

Phase Estimation via Quantum Interferometry for Noisy Detectors

Nicolò Spagnolo,^{1,2} Chiara Vitelli,¹ Vito Giovanni Lucivero,¹ Vittorio Giovannetti,³
Lorenzo Maccone,⁴ and Fabio Sciarrino^{1,*}

¹*Dipartimento di Fisica, Sapienza Università di Roma, piazzale Aldo Moro 5, I-00185 Roma, Italy*

²*Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, piazzale Aldo Moro 5, I-00185 Roma, Italy*

³*NEST, Scuola Normale Superiore and Istituto Nanoscienze-CNR, Piazza dei Cavalieri 7, I-56126 Pisa, Italy*

⁴*Dip. Fisica "A. Volta," INFN Sez. Pavia, Università di Pavia, via Bassi 6, I-27100 Pavia, Italy*

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The sensitivity in optical interferometry is strongly affected by losses during the signal propagation or at the detection stage. The optimal quantum states of the probing signals in the presence of loss were recently found. However, in many cases of practical interest, their associated accuracy is worse than the one obtainable without employing quantum resources (e.g., entanglement and squeezing) but neglecting the detector's loss. Here, we detail an experiment that can reach the latter even in the presence of imperfect detectors: it employs a phase-sensitive amplification of the signals after the phase sensing, before the detection. We experimentally demonstrated the feasibility of a phase estimation experiment able to reach its optimal working regime. Since our method uses coherent states as input signals, it is a practical technique that can be used for high-sensitivity interferometry and, in contrast to the optimal strategies, does not require one to have an exact characterization of the loss beforehand.

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From the investigation of fragile biological samples, such as tissues [1] or blood proteins in aqueous buffer solution [2], to gravitational wave measurements [3,4], the estimation of an optical phase ϕ through interferometric experiments is an ubiquitous technique. For each input state of the probe, the maximum accuracy of the process, optimized over all possible measurement strategies, is provided by the quantum Fisher information I_ϕ^q through the quantum Cramér-Rao (QCR) bound [5,6]. The QCR sets an asymptotically achievable lower bound on the mean square error of the estimation $\delta\phi \geq (MI_\phi^q)^{-1/2}$, where M is the number of repeated experiments. In the absence of noise and when no quantum effects (like entanglement or squeezing) are exploited in the probe preparation, the QCR bound scales as the inverse of the square root of the mean photon number, the standard quantum limit (SQL). Better performances are known to be achievable when using entangled input signals [7–12]. However, all experiments up to now have been performed using postselection and cannot claim a sub-SQL sensitivity [13]. An alternative approach, exploited in gravitational wave interferometry, relies on combining an intense coherent beam with squeezed light on a beam splitter, obtaining an enhancement in the sensitivity of a constant factor proportional to the squeezing factor [3,4,14]. Additionally, in the presence of loss, the SQL can be asymptotically beaten only by a constant factor [9,15,16], so that sophisticated sub-SQL strategies [11,17] (implemented up to now only for few photons) may not be worth the effort. This implies that, for practical high-sensitivity interferometry, the best resource exploitation (or, equivalently, the minimally invasive scenarios) currently entail strategies based on the use of a

coherent state $|\alpha\rangle$, i.e., a classical signal. Its QCR bound takes the form $\delta\phi \geq (2M\eta\xi|\alpha|^2)^{-1/2}$, where we consider separately the loss $\mathcal{L}_\xi = 1 - \xi$ in the sensing stage and the loss $\mathcal{L}_\eta = 1 - \eta$ in the overall detection process. Here, we present the experimental realization of a robust phase estimation protocol that improves the above accuracy up to $\sim(2M\xi|\alpha|^2)^{-1}$, while still using coherent signals as input. It achieves the SQL of a system only affected by the propagation loss \mathcal{L}_ξ , and not by the detection stage \mathcal{L}_η .

