), followed by a sequence of 50/50 beam splitters connecting each QFT output mode with a different single mode from the bottom half of the interferometer (layer B). Note that all beam splitters in layer B can be implemented in parallel. Intuitively, this interferometer is similar to a set of $n-1$ HOM tests in parallel, between the top input photon (which we call the reference photon r) and every other photon.

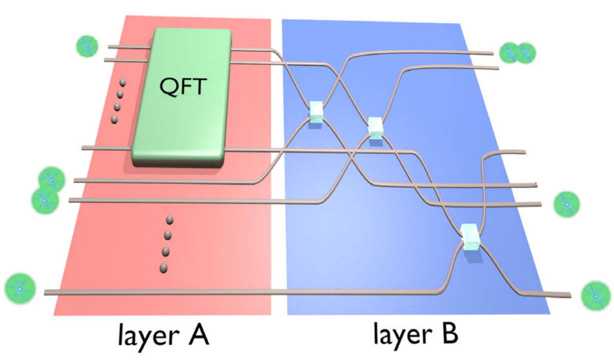

FIG. 1. Family of interferometers capable of witnessing genuine n-photon indistinguishability. QFT is the quantum Fourier transform acting on $(n-1)$ modes. Each output of the QFT is connected to a 50/50 beam splitter. As discussed in the main text, no mixture of states with less than n indistinguishable photons can result in a probability of bunching $p_b > [(2n-3)/(2n-2)]$.

Now let p_b be the overall probability of bunching in this interferometer and $p_b^{\mathrm{HOM}}(r,j)$ be the probability of bunching in a two-photon HOM test between photons r and j. The reference photon r interferes with each of the bottom $(n-1)$ input photons with uniform probability $1/(n-1)$; so p_b will be the average of $p_b^{\mathrm{HOM}}(r,j)$ for all $n-1$ photons j, namely $p_b = (n-1)^{-1}\sum_{j=1}^{n-1} p_b^{\mathrm{HOM}}(r,j)$. The HOM effect predicts a two-photon probability of bunching $p_b^{\mathrm{HOM}} = 1/2$ (1) for distinguishable (indistinguishable) photon pairs. Hence, if all photons are perfectly indistinguishable we have $p_b = 1$.

Suppose now there are exactly two mutually orthogonal photons. Two different scenarios may arise. If a single photon k is distinguishable from the reference photon r, p_b will decrease to $p_b^{\perp} = 1 - [1/2(n-1)] = [(2n-3)/(2n-2)]$. A second scenario is obtained when photons $(k,l) \neq r$ are mutually orthogonal, that is, the two mutually distinguishable photons do not include the reference one. In this case, the probability of projecting state $|r\rangle$ onto the subspace spanned by $\{|k\rangle, |l\rangle\}$ must satisfy

$$\langle r|(|k\rangle\langle k| + |l\rangle\langle l|)|r\rangle = |\langle r|k\rangle|^2 + |\langle r|l\rangle|^2 \leq 1. \quad (2)$$

Now recall that, for each photon pair, $p_b^{\mathrm{HOM}}(r,j)$ can be given in terms of the two-photon overlap as $p_b^{\mathrm{HOM}}(r,j) = (1 + |\langle r|j\rangle|^2)/2$ [57]. Equation (2) then implies that p_b will again decrease to at most $p_b^{\perp} = [(2n-3)/(2n-2)]$.

By convexity, it follows any mixture of states having at least two mutually orthogonal photons must have a bunching probability satisfying

$$p_b \leq p_b^{\perp} \equiv \frac{2n-3}{2n-2}, \quad (3)$$

where we identify the threshold value $p_b^{\perp}$ for the bunching probability p_b. By picking different reference photons, we actually obtain n different inequalities of form [Eq. (3)]; a violation of any of them serves as a witness of genuine n-photon indistinguishability, that is, guarantees that $c_1 > 0$ in Eq. (1). For $n > 3$ photons it is possible to design interferometers implementing witnesses different from those described by Eq. (3), see Ref. [58].

The amount of violation of any of our witnesses [Eq. (3)] can be translated into bounds for the degree of genuine n-photon indistinguishability. To that end, assume first the worst-case scenario with a state of the type

$$\rho = c_1\rho^{\parallel} + (1-c_1)\rho^{\perp}, \quad (4)$$

where $\rho^{\perp}$ is some state that *saturates* the bound of Eq. (3). The probability of bunching p_b is then

$$p_b = c_1 + (1 - c_1)p_b^{\perp}. \quad (5)$$

This gives a lower bound on the value of c_1. An analogous argument using the best-case scenario where photons are either all identical or all distinguishable leads to an upper bound. These together give the following range of values for c_1:

$$\frac{p_b - p_b^{\perp}}{1 - p_b^{\perp}} \leq c_1 \leq 2p_b - 1, \quad (6)$$

Thus, the amount of violation $p_b - p_b^{\perp}$ quantifies the amount of genuine n-photon indistinguishability in states of the form we consider. Note that c_1 gives us a lower bound for the probability of bunching of all photon pairs, including those which were not directly measured. Note also that asymptotically (in n) we have $p_b^{\perp} = 1 - \mathrm{O}(1/n)$. This means that, although our test becomes more stringent as n increases, it is also efficient, in the sense that we expect to be able to resolve the gap between p_b and $p_b^{\perp}$ with a number of events that scales like a polynomial in n.

