Experimental Generation of Robust Entanglement from Classical Correlations via Local Dissipation

Adeline Orieux,1,* Mario A. Ciarpani,1 Paolo Mataloni,1,2 Dagmar Bruß,3 Matteo Rossi,4 and Chiara Macchiavello4

1Dipartimento di Fisica, Sapienza Università di Roma, Piazzale Aldo Moro, 5, I-00185 Roma, Italy
2Istituto Nazionale di Ottica, Consiglio Nazionale delle Ricerche (INO-CNR), Largo Enrico Fermi, 6, I-50125 Firenze, Italy
3Institut für Theoretische Physik III, Heinrich-Heine-Universität Düsseldorf, D-40225 Düsseldorf, Germany
4Dipartimento di Fisica and INFN-Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy

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We experimentally show how classical correlations can be turned into quantum entanglement, via the presence of dissipation and the action of a CNOT gate. We first implement a simple two-qubit protocol in which entanglement production is not possible in the absence of such kind of noise, while it arises with its introduction, and is proportional to its amount. We then perform a more elaborate four-qubit experiment, by employing two hyperentangled photons initially carrying only classical correlations. We demonstrate a scheme where the entanglement is generated via local dissipation, with the advantage of being robust against local unitaries performed by an adversary.

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Introduction.—Entanglement is the most precious resource in quantum information processing [1,2]. However, what is most precious is often also most fragile. Indeed, a profound opponent of entanglement is noise: decoherence and dissipation both typically decrease entanglement, unless they are tailored specifically in a collective way, such as, for example, in correlated noisy channels [3,4], or in engineered dissipation of coupled systems [5] or in the presence of tunable noise [6]. Hence, in general, except for peculiar experimental conditions, noise represents a strong obstacle for entanglement.

In this work we introduce a scenario where, with fixed given resources of input states and gates, no entanglement can be produced, unless one switches on any local non-unital noise, such as dissipation. Note that not all types of noise would work: for instance dephasing, which is unital, would not be helpful here. In this setting entanglement is zero without noise, and it grows with the amount of the introduced local noise. An important aspect of this quantum effect is the choice of the dimension of the underlying Hilbert space: as long as only two qubits are present, we will show that an adversary (Eve) is always able to prevent the production of entanglement by applying a suitable local rotation. We will also show that, however, by embedding the protocol into a higher-dimensional Hilbert space, concretely by using four qubits, the creation of entanglement via local noise can be made robust against any possible unitary action of Eve. Interestingly, in order to achieve this robustness it is sufficient to have input states that exhibit just classical correlations. Experimentally, the different dimensional settings of two and four qubits are implemented using the path and polarization degrees of freedom of photons and the fragility of the effect in low dimensions is demonstrated.

The reason behind this effect is an intricate relationship between the concepts of separability, quantum correlations, and entanglement: a necessary ingredient for the production of entanglement in our scenario is the generation of nonvanishing off-diagonal terms (“quantum coherences”) in the initial density matrix, which may arise via a local nonunital channel. However, the presence of quantum coherences is not sufficient for robustness of the protocol, in the sense explained above. A sufficient ingredient for robustness is the presence of classical correlations within the initial state: they are turned into correlations of a quantum nature (often referred to as “discord”) via a local nonunital channel, which still keeps the quantum state separable. Finally, the quantum correlations are activated into nonvanishing entanglement with a CNOT gate.

Generation of quantum correlations from classical correlations by local noise was theoretically investigated in Ref. [7], see also Refs. [8,9], and experimentally demonstrated with trapped ions [10], while entanglement activation from discord was theoretically proposed in Ref. [11] (for a different interpretation see Ref. [12]), and experimentally demonstrated in Ref. [13]. In the scenario that we propose here these two effects are combined, resulting in a scheme where we are able to generate entanglement in a robust way, starting from classically correlated states at the input and switching on a local noisy device, acting on a single qubit.

