Enhanced Resolution of Lossy Interferometry by Coherent Amplification of Single Photons

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(Received 16 April 2010; revised manuscript received 14 July 2010; published 8 September 2010)

In the quantum sensing context most of the efforts to design novel quantum techniques of sensing have been constrained to idealized, noise-free scenarios, in which effects of environmental disturbances could be neglected. In this work, we propose to exploit optical parametric amplification to boost interferometry sensitivity in the presence of losses in a minimally invasive scenario. By performing the amplification process on the microscopic probe after the interaction with the sample, we can beat the losses' detrimental effect on the phase measurement which affects the single-photon state after its interaction with the sample, and thus improve the achievable sensitivity.

DOI: 10.1103/PhysRevLett.105.113602 PACS numbers: 42.50.Ex, 42.50.Dv, 42.50.St

The aim of quantum sensing is to develop methods to extract the maximum amount of information from a system with a minimal disturbance on it. Indeed, the possibility of performing precision measurements by adopting quantum resources can increase the achievable precision going beyond the semiclassical regime of operation [1–3]. In the case of interferometry, this can be achieved by the use of the so-called N00N states, which are quantum mechanical superpositions of just two terms, corresponding to all the available photons N placed in either the signal arm or the reference arm. The use of N00N states can enhance the precision in phase estimation to $1/N$, thus improving the scaling of the achievable precision with respect to the employed resources [4,5]. This approach can have wide applications for minimally invasive sensing methods in order to extract the maximum amount of information from a system with minimal disturbance. The experimental realization of protocols involving N00N states containing up to 4 photons have been realized in the past few years [6–10]. Other approaches [11,12] have focused on exploiting coherent and squeezed light to generated fields which approximate the features of N00N states. Nevertheless, these quantum states turn out to be extremely fragile under losses and decoherence [13], unavoidable in experimental implementations. A sample, whose phase shift is to be measured, may at the same time introduce high attenuation. Since quantum-enhanced modes of operations exploit fragile quantum mechanical features, the impact of environmental effects can be much more deleterious than in semiclassical schemes, destroying completely quantum benefits [14,15]. This scenario puts the beating of realistic, noisy environments as the main challenge in developing quantum sensing. Very recently, the theoretical and experimental investigations of quantum states of light resilient to losses have attracted much attention, leading to the best possible precision in optical two-mode interferometry, even in the presence of experimental imperfections [16–21].

In this work, we adopt a hybrid approach based on a high gain optical parametric amplifier operating for any polarization state in order to transfer quantum properties of different microscopic quantum states in the macroscopic regime [22,23]. By performing the amplification process of the microscopic probe after the interaction with the sample, we can beat the losses' detrimental effect on the phase measurement which affects the single-photon state after the sample. Our approach may be adopted in a minimally invasive scenario where a fragile sample, such as biological or artifacts systems, requires as few photons as possible impinging on it in order to prevent damages. The action of the amplifier, i.e., the process of optimal phase covariant quantum cloning, is to broadcast the phase information codified in a single photon into a large number of particles. Such multiphoton states have been shown to exhibit a high resilience to losses [24–26] and can be manipulated by exploiting a detection scheme which combines features of discrete and continuous variables. The effect of losses on the macroscopic field consists in the reduction of the detected signal and not in the complete cancellation of the phase information as would happen in the single-photon probe case, thus improving the achievable sensitivity. This improvement does not consist in a scaling factor but turns out to be a constant factor in the sensitivity depending on the optical amplifier gain. Hence, the sensitivity still scales as $\sqrt{N}$, where N is the number of photons impinging on the sample, but the effect of the amplification process is to reduce the detrimental effect of losses by a factor proportional to the number of generated photons.

Let us review the adoption of single photons in order to evaluate the unknown phase φ, Fig. 1(a). The phase φ introduced in the path k_2 is probed by sending to the sample N input photons, each one in the state

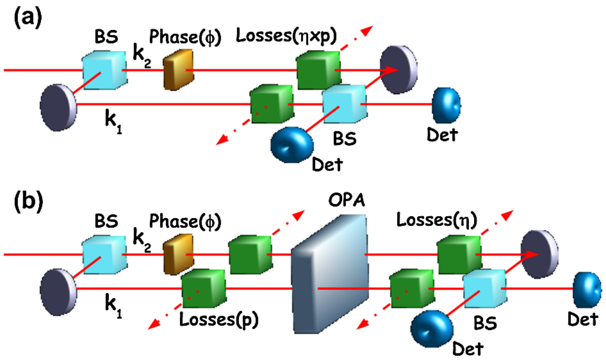

FIG. 1 (color online). Scheme for the phase measurement. (a) Interferometric scheme adopted to estimate the phase (ϕ) introduced in the mode k_2. (b) Interferometric scheme adopting a single photon and the optical parametric amplifier: the amplification of the single-photon state is performed before dominant losses.