Our scheme employs a conventional interferometric phase sensing stage that uses coherent-state probes. These are amplified with an optical parametric amplifier (OPA) carrying the phase after the interaction with the sample, but before the lossy detectors. No postselection is employed to filter [12,13] the output signal. The OPA (an optimal phase covariant quantum cloning machine [18]) transfers the properties of the injected state into a field with a larger number of particles, robust under losses and decoherence [19]. Previous works addressed quantum signal amplification, namely quadrature signal, in a lossy environment adopting nonlinear methods [20] and feedforward techniques [21]. At variance with these approaches our Letter analyzes how the amplification of coherent states can be adopted for phase estimation purposes in a lossy environment. Specifically by studying the quantum Fisher information problem, we show that, by adopting the amplification-based strategy, the extracted information can achieve the quantum Cramér-Rao bound associated to the coherent probe state measured with a perfect detection apparatus. Since the amplification acts after the interaction of the probe state with the sample, our scheme is suitable for the analysis of fragile samples, e.g., optical

microscopy of biological cells [22], or for single-photon interferometry [23] (where the small intensity of the probes achieved only limited accuracy).

Theory.—The probe is a horizontally polarized (H) coherent state $|\alpha\rangle_H|0\rangle_V$, with $\alpha = |\alpha|e^{i\theta}$. The state is rotated in the $\vec{\pi}_\pm = 2^{-1/2}(\vec{\pi}_H \pm \vec{\pi}_V)$ polarization basis, and the interaction with the sample induces a phase shift ϕ between the $\vec{\pi}_\pm$ polarization components: U_ϕ . The sample loss \mathcal{L}_ξ reduces the state amplitude to $\beta = \sqrt{\xi}\alpha$. The maximum amount of information which can be extracted on the coherent probe state is encoded in the corresponding QCR bound $\delta\phi \geq (MI_{\text{SQL}}^q)^{-1/2}$, where $I_{\text{SQL}}^q = 2|\beta|^2$. In the absence of amplification, the detection losses \mathcal{L}_η would increase the QCR to $\delta\phi \geq (M\eta I_{\text{SQL}}^q)^{-1/2}$. To prevent this and to attain the previous bound, we implemented the operations shown in Fig. 1(a): a $\lambda/4$ wave-plate with optical axis at 45° and the OPA, described by the unitary $U_{\text{OPA}} = \exp[g(a_H^{\dagger 2} - a_V^{\dagger 2})/2 + \text{H.c.}]$, where $g = |g|e^{i\lambda}$ is the amplifier gain, and a_H and a_V are the annihilation operators of the two polarization modes. After the action of detection losses $1 - \eta$, the state evolves into $\rho_\phi^{\beta, g, \eta}$. The quantum Fisher information I_{ampl}^q of the amplification strategy, evaluated on the state $\rho_\phi^{\beta, g, \eta}$ and quantifying the optimal performances of the scheme, reads

$$I_{\text{ampl}}^q(|\beta|, g, \eta) = 2|\beta|^2 \eta \frac{e^{2(g - g_{\text{eff}})}}{\sqrt{1 + 4\eta(1 - \eta)\bar{n}}}, \quad (1)$$

where $g_{\text{eff}} = 1/4 \log[(\eta e^{2g} + 1 - \eta)/(\eta e^{-2g} + 1 - \eta)]$, $\bar{n} = \sinh^2 g$, and we maximized the ϕ -dependent quantum Fisher information by choosing $\phi = \pi/2 - \lambda/2 + \theta$ [24]. For $\bar{n} \gg (8\eta)^{-1}$ and $|\beta|^2 \gg 1/2$, we observe that I_{ampl}^q approaches the SQL limit I_{SQL}^q [dash-dotted line in Fig. 1(e)]. In other words, increasing the amplifier gain, the effects of the detector loss can be asymptotically removed [25].