Theory.—Let us briefly describe the fixed resources for the two-qubit scheme [see Fig. 1(a)]: we are given two qubits in a product state, where a qubit is in the maximally mixed state \( \rho_A = \frac{1}{2} \) and an ancilla is in the state \( \rho_C = |0}\rangle\langle 0| \). We are also given a CNOT gate with \( A(C) \) the control (target) qubit and we are allowed to perform any possible local unitary operation on \( A \) and \( C \). The goal is to create
entanglement in the total state $\rho_{out}$ of qubits $A$ and $C$. It is
clear that with only these fixed resources at our disposal it is
not possible to create entanglement in the bipartite system
considered here.

If in addition we are allowed to use a local noisy device
acting only on qubit $A$, such as an amplitude damping
channel (see later for details), as reported in Fig. 1(a), the
situation changes. Even if at first glance we might expect
that our ability to generate entanglement could not be
improved by adding such a resource, since the operation is
local and noisy channels are usually regarded as detrimen-
tal for quantum features, it turns out that if this extra
resource is nonunit (i.e., if it does not preserve the
identity) it allows us to produce an entangled output state
$\rho_{out}$. Indeed, if $\rho_A$ is not the identity anymore, off-diagonal
elements can appear in it after the application of a unitary
$U_A$, resulting in entanglement in $\rho_{out}$ after the CNOT
gate. One can also show that the generation of entanglement in
this protocol can always be prevented by a suitable local
rotation (that makes $\rho_A$ diagonal again) performed by some
adversary after the action of the CNOT gate, as shown in
Fig. 1(b). Note that in this two-qubit scenario, no classical
correlations are present in the initial state; thus, no quantum
correlations are created by the nonunit channel, which is
why the entanglement activation is not robust [13].

We will now present a four-qubit scheme where the
entanglement will not be vulnerable anymore to local
rotations. We begin with a two-qubit state $\rho'_{AB}$, which is
diagonal in the computational basis (which is a factorized
basis) and therefore exhibits at most classical correlations,
and two ancilla qubits $C$ and $D$, both initialized in the state
$|0\rangle\langle 0|$. We can then apply two CNOT gates operating on
qubits $AC$ and $BD$ [see Fig. 1(c) with the noisy device
switched off]. Under these conditions the output state will
always be a separable state [11].

We now introduce a local nonunit noisy device, acting
on qubit $A$, given by the block of gates $H\Lambda_pH$, where $H$
is the Hadamard transform and $\Lambda_p$ is an amplitude damping
channel, where the noise parameter $\eta$ can be tuned by a
knob. Its action is explicitly described by $\Lambda_\eta(p) =
\sum_{j=1}^2 A_j p A_j^\dagger$, with the Kraus operators $A_1 = |0\rangle\langle 0| +
\sqrt{1-\eta}|1\rangle\langle 1|$, $A_2 = \sqrt{\eta}|0\rangle\langle 1|$ with $\eta \in [0,1]$, where $\eta = 0$
corresponds to the noiseless case. Note that this channel is
a model for dissipation.

We show in the following that by adding only this extra
local noisy resource entanglement is switched on at the
output of the circuit in the bipartition $AB|CD$, starting from
input states which are at most classically correlated. Such
states read

$$\rho_{in} = [p|00\rangle\langle 00| + (1-p)|11\rangle\langle 11|]_{AB} \otimes |00\rangle\langle 00|_{CD}. \tag{1}$$

They are transformed by the circuit of Fig. 1(c) into block-
diagonal output states:

$$\rho_{out} = p\rho^+_{AC} \otimes |00\rangle\langle 00|_{BD} + (1-p)\rho^-_{AC} \otimes |11\rangle\langle 11|_{BD}. \tag{2}$$