$2^{-1/2}(|1\rangle_{k1} + |1\rangle_{k2})$. After the propagation, the sample introduces a phase φ on the probe beam and each photon is found in the state $\frac{1}{\sqrt{2}}(|1\rangle_{k1} + e^{i\varphi}|1\rangle_{k2})$. The two modes k_1 and k_2 are then combined on a beam splitter (BS) and detected by (D_1', D_2') with an overall detection efficiency equal to t. N performed experiments leads to an output signal equal to $I = I(D_1') - I(D_2') = tN\cos\varphi$, whose fluctuations are given by $\sigma = (tN)^{1/2}$. The uncertainty on the phase measurement around the value $\frac{\pi}{2}$ can hence be estimated as $\Delta\varphi = (\frac{\partial I}{\partial \varphi})^{-1}\Delta I = \frac{1}{\sqrt{tN}}$, the semiclassical shot noise limit, and the sensitivity of the interferometer can be evaluated as $S_{1\,\text{phot}} = \frac{1}{\Delta\varphi} = \sqrt{tN}$.

In order to avoid the detrimental effect of a low value of t, our strategy involves the amplification of the single-photon probe, Fig. 1(b). In the theory and experiment described here, the two modes k_1 and k_2 correspond to two orthogonal polarization modes: horizontal (H) and vertical (V) associated to the same longitudinal spatial mode k. The input single photon is prepared in the polarization state: $|+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$. After the propagation over the interferometer, the photon acquires the unknown phase φ: $|\varphi\rangle = \frac{1}{\sqrt{2}} \times (|H\rangle + e^{\imath\varphi}|V\rangle)$. The amplification performed by the optical parametric device generates the output state $|\Phi^\varphi\rangle = \hat{U}_{\text{OPA}}|\varphi\rangle = \cos\frac{\varphi}{2}|\Phi^+\rangle + \imath\sin\frac{\varphi}{2}|\Phi^-\rangle$, where $|\Phi^{+,-}\rangle$ are the wave functions described in Ref. [24]. Precisely, the state $|\Phi^+\rangle$ ($|\Phi^-\rangle$) presents a Planckian probability distribution as a function of photons polarized $\vec{\pi}_-$ ($\vec{\pi}_+$) and a long tail distribution as a function of photons polarized $\vec{\pi}_+$ ($\vec{\pi}_-$). The two distributions belonging to the state $|\Phi^+\rangle$ and $|\Phi^-\rangle$ partially overlap, but become distinct on the border of the Fock states' plane [27]. For the state $|\Phi^\varphi\rangle$, the average number of photons emitted over the polarization mode $\vec{\pi}_+$ is equal to $\langle n_+\rangle = \bar{n} + \cos^2\frac{\varphi}{2}(2\bar{n}+1)$ with $\bar{n} = \sinh^2 g$ and g the gain of the amplifier, while the average number of photons emitted over the polarization mode $\vec{\pi}_-$ is equal to $\langle n_-\rangle = \bar{n} + \sin^2\frac{\varphi}{2}(2\bar{n}+1)$. The previous expressions lead to a phase-dependent intensity with a visibility $V = \frac{\langle n_+\rangle - \langle n_-\rangle}{\langle n_+\rangle + \langle n_-\rangle} \rightarrow 0.50$ for $g \rightarrow \infty$. The resilience to losses of such multiphoton fields [25] renders them suitable for the implementation of quantum information applications in which noisy channels and low detection efficiency are involved. We consider the case in which the losses are unavoidable during the detection process and happen after the single-photon amplification (Fig. 1). After the propagation over a lossy channel, the state evolves from $|\Phi^\varphi\rangle\langle\Phi^\varphi|$ into a mixed state $\hat{\rho}^\varphi_\eta$. For details on the explicit expressions of the coefficients of the density matrix $\hat{\rho}^\varphi_\eta$, see [25].