Achieving the accuracy associated with quantum Fisher bound I_{ampl}^q of Eq. (1) would need to use an optimal estimation strategy which is difficult to characterize [6] and most likely challenging to implement. To experimentally test our proposal we decided hence to recover ϕ by measuring (via lossy detectors) the photon number difference $D = n_H - n_V$ between the two modes on the output state $\rho_\phi^{\beta, g, \eta}$ after losses, with $n_x \equiv a_x^\dagger a_x$. Even though in general this scheme fails to reach the accuracy bound of I_{ampl}^q in the limit of high gain g and high amplitude β it allows us to reach the value of I_{SQL}^q (and hence of I_{ampl}^q). Indeed, the resulting uncertainty can be evaluated [5] as $\delta\phi = \sigma(\langle D \rangle) \left| \frac{\partial \langle D \rangle}{\partial \phi} \right|^{-1}$, where $\langle D \rangle$ is the expectation value

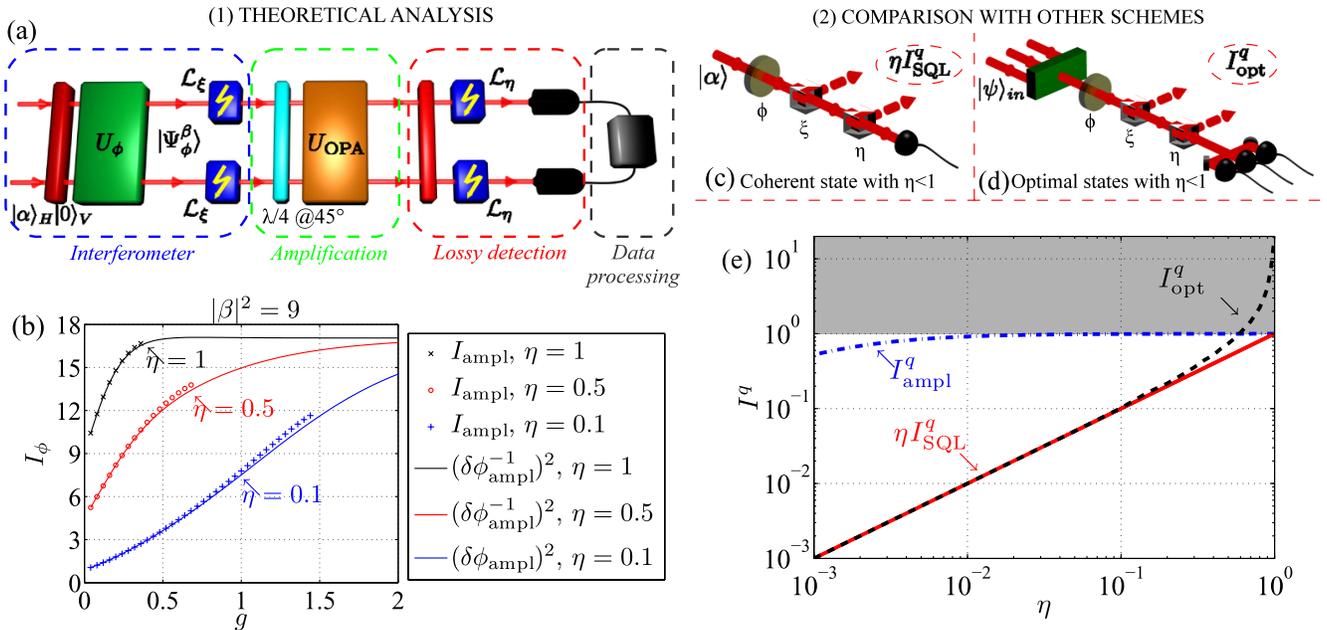


FIG. 1 (color online). *Theoretical analysis:* (a) Scheme of the amplifier based protocol. (b) Comparison between the classical Fisher information I_{ampl} (points) and the sensitivity $(\delta\phi_{\text{ampl}}^{-1})^2$ (lines) for $|\beta|^2 = 9$. *Comparison with other schemes:* (c) Conventional (unamplified) coherent-state interferometry with sample and detection loss \mathcal{L}_ξ and \mathcal{L}_η respectively: it can achieve the SQL bound connected to the quantum Fisher information (QFI) ηI_{SQL}^q . (d) Interferometry based on the states that optimize the QFI in the presence of loss, proposed in [8], with the corresponding QFI, I_{opt}^q . (e) Comparison between the QFI for the three strategies (a), (c) and (d), for $|\beta|^2 = 20$ and $g = 3.5$, normalized with respect to I_{SQL}^q . Blue dash-dotted line: QFI of our method I_{ampl}^q . Red solid line: QFI of the coherent state phase estimation with loss of Fig. 1(c), ηI_{SQL}^q . Green dashed line: QFI of the optimal strategy of Fig. 1(d), I_{opt}^q [8].

of D on the output state. A calculation of the estimation error $\delta\phi$ of the whole procedure shows that it depends on the value of the phase ϕ to be estimated. The maximum sensitivity, that is, the minimum uncertainty $\delta\phi_{\text{ampl}}$, is obtained for $\phi = \pi/2$ by setting $\lambda = 2\theta$,

$$\delta\phi_{\text{ampl}} = \frac{a^{1/2}(\bar{n}, \eta)}{|\beta|^2 \sqrt{\eta} [1 + 2\bar{n} + 2\sqrt{\bar{n}(1 + \bar{n})}]}, \quad (2)$$