Here we have defined

$$\rho^\pm = \frac{1}{2}[(1 \pm \sqrt{\bar{\eta}})|00\rangle\langle 00| + (1 \pm \sqrt{\bar{\eta}})|11\rangle\langle 11|
+ \eta(|00\rangle\langle 11| + |11\rangle\langle 00|)] \tag{3}$$

with $\bar{\eta} = 1 - \eta$. In order to quantify the entanglement of
$\rho_{out}$ with respect to the splitting $AB|CD$, we apply the
partial transposition $T_{AB}$ with respect to $AB$, and study
the negative eigenvalues of $\rho_{out}^{T_{AB}}$. For $\eta \neq 0$ there
are two negative eigenvalues, namely, $\lambda_1 = -p(\eta/2)$ and
$\lambda_2 = -(1-p)\eta/2$. We quantify the amount of entangle-
ment in terms of the negativity, defined as $N(\rho) = \sum_1^2 |\lambda_1|$. Thus,
for any state $\rho_{in}$ as in Eq. (1) the negativity of the
output state as in Eq. (2) is given by

$$N(\rho_{out}) = \frac{\eta}{2}. \tag{4}$$

Therefore, the entanglement increases linearly by increas-
ing the noise parameter. Note that the negativity in Eq. (4)
is independent of $p$; however, the amount of classical
correlations in the initial state is not. Indeed, in this
four-qubit scheme, an adversary using two local unitaries
$V_A$ and $V_B$ just before the CNOT gates cannot prevent the
presence of entanglement in $\rho_{out}$ as long as classical
correlations are present in the initial state ($p \neq 0, 1$) and
as long as they are transformed into discord by the
nonunit channel ($\eta \neq 0, 1$).—Theoretical details of the
different robustness properties of both protocols are
reported in the Supplemental Material [14].

FIG. 1 (color online). (a) Two-qubit scheme: the entangled
output state $\rho_{out}$ is generated by starting from the initial state
$\frac{1}{2} \otimes |00\rangle\langle 00|$, and using local unitaries $U_A$ and $U_C$ and a CNOT
gate. A local nonunit noisy element can be switched on by turning
the knob. (b) The same protocol in the presence of an adversary
player using a local unitary $V_A$ to try and prevent the generation
of entanglement. (c) Four-qubit scheme: here $\rho_{out}$ is generated by starting from the initial state
$\frac{1}{2} \otimes |00\rangle\langle 00|$, and using local unitaries $U_A$ and $U_C$ and a CNOT gate.

The theoretical details of the


PRL 115, 160503 (2015) PHYSICAL REVIEW LETTERS week ending
16 OCTOBER 2015

160503-2
with the path qubit in the state $\rho^A_0 = \frac{1}{2} (|0\rangle_0 |0\rangle_A + |1\rangle_1 |1\rangle_A)$, and the polarization qubit was in the state $|H\rangle_C \langle H|$, where $H(V)$ designs the horizontal (vertical) polarization and corresponds to the state $0 (1)$ of the computational basis.

Both Hadamard gates $H$ were realized with balanced beam splitters (BSs). The amplitude damping channel $\Lambda_{\eta}$ was achieved by a combination of two BSs (BS$_1$ and BS$_2$) and three attenuation filters allowing us to transfer, in a tunable way, a portion of mode $|1\rangle$ onto mode $|0\rangle$ of the path qubit $A$. The relationship between the noise parameter $\eta$ and the intensity transmission coefficients $\alpha_0$, $\alpha_1$, and $\alpha_2$ of the attenuators is (see calculation in the Supplemental Material [14]): $\alpha_0 = R^2/T = 31\%$, $\alpha_1 = \eta$ and $\alpha_2 = \alpha_0(1-\eta) = R^2/T(1-\eta)$, with $T = 57.5\%$ ($R = 42.5\%$) the measured intensity transmission (reflection) coefficient of the BSs. For each setting of $\eta$, the transmission of each attenuator was tuned to these values by adjusting their position relative to the beam. The CNOT gate was implemented by inserting a half-wave plate (HWP) at $45^\circ$ in the path mode corresponding to $|1\rangle_A$. A HWP at $0^\circ$ was also inserted in the other path mode to maintain the same optical length for both modes.