After the amplification stage and the transmission losses, the received field is analyzed through single-photon detectors (D_1', D_2') in the $\{\vec{\pi}_+, \vec{\pi}_-\}$ polarization basis. Our aim is to compare the achievable sensitivity *with* and *without* the optical amplifier ($g = 0$). To take into account experimental imperfections, we divide the losses t in two contributions: the first one includes all the losses between the sample and the optical amplifier (p), while the second parameter takes into account all the inefficiencies up to the detection stage (η): $t = p \times \eta$. Our strategy cannot compensate for losses that occur before the amplifier (p), but can compensate for large (even very large, if the gain is high enough) losses after the amplification (η). A first insight on this property of the optical parametric amplifier has been given in Ref. [28] by analyzing the signal to noise of the amplification of a coherent state signal in lossy conditions. The sensitivity S_{ampl}, obtained by measuring the difference $\langle D\rangle = \langle n_+\rangle - \langle n_-\rangle$ intensity signals provided by the detectors around the phase value $\varphi = \frac{\pi}{2}$, is found to be

$$S_{\text{ampl}} = \frac{\sqrt{N}p\eta c}{\{\eta^2[p\bar{n}(4c+2) + 2\bar{n}c] + \eta[pc + 2\bar{n}]\}^{1/2}}, \quad (1)$$

with $c = 2\bar{n} + 1$.

Let us first consider the case $p = 0.5$: Fig. 2(a) reports the logarithm of the enhancement of the squared sensitivity $E = (\frac{S_{\text{ampl}}}{S_{1\,\text{phot}}})^2$ versus g and η. E represents the reduction factor in the number of photons sent onto the sample in order to obtain the same information on the phase φ, by exploiting the amplification strategy with respect to the single-photon probe scheme. As it can be observed in Fig. 2(a), a large improvement can be obtained in the regime of high losses and large gain of the amplifier. The motivation of such behavior is the following: the present approach allows us to increase the number of detected photons by a factor $4\bar{n}$ with respect to the single-photon case keeping a visibility of the fringe patterns reduced only to 50% (for $g \rightarrow \infty$, $p = 1$). The enhancement E is then slightly affected by the reduction in the visibility, due to the amplification noise [29], while it is significantly improved by the increase in the detected signal.

In Fig. 2(b) we report the trend of the enhancement as a function of losses $0.15 \leq p \leq 0.5$ and $0.05 \leq \eta \leq 0.2$ for a nonlinear gain of $g = 4.5$. Such a range corresponds to

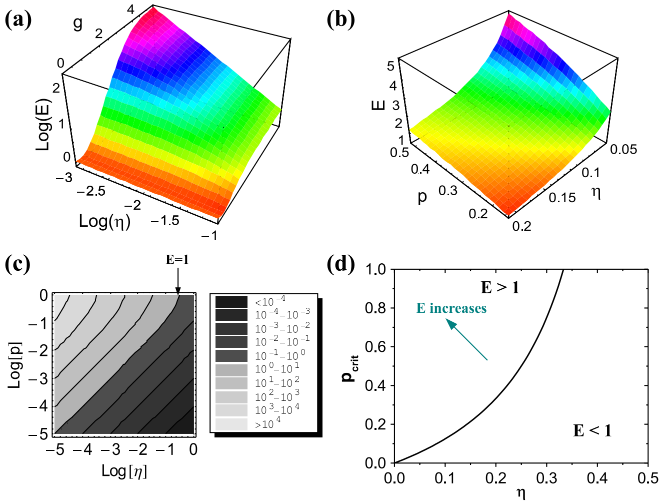

FIG. 2 (color online). (a) Logarithm of the enhancement versus the nonlinear gain g and the transmittivity of the lossy channel η for a value of losses between the phase shifter and the amplifier equal to $p = 0.5$. (b) Trend of E as a function of losses before amplification $0.15 \leq p \leq 0.5$ and after amplification $0.05 \leq \eta \leq 0.2$ ($g = 4.5$). (c) Contour plot of the enhancement as a function of the logarithm of p and η. The lighter region corresponds to $E > 1$, the darker one to $E < 1$. (d) Nonideal case $p \neq 1$: trend of the injection probability critical value for which $E > 1$ as a function of the detection efficiency.

typical values of losses and detection efficiency achievable in practical schemes. We observe that also in this regime an enhancement greater that 1 can be obtained. For large values of the gain g the enhancement saturates to the value $E_{\lim} = \frac{p}{\eta(2p+1)}$ [contour plot in Fig. 2(c)]; we can then identify a critical value of p above which the enhancement is greater than 1: $p_{\rm crit} = \frac{\eta}{1-2\eta}$. For $\eta \geq 0.33$ no enhancement can be achieved by exploiting the amplification strategy [see Fig. 2(d)].