where $a(\bar{n}, \eta) = 2\bar{n}(1 + \eta + 2\eta\bar{n}) + |\beta|^2[1 + 2\bar{n} + \eta\bar{n}(6 + 8\bar{n})]$. It is then clear that for $\bar{n} \gg (2\eta)^{-1}$ and $|\beta|^2 \gg 1/2$ we have $\delta\phi_{\text{ampl}} \simeq (2|\beta|^2)^{-1/2}$, that is, the QCR bound of the state $|\Psi_{\phi}^{\beta}\rangle$ (before the amplification and the detector loss) can be attained by our detection strategy. We also notice that the adopted data processing is optimal for a wide range of parameters. This can be shown by evaluating the classical Fisher information I_{ampl} , which represents the maximum amount of information that can be extracted from the probe state using our choice of measurement, optimizing over all possible data processing. In the present strategy, the sensitivity $(\delta\phi_{\text{ampl}}^{-1})^2$ closely tracks the I_{ampl} both for small and intermediate values of \bar{n} . Furthermore, the trend of the two curves suggest a close resemblance also in the high photon number regime [see Fig. 1(b)].

Because of the dependence of $\delta\phi$ on ϕ , to achieve the minimum error $\delta\phi_{\text{ampl}}$ an adaptive strategy [26] is necessary. In the Supplemental Material [27], we show that it is sufficient to use a simple two-stage strategy in which we first find a rough estimate of the phase ϕ_{est} employing conventional phase estimation methods, and then we use it to tune the zero reference so that our scheme operates at its optimal working point detailed above. We also show that the resources employed in the first stage of this adaptive strategy are asymptotically negligible with respect to the resources employed in the second high-resolution stage.

Efficiency of the phase estimation.—We now compare our method to other strategies, using as a benchmark the SQL $\delta\phi \geq (M I_{\text{SQL}}^q)^{-1/2}$, which would be achieved by a probe coherent state with $|\beta|^2$ average photons using lossless detectors. Consider now the case with no amplification, where a coherent state is subject to both the sample and detector loss [Fig. 1(c)]. This is the strategy conventionally used in interferometry [28]. Our method clearly always outperforms it, see the continuous line in Fig. 1(e). Furthermore, in a lossy scenario the present amplifier-based method achieves better performances than any quantum strategy. Recently, the optimal strategy in the presence of loss was derived [8] [Fig. 1(d)]. It employs the state that maximizes the quantum Fisher information in lossy conditions. Of course, this strategy cannot be beaten if one could access the optimal measurement that attains the QCR bound. Even though elegant proof of principle experiments exist [10], both this measurement and the creation of these states without using postselection are beyond the reach of