Finally, the entanglement in the output state of $AC$ was estimated by a standard tomographic reconstruction measurement of the two-qubit state [17] from which we recovered the negativity $N$: in Fig. 3(a) we report its measured value obtained for different values of $\eta$ (blue squares), together with its theoretical value given by Eq. (4) (dark line). The dashed blue line corresponds to the expected theoretical value, taking into account the finite interference visibility achieved in the interferometers (see Supplemental Material [14]). The figure shows a good agreement between the measured and theoretical behaviors, demonstrating that the amount of entanglement created increases with the noise. The deviation from the theoretical behavior, in the experimental value of $N$ for $\eta = 0.5$ in Fig. 3(a), is caused by an unexpected discontinuity in the attenuator $\alpha_1$.

As expected, the creation of entanglement can be prevented by Eve for any amount of noise. With the protocol of Fig. 1(b) the best strategy for Eve is to perform another Hadamard gate $(V_A = H_{Eve})$ to cancel the action of the second Hadamard. Indeed, in this way, even though the noise is nonunitary, the density matrix of qubit $A$ remains diagonal (in the computational basis), so no entanglement is generated. In order to simulate Eve’s attack experimentally, we simply removed BS$_3$ from our setup (see Fig. 2). As expected, the negativity measured in this case is always vanishing [see Fig. 3(a), red crosses].

**Experiment: two-qubit protocol.**—The two-qubit protocol was implemented with the optical setup shown in Fig. 2(a) where qubits $A$ and $C$ (encoded, respectively, in the path and the polarization degrees of freedom (DOFs) of the signal photon) were used. Note that the actual setup consisted in a folded version (a Sagnac configuration) which guarantees an intrinsic phase stability for the path qubits. Horizontally polarized degenerate photon pairs [see Fig. 2(b)] are emitted by spontaneous parametric down-conversion (SPDC) on the surface of a cone, two photons of the same pair being diametrically opposite [16]. A 4-hole mask intercepted the emission cone, selecting the two pairs of modes $|0\rangle_0 |0\rangle_{\text{signal/idler}}$ or $|1\rangle_1 |1\rangle_{\text{signal/idler}}$. In this configuration, the source naturally generates a path-entangled Bell state. In order to ensure that only classical correlations are present in the input state for the experiments at hand, the phase coherence between these two possible pairs is destroyed by having an optical path difference between any two path modes larger than the photon’s coherence time. Thus, the input state was prepared

**Experiment: four-qubit protocol.**—We then implemented the four-qubit scheme shown in Fig. 2(a): two extra qubits $B$ and $D$, encoded respectively in the path and the polarization DOFs of the idler photon, were added. The prepared input state was $\rho^A_B = p |0\rangle_0 |0\rangle_{AB} + (1-p) |1\rangle_1 |1\rangle_{AB}$, with $p = 0.5$ for the path qubits, and
FIG. 3 (color online). Experimental results. (a) Two-qubit protocol: negativity $N(\rho_{\text{out}})$ vs the noise parameter $\eta$. (b) Four-qubit protocol: expectation value $\langle W' \rangle = -\langle W \rangle$ of the entanglement witness applied to $\rho_{\text{out}}$ as a function of $\eta$. The dashed lines correspond to a theoretical calculation that takes into account the experimental imperfections and the measured value of $\rho$ in the input state (see the Supplemental Material [14]). Our results are in agreement with Eq. (4): while there is no entanglement when local noise is absent ($\eta = 0$), it is generated as the noise increases. The experimental demonstration of the robustness of the protocol would require the actual measurement of a nonvanishing entanglement measure for every possible unitary operation that Eve can apply on qubits $A$ and $B$. As this is not practically feasible in our setup, instead we numerically computed the minimal amount of entanglement that is generated by the protocol when Eve adopts the local rotations on qubits $A$ and $B$ that best reduce this entanglement. This result [see red dots in Fig. 3(b)] is obtained, for each value of $\eta$, by a minimization of the negativity over all possible unitaries $V_A$ and $V_B$. Notice that when the noise level is low, i.e., $\eta$ is close to zero, Eve’s best attack cannot reduce much the amount of created entanglement. Only in the extreme cases $\eta = 0, 1$ she can totally prevent the creation of an entangled output state.