Interpretation in terms of set theory.—We now present a simple and intriguing interpretation of coefficient c_1 in Eq. (1) in terms of set theory. Let us associate a set with each single-photon state, and associate the size of the pairwise intersection of two sets $|A \cap B|$ with $|\langle A|B\rangle|^2$, i.e., the probability that a photon in state $|A\rangle$ passes for a photon in state $|B\rangle$ [59]. The sets used to model single-photon states then have size one, as $|A| = |A \cap A| = |\langle A|A\rangle|^2 = 1$ for any A. Now let us take n sets A_i $(i = 1, 2, \ldots, n)$ to represent the states of n photons, and consider the $(n-1)$ pairwise intersections of a single reference set (call it A_r) with all others. In the Supplemental Material [51] we prove the following theorem:

Theorem 1.—Consider n sets A_i $(i = 1, 2, \ldots, n)$, each of size 1. For any $r \in \{1, 2, \ldots, n\}$, define

$$\mathcal{I}_r = \sum_{j|j\neq r} |A_r \cap A_j|. \quad (7)$$

Then, for all $r \in \{1, 2, \ldots, n\}$, the size of the common intersection of all sets satisfies

$$2 - n + \mathcal{I}_r \leq \left| \bigcap_{j=1}^{n} A_j \right| \leq \frac{1}{n-1}\mathcal{I}_r. \quad (8)$$

Let us now interpret this theorem in the light of our proposed association between pairwise set intersections and two-photon overlaps. As we remarked previously, p_b can be written in terms of two-photon wave function overlaps [57,59]. If we use this to rewrite the bounds of Eq. (8) in terms pairwise photon overlaps, we find that $|\cap_{j=1}^{n} A_j|$ satisfies exactly the same bounds as c_1 in Eq. (6). This suggests that we identify these two quantities, i.e., the size of the intersection $|\cap_{j=1}^{n} A_j|$ on the set-theoretic side of the argument with the value of c_1, characterizing genuine n-photon indistinguishability. Our intuitive, operational interpretation for pairwise set intersections leads us to a nontrivial but consistent operational interpretation of $|\cap_{j=1}^{n} A_j|$ as well.

Experimental implementation.—We now describe the experimental scheme designed to witness and quantify genuine three-photon indistinguishability. To generate the three-photon states we used fourfold coincidence events from a parametric down-conversion (PDC) source. One of the four photons acts as a trigger, heralding the other three that are injected into the interferometer after proper synchronization. In the Supplemental Material [51], we show that the obtained input state is well approximated by a state of the form of Eq. (1). The interferometer is made up of two layers: layer A consists of a single-mode in-fiber balanced beam splitter, while layer B comprises two beam splitters, each connected to layer A via one input port. A schematic representation of the setup is shown in Fig. 2, with further information in the Supplemental Material [51].

We measured the probability of bunching events p_b for all five extremal distinguishability states of three photons, i.e., states in which every pair of photons is either perfectly