practical implementations for the foreseeable future, especially for states with large average photon numbers. In addition, the form of these states strongly depends on the value of the loss \mathcal{L}_{ξ} : it may be unknown and its experimental evaluation typically requires irradiating the sample, which removes the advantage of using the optimal minimally invasive states. In contrast, the present amplifier-based protocol uses readily available input states and detection strategies, and does not require *a priori* knowledge since the choice of the coherent state is independent of the value of the loss. Since our method is devised especially to counter the detector loss \mathcal{L}_{η} , we compare the performance of our states with the optimal state calculated for the total amount of loss $\mathcal{L}_{\xi\eta}$, showing that our method can achieve better performance for the practically relevant case of low values of η [see dashed line in Fig. 1(e)], where the detection strategy is clearly not optimized to achieve the QCR bound of the optimal states.

Experimental setup.—We now describe the experimental implementation in highly lossy conditions, showing that we can achieve a significant phase-sensitivity enhancement with respect to the coherent probe based strategy. The optical setup is reported in Fig. 2. To acquire the phase shift to be measured, the probe coherent state is injected into the sample, which is simulated by a Babinet-Soleil compensator that introduces a tunable phase shift ϕ between the H and V polarizations. Subsequently, the probe state is superimposed spatially and temporally with a pump and injected into the OPA. In this experimental realization the phases of the pump and of the coherent state are not stabilized: this will reduce the achievable enhancement by a fixed numerical factor of 4. Note that such condition corresponds to the

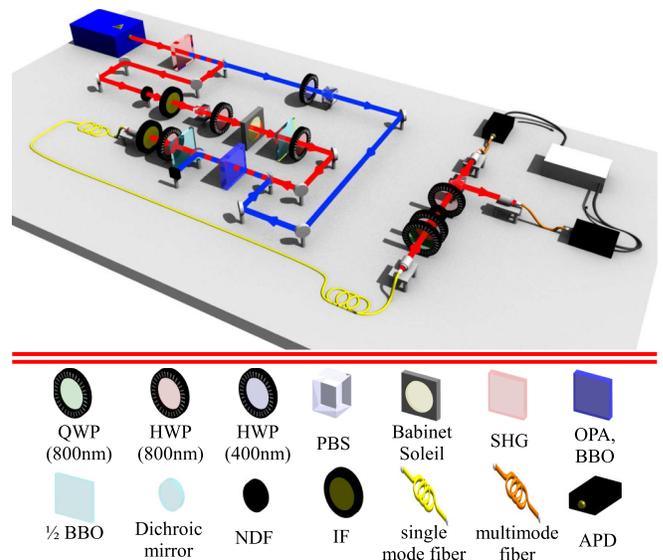


FIG. 2 (color online). Experimental setup for the practical implementation of the protocol. For details on the experimental setup refer to the description in the text and to the Supplemental Material [27].