The previous theoretical predictions have been experimentally tested by adopting a high gain optical parametric amplifier with a maximum gain $g = 4.5$. A detailed description of the apparatus can be found in Refs. [24,27]. The single-photon probe was generated in a first nonlinear crystal and heralded by a trigger detector D_T; hence, the phase shifting φ was introduced via a Soleil-Babinet compensator. Then the probe photon was superposed to an ultraviolet pump beam and injected into a second nonlinear crystal realizing the optical parametric amplifier. The output radiation was then coupled with a single mode fiber and detected with single-photon detectors. Additional controlled losses were introduced by adopting neutral optical filters. For the sake of simplicity we propose working in the single-photon counting regime; in order to describe the detection apparatus in a linear regime, the following condition for the average number of detected photons must be satisfied $\eta\langle n_{\pm}\rangle \ll 1$ (experimental details can be found in the supplementary material [30]). We found experimentally a value of p equal to 0.15 due to spatial and spectral mismatch between the injected single photon and the ultraviolet pump beam (k'_p). The output fringe patterns have been recorded for different values of the gain g and hence of the generated number of photons in the amplifier. In the extreme condition with $\eta = 3 \times 10^{-4}$ and $g = 4.5$, we observed an enhancement of a factor $\sim$210, as shown in Fig. 3(a), in which is reported the trend of E as a function of the amplifier gain, compared with the theoretical prediction.

We now discuss the optimality of the measurements performed on the multiphoton state, in order to extract the maximum information about the phase φ codified in the optical field. This quantity is expressed by the quantum Fisher information [3,31], defined as $H(\varphi) = \mathrm{Tr}[\hat{\rho}^{\varphi}\hat{L}_{\varphi}]$, where $\hat{L}_{\varphi}$ is the symmetric logarithmic derivative $\partial_{\varphi}\hat{\rho}^{\varphi} = \frac{\hat{L}_{\varphi}\hat{\rho}^{\varphi}+\hat{\rho}^{\varphi}\hat{L}_{\varphi}}{2}$ and $\hat{\rho}^{\varphi}$ is the density matrix of the state in which the phase is codified. The quantum Cramer-Rao bound [3] quantifies the maximum precision achievable on the estimation of the phase φ optimized over all possible measurements as $\Delta^2\varphi \geq 1/H(\varphi)$. In the high lossy regime $\eta\langle n_{\pm}\rangle \ll 1$, the single-photon amplified states lead to a quantum Fisher information equal to $H_{\rm ampl}(\varphi) \approx 2\bar{n}\eta p(1 + p^{-1})^{-1}$, to be compared with the single-photon case, which gives $H_{1\,\rm phot}(\varphi) = \eta p$. This result allows us to investigate the optimality of the counting measurement strategy. The sensitivity achieved with this scheme, given by Eq. (1), can be written in the high lossy regime as $S^2_{\rm ampl} = (\Delta^2\varphi)^{-1} \approx 2\bar{n}\eta p(1 + p^{-1})^{-1}$, thus saturating the Cramer-Rao bound and ensuring the optimality of this scheme in the high lossy regime.