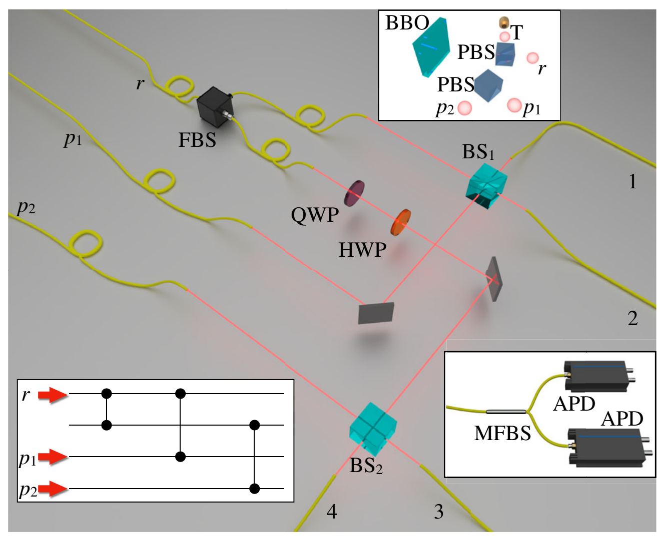

FIG. 2. Scheme of the experimental apparatus to test for genuine three-photon indistinguishability. Three-photon states are generated by a parametric down-conversion source operating in the double-pair emission regime (top right inset). Photons are made indistinguishable in polarization, spectrum, and temporal delay, which was employed to control the degree of distinguishability. One photon (r) is split via a fiber beam splitter, and each output path interferes with one of the other photons in a second layer of beam splitters (see bottom left inset for a scheme of the circuit). Each output mode is measured by a detection block (bottom right inset) able to discriminate one- and two-photon contributions. Legend: BBO—Beta Barium Borate crystal, HWP—half-wave plate, PBS—polarizing beam splitter, FBS—fiber beam splitter, QWP—quarter-wave plate, BS—beam splitter, MFBS—multimode fiber beam splitter, APD—avalanche photodiode.

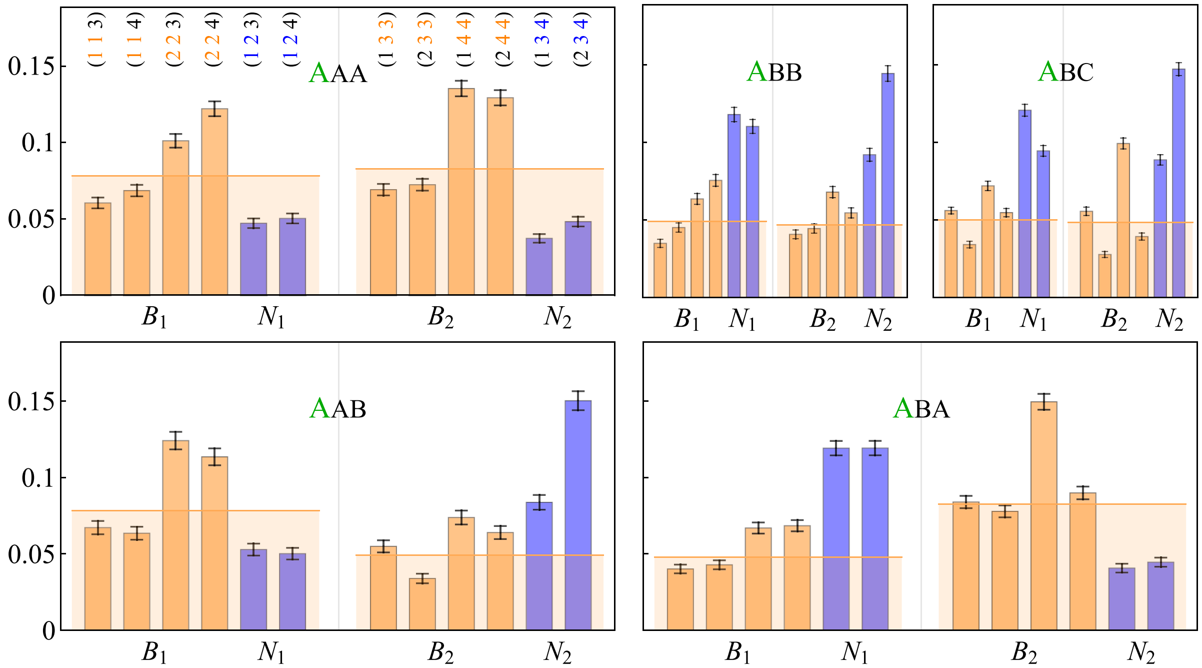

FIG. 3. Histograms of observed probability distributions over outputs for different distinguishability regimes (top of each panel), obtained via relative delays between input photons. In all histograms events are ordered first by output beam splitter (BS_1, modes 1 and 2: left; BS_2, modes 3 and 4: right) and then by type of event (bunching: orange B; no bunching: blue N). Bars are sorted following the mode assignment list shown in panel AAA, with mode numbers colored according to type of event. For each BS, orange shaded regions indicate the bunching probability $p_b^{(\mathrm{BS})}$ (rescaled by 0.1 to fit the plot). p_b for BS_1 and BS_2 helps identify the distinguishability between their photons and the reference one, i.e., $p_b^{(\mathrm{BS}_1)} \sim p_b^{(\mathrm{BS}_2)}$ for AAA, ABB, ABC, $p_b^{(\mathrm{BS}_1)} > p_b^{(\mathrm{BS}_2)}$ for AAB, $p_b^{(\mathrm{BS}_1)} < p_b^{(\mathrm{BS}_2)}$ for ABA. Data are corrected by excluding events with $n > 3$ photons due to PDC multipair emission (see the Supplemental Material [51]).

distinguishable or indistinguishable. This was done by introducing appropriate temporal delays and collecting up to $N \geq 2000$ events per state. In Fig. 3 we report the full set of output measurements for all extremal states of distinguishability, where different letters A, B, and C describe mutually orthogonal state functions. In Fig. 4 we report the measured values of p_b for all extremal states. For the fully indistinguishable state, our measured witness was $p_b = 0.805 \pm 0.012$, which according to our bounds from Eq. (3) guarantees that c_1 must lie in the interval $0.22 \pm 0.04 \leq c_1 \leq 0.61 \pm 0.02$, thus confirming genuine three-photon indistinguishability. As discussed earlier, this lower bound of c_1 is also a lower bound for the (unmeasured) probability of bunching of the second and third photons.