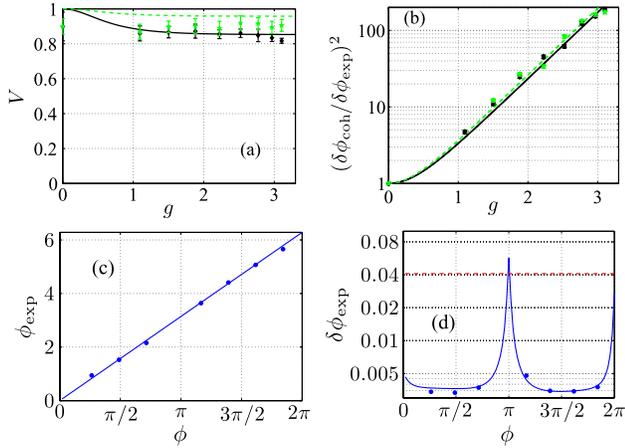


FIG. 3 (color online). Experimental results. (a) Fringe pattern visibility and (b) experimental enhancement $(\delta\phi_{\text{coh}}/\delta\phi_{\text{exp}})^2$ evaluated at $\phi = \pi/2$ as a function of the nonlinear gain g for $|\beta|^2 \sim 5.8$, $\eta \sim 1.46 \times 10^{-4}$ (experiment: black diamond points; theory: black solid line) and $|\beta|^2 \sim 22.8$, $\eta \sim 3.48 \times 10^{-5}$ (experiment: green star points; theory: green dashed line). (c) and (d) Experimental results for the phase estimation experiment performed with the amplifier based strategy ($g = 3.3$, $|\beta|^2 \sim 22.8$, $\eta \sim 3.48 \times 10^{-5}$) for different values of the phase. Estimated values of the phase ϕ_{exp} (c) and corresponding error $\delta\phi_{\text{exp}}$ (d). Points: experimental results. Blue solid lines: theoretical prediction given respectively by the true value of the phase ϕ (c) and by the classical Fisher information (d). Red dashed line corresponds to the classical Fisher information for the adopted coherent state without amplification.

absence of an external phase reference. In contrast to previous realizations of parametric amplification of coherent states [29] which focused on the single-photon excitation regime, we could achieve a large value for the nonlinear gain, up to $g = 3.3$, corresponding to a number of generated photons per mode $\bar{n} \sim 180$ in spontaneous emission. In addition, our scheme is also able to exploit the polarization degree of freedom. After the amplification, the two output orthogonal polarizations were spatially divided and detected by two avalanche photodiodes. Their count rates are then subtracted to obtain the value of $\langle D \rangle$, and recorded as a function of the phase ϕ , introduced by the Babinet.

Experimental phase estimation.—The results of the experiment are reported in Fig. 3. An enhancement of ~ 200 in the counts rate for the former case is observed without significantly affecting the visibility of the fringe pattern [Fig. 3(a)], leading to an increased phase resolution. We measured the enhancement $(\delta\phi_{\text{coh}}/\delta\phi_{\text{exp}})^2$ achievable with our protocol $\delta\phi_{\text{exp}}$ with respect to the conventional unamplified interferometry $\delta\phi_{\text{coh}}$, in the $\phi = \pi/2$ working point [see Fig. 3(b)]. The quantity $(\delta\phi_{\text{coh}}/\delta\phi_{\text{exp}})^2$ represents the fraction of additional runs \bar{M} of a coherent state phase estimation experiment in order to achieve the same performances of the amplifier-based strategy, with the two

protocols compared for the same values of $|\beta|^2$ and η . Our measurement shows a good agreement with the theoretical predictions. A significant enhancement up to a value of $(\delta\phi_{\text{coh}}/\delta\phi_{\text{exp}})^2 = 186.3 \pm 9.3$ has been achieved.

We then performed a phase estimation experiment with the amplifier-based strategy for different values of the phase shift ϕ . To this end, for each chosen value of the phase we recorded the photon counts in the two output detectors for $M_{\text{expt}} = 7.5 \times 10^5$ subsequent pulses of the coherent state. Then, we adopted a Bayesian approach in order to obtain an estimate ϕ_{expt} for the phase and to evaluate the associated error $\delta\phi_{\text{expt}}$. The results are reported in Figs. 3(c) and 3(d). We observe that the estimated values of the phase ϕ_{expt} are in good agreement with the corresponding true values ϕ , and that the estimation process reaches the Cramér-Rao bound. Furthermore, the obtained results clearly outperforms the coherent state strategy when no amplification is performed [red dashed line in Fig. 3(d)].

Conclusions and perspectives.—We discuss a strategy for phase estimation in the presence of noisy detectors that can reach the performance of a lossless probe. This approach involves coherent states as input signals, thus not requiring any *a priori* characterization of the amount of losses, and phase sensitive amplification after the interaction with the sample and before detection losses. As a further perspective, our method could be exploited with different classes of probe states, including quantum resources such as squeezing, leading to sub-SQL phase estimation experiments in lossy conditions.

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*fabio.sciarrino@uniroma1.it
http://quantumoptics.phys.uniroma1.it

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