As a more sophisticated strategy, it is possible to elaborate on an approach which leads to higher visibility of the detected fringe patterns at the cost of a reduced detection rate of the signal: the output radiation is measured in polarization with two linear detectors, for instance, photomultipliers. The intensity signals generated by the detectors proportional to the orthogonally polarized number of photons are compared shot by shot by the orthogonality-filter (OF) electronic device introduced in Ref. [24]. When the number of photons m_{φ}, detected in the $\vec{\pi}_{\varphi}$ polarization, exceeds $n_{\varphi_{\perp}}$, detected in the $\vec{\pi}_{\varphi\perp}$ polarization, over a

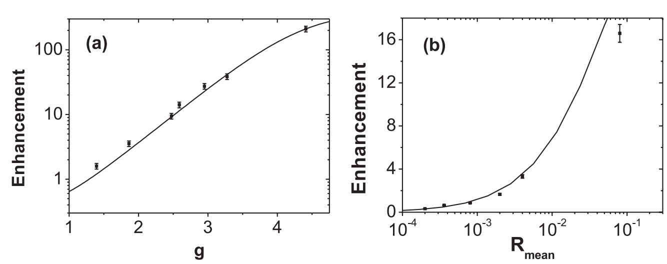

FIG. 3. (a) Experimental results of the enhancement E versus the nonlinear gain for the counting detection strategy. Continuous line: theoretical prediction for the expected enhancement with $\eta = 3 \times 10^{-4}$, $p = 0.15$. (b) Experimental results of the enhancement E versus the signal rate for the OF strategy; the continuous line reports the theoretical prediction for $p = 0.14$, $\eta = 0.005$.

certain adjustable threshold k, i.e., $m_{\varphi} - n_{\varphi_{\perp}} > k$, the $(+1)$ outcome is assigned to the event and the state $|\Phi^{\varphi}\rangle$ is detected. On the contrary, when the condition $n_{\varphi_{\perp}} - m_{\varphi} > k$ is satisfied, the (-1) outcome is assigned and the state $|\Phi^{\varphi\perp}\rangle$ is detected. Finally, an inconclusive result (0) is obtained when the unbalance between detected pulses does not exceed the threshold k. As the gain is increased, the number of transmitted photons $\eta\langle n\rangle$ becomes sufficient to detect all the N repeated trials. In the high losses regime, at variance with the single-photon case, all pulses can be exploited to extract information about the phase φ. The action of the OF is then to select those events that can be discriminated with higher fidelity, leading to an increase in the visibility, at the cost of discarding part of the data.

According to these considerations, the "detection" efficiency of the scheme, i.e., the percentage of detected events, is given by the average signal $\bar{\eta} = R_{\text{mean}}(k)$ filtered by the OF device. This parameter $\bar{\eta}$ corresponds to the overall efficiency of the amplification-OF-based detection scheme. We calculated the phase measurement uncertainty through the standard definition $\Delta\varphi_{\text{OF}} = \Delta R_{\text{OF}}(\varphi)|\frac{\partial R_{\text{OF}}(\varphi)}{\partial\varphi}|^{-1}$. The minimum uncertainty is achieved for $\varphi = \frac{\pi}{2}$. The resulting sensitivity averaged over N trials is thus $S_{\text{OF}} = V\sqrt{R_{\text{mean}}}\sqrt{N}$, where V is the visibility of the fringe pattern. This expression shows that the phase fluctuations no longer depend on the efficiency η of the channel, but only on the average percentage of detected pulses R_{mean}.

We have experimentally tested the enhancement obtained by the OF strategy. We report in Fig. 3(b) the experimental trend of the enhancement as a function of the signal rate compared with the expected theoretical trend ($p = 0.14$, $\eta = 0.005$). In the adopted apparatus the single-photon fringe pattern shows a visibility ~50% due to the generation of more than a single-photon pair by the first nonlinear crystal adopted as the heralded single-photon source. This seed visibility value is also responsible for a reduction of the amplified state visibility and has been taken into account in the comparison between the two strategies. By comparing the enhancement obtained through the counting and the OF-based detection methods, we can conclude that the first one allows us to achieve a higher enhancement.

In conclusion, the ability to generate suitable quantum light probes and quantum detectors is a crucial prerequisite for the operation of any quantum sensor. The optimal probes maximizing the sensitivity and performance of the sensors can be theoretically determined, but the resulting quantum states are often very complicated, difficult to generate, and extremely sensitive to losses and noise. We proposed and realized a simple conceptual strategy to apply in a lossy scenario that can be engineered with the existing quantum-optics technology. Our results show that a large sensitivity improvement can be achieved even in a high losses condition in which the dominant losses act after the interaction of the probe with the sample, hence including all the inefficiencies in the detection of the probe (spatial and spectral filtering, transmission, efficiency of the detectors).

We acknowledge support by the "Futuro in Ricerca" Project HYTEQ and Progetto d'Ateneo of Sapienza Università di Roma.

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