In fact, from the experimental data in Fig. 3, we can obtain a tighter upper bound for c_1 by examining the bunching probabilities for each individual beam splitter in layer B. This additional analysis can be performed in a general scenario, provided that the initial assumption [Eq. (1)] on the state holds. It is easy to check that the probability of bunching of state [Eq. (1)] for beam splitter BS_1 satisfies $2p_b^{\mathrm{HOM}}(\mathrm{BS}_1) - 1 = c_1 + c_2$, where c_2 is associated with state ρ_{AAB}. Since all $c_j \geq 0$, $c_1 \leq 2p_b^{\mathrm{HOM}}(\mathrm{BS}_1) - 1$. An analogous reasoning for BS_2

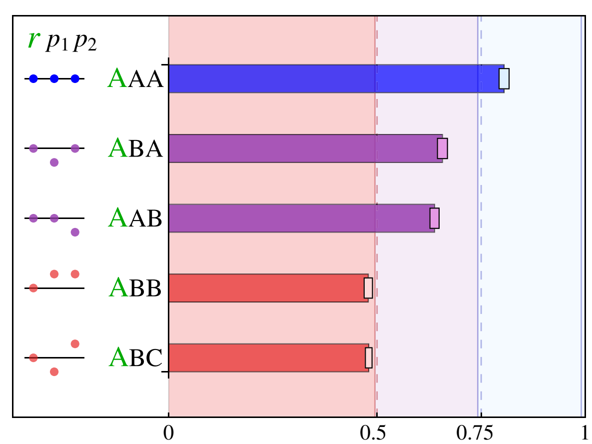

FIG. 4. Measured bunching probabilities p_b for different regimes of distinguishability, i.e., when the reference photon r is synchronized temporally with all the others (p_1, p_2) (blue), only one (purple), or none (red) (shown on the right). Red bars exhibit similar behaviors since the test does not discriminate the two cases. Purple bars are slightly different due to the overlap between photons belonging to the same or different pair. The dashed blue (red) vertical line is the threshold for indistinguishable (distinguishable) particles. Solid lines are the thresholds that account for the nonideal unitary implementation (see the Supplemental Material [51]).

gives $c_1 \leq 2p_b^{\mathrm{HOM}}(\mathrm{BS}_2) - 1$. We can read out p_b^{HOM} for BS_1 and BS_2 in Fig. 3(a); the smaller of the two is $p_b^{\mathrm{HOM}}(\mathrm{BS}_1) = 0.78 \pm 0.01$, which results in the tighter bound $c_1 \leq 0.57 \pm 0.02$. Incidentally, these tighter upper bounds for c_1 also have a precise counterpart in our proposed set-theoretical model (see the Supplemental Material [51]). As expected, for all other scenarios the threshold $p_b^{\perp}$ for genuine three-photon indistinguishability is not violated (see Fig. 4).

In order to investigate in more detail how p_b depends on degrees of freedom other than time of arrival, we reduced the spectral indistinguishability by removing the interferential filters at the source, while keeping the photons temporally synchronized. In particular, the absence of filters leads to an increase in the amount of spectral correlations within photons belonging to the same PDC pair, thus reducing the purity of the input photon states (see the Supplemental Material [51] for more details). With this setup, our measured probability of bunching was $p_b = 0.66 \pm 0.01$, well below the minimum threshold necessary to witness genuine multiphoton indistinguishability.

Discussion.—Multiparticle interference is a key resource for metrology and optical quantum computation. In this Letter, we tackle the problem of testing whether a photon source prepares states with genuine n-photon indistinguishability, as opposed to convex combinations of states in which effectively only a smaller number of photons interfere. Our scheme is simple, scalable, and, when applied to single-photon states, can be implemented using only elementary linear-optical elements. We experimentally demonstrate the protocol by characterizing three-photon states in all extremal regimes of distinguishability. Furthermore, we propose a model for multiboson indistinguishability in terms of set theory, a connection that we expect will foster further theoretical investigations. Ultimately, its relatively low technological requirements make the implementation of our test suitable for miniaturization using integrated photonic circuits, which encourages its adoption in the characterization of large-scale multiphoton applications.

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