**Conclusion.**—We have shown that quantum entanglement can be switched on by the help of a local noisy device, and increases with the amount of noise introduced on purpose. Using a single photon, we experimentally created entanglement between its path and polarization degrees of freedom, demonstrating the idea in terms of a simple two-qubit protocol. A more sophisticated four-qubit scenario had the advantage over the previous one of being robust against local unitaries performed by an adversary player thanks to the initial presence of classical correlations. Our experiments have thus implemented some subtle features of the quantum world: while, obviously, no local action can produce correlations, a local nonunital quantum channel can turn classical correlations into quantum correlations [7]. The local amplitude damping channel (as a model for dissipation) that we realized performed this task, while preserving separability of the state. The quantum correlations were then activated into entanglement [11], by non-local gates. We emphasize that without this dissipative channel no entanglement could have been produced, and that its robustness crucially depends on the underlying Hilbert space dimension.

Measurement settings $\{\sigma_x, \sigma_y, \sigma_z, \sigma_x, \sigma_y, \sigma_z, \sigma_x, \sigma_y, \sigma_z\}$ (in lexicographic order $ABCD$) that reduced to 32 projective measurements (see Eq. S15 in the Supplemental Material [14]). Pauli operations on qubits $A$ and $B$ were performed by a standard polarization analysis setup, while qubits $C$ and $D$ were analyzed by selecting proper phase values between BS$_3$ and BS$_4$ [see Fig. 2(a)].

In Fig. 3(b) we show the measured expectation value $\langle W' \rangle = -\langle W \rangle$ of the witness as a function of $\eta$ (blue squares). This quantity provides a lower bound for the negativity of $\rho_{\text{out}}$: $N(\rho_{\text{out}}) \geq \langle W' \rangle$ [19]. The blue dashed line corresponds to the theoretical evolution computed with the experimental imperfections and the measured value of $\rho$ in the input state (see the Supplemental Material [14]).

$|H\rangle\langle H|_{CD}$ for both polarization qubits. All the gates were implemented as in the two-qubit setup. The additional CNOT gate between qubits $B$ and $D$ was implemented by a HWP at $45^\circ$ in the path mode $|1\rangle_B$.

We checked for entanglement in the splitting $AB|CD$ by using the entanglement witness

$$W = \frac{1}{8}(1 \otimes 1 - \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z)_{AC}$$

$$\otimes (1 \otimes 1 + \sigma_z \otimes \sigma_z)_{BD},$$

(5)

where $\sigma_x$, $\sigma_y$, and $\sigma_z$ are the Pauli matrices. This witness certifies the presence of bipartite entanglement in the final state $\rho_{\text{out}}$, whenever $\langle W \rangle < 0$ [18]. It was constructed as follows: the eigenvectors of $\rho_{\text{out}}^T$, see Eq. (2), are $|\lambda_1^+\rangle = |\Psi^-\rangle_{AC} \otimes |00\rangle_{BD}$ and $|\lambda_2^+\rangle = |\Psi^-\rangle_{AC} \otimes |11\rangle_{BD}$, with $|\Psi^-\rangle = 1/\sqrt{2}(|01\rangle - |10\rangle)$; therefore, an entanglement witness with respect to the splitting $AB|CD$ can be defined as $W = (|\lambda_1^+\rangle \langle \lambda_1^+| + |\lambda_2^+\rangle \langle \lambda_2^+|)^T_{x,y}$, which, if decomposed in terms of local operators, takes the form Eq. (5). Notice that the experimental detection of $W$ requires only the three
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*Present address: Télécom ParisTech, CNRS-LTCI, 46 rue Barrault, F-75634 Paris Cedex 13, France. adeline.orieux@gmail.com


