Enhanced Resolution of Lossy Interferometry by Coherent Amplification of Single Photons

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In the quantum sensing context most of the efforts to design novel quantum techniques of sensing have been constrained to idealized, noise-free scenarios, in which effects of environmental disturbances could be neglected. In this work, we propose to exploit optical parametric amplification to boost interferometry sensitivity in the presence of losses in a minimally invasive scenario. By performing the amplification process on the microscopic probe after the interaction with the sample, we can beat the losses’ detrimental effect on the phase measurement which affects the single-photon state after its interaction with the sample, and thus improve the achievable sensitivity.

The aim of quantum sensing is to develop methods to extract the maximum amount of information from a system with a minimal disturbance on it. Indeed, the possibility of performing precision measurements by adopting quantum resources can increase the achievable precision going beyond the semiclassical regime of operation [1–3]. In the case of interferometry, this can be achieved by the use of the so-called N00N states, which are quantum mechanical superpositions of just two terms, corresponding to all the available photons \(N\) placed in either the signal arm or the reference arm. The use of N00N states can enhance the precision in phase estimation to \(1/N\), thus improving the scaling of the achievable precision with respect to the employed resources [4,5]. This approach can have wide applications for minimally invasive sensing methods in order to extract the maximum amount of information from a system with minimal disturbance. The experimental realization of protocols involving N00N states containing up to 4 photons have been realized in the past few years [6–10]. Other approaches [11,12] have focused on exploiting coherent and squeezed light to generated fields which approximate the features of N00N states. Nevertheless, these quantum states turn out to be extremely fragile under losses and decoherence [13], unavoidable in experimental implementations. A sample, whose phase shift is to be measured, may at the same time introduce high attenuation. Since quantum-enhanced modes of operations exploit fragile quantum mechanical features, the impact of environmental effects can be much more deleterious than in semiclassical schemes, destroying completely quantum benefits [14,15]. This scenario puts the beating of realistic, noisy environments as the main challenge in developing quantum sensing. Very recently, the theoretical and experimental investigations of quantum states of light resilient to losses have attracted much attention, leading to the best possible precision in optical two-mode interferometry, even in the presence of experimental imperfections [16–21].

In this work, we adopt a hybrid approach based on a high gain optical parametric amplifier operating for any polarization state in order to transfer quantum properties of different microscopic quantum states in the macroscopic regime [22,23]. By performing the amplification process of the microscopic probe after the interaction with the sample, we can beat the losses’ detrimental effect on the phase measurement which affects the single-photon state after the sample. Our approach may be adopted in a minimally invasive scenario where a fragile sample, such as biological or artifacts systems, requires as few photons as possible impinging on it in order to prevent damages. The action of the amplifier, i.e., the process of optimal phase covariant quantum cloning, is to broadcast the phase information codified in a single photon into a large number of particles. Such multiphoton states have been shown to exhibit a high resilience to losses [24–26] and can be manipulated by exploiting a detection scheme which combines features of discrete and continuous variables. The effect of losses on the macroscopic field consists in the reduction of the detected signal and not in the complete cancellation of the phase information as would happen in the single-photon probe case, thus improving the achievable sensitivity. This improvement does not consist in a scaling factor but turns out to be a constant factor in the sensitivity depending on the optical amplifier gain. Hence, the sensitivity still scales as \(\sqrt{N}\), where \(N\) is the number of photons impinging on the sample, but the effect of the amplification process is to reduce the detrimental effect of losses by a factor proportional to the number of generated photons.

Let us review the adoption of single photons in order to evaluate the unknown phase \(\varphi\), Fig. 1(a). The phase \(\varphi\) introduced in the path \(k_2\) is probed by sending to the sample \(N\) input photons, each one in the state
situations are given by

\[ \frac{1}{2} \left( |1\rangle_{k1} + |1\rangle_{k2} \right). \]

After the propagation, the sample introduces a phase \( \varphi \) on the probe beam and each photon is found in the state \( \frac{1}{\sqrt{2}} (|1\rangle_{k1} + e^{i\varphi} |1\rangle_{k2}) \). The two modes \( k_1 \) and \( k_2 \) are then combined on a beam splitter (BS) and detected by \( (D'_1, D'_2) \) with an overall detection efficiency equal to \( \eta \). Performing experiments leads to an output signal equal to \( I = I(D'_1) - I(D'_2) = \eta N \cos \varphi \), whose fluctuations are given by \( \sigma = (\eta N)^{1/2} \). The uncertainty on the phase measurement around the value \( \varphi \) can hence be estimated as \( \Delta \varphi = \frac{(\eta N)^{-1/2} \Delta I}{\eta} \), the semiclassical shot noise limit, and the sensitivity of the interferometer can be evaluated as \( S_1 \text{phot} = \frac{1}{\Delta \varphi} \sqrt{\frac{N}{\eta}} \).

In order to avoid the detrimental effect of a low value of \( \eta \), our strategy involves the amplification of the single-photon probe, Fig. 1(b). In the theory and experiment described here, the two modes \( k_1 \) and \( k_2 \) correspond to two orthogonal polarization modes: horizontal \( (H) \) and vertical \( (V) \) associated to the same longitudinal spatial mode \( k \). The input single photon is prepared in the polarization state: \( |+\rangle = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle) \). After the propagation over the interferometer, the photon acquires the unknown phase \( \varphi \): \( |\varphi\rangle = \frac{1}{\sqrt{2}} \times (|H\rangle + e^{i\varphi} |V\rangle) \). The amplification performed by the optical parametric device generates the output state \( |\Phi\rangle = \hat{U}_{\text{OPA}} |\varphi\rangle = \cos \frac{\varphi}{2} |\Phi^+\rangle + i \sin \frac{\varphi}{2} |\Phi^-\rangle \), where \( |\Phi^+\rangle \) and \( |\Phi^-\rangle \) are the wave functions described in Ref. [24]. Precisely, the state \( |\Phi^+\rangle \) presents a Planckian probability distribution as a function of photons polarized \( \hat{\pi}_- \) \( \langle \hat{\pi}_- \rangle \) and a long tail distribution as a function of photons polarized \( \hat{\pi}_+ \) \( \langle \hat{\pi}_+ \rangle \). The two distributions belonging to the state \( |\Phi^+\rangle \) and \( |\Phi^-\rangle \) partially overlap, but become distinct on the border of the Fock states’ plane [27]. For the state \( |\Phi^+\rangle \), the average number of photons emitted over the polarization mode \( \hat{\pi}_- \) is equal to \( \langle n_- \rangle = \bar{n} + \cos^2 \frac{\varphi}{2} (2\bar{n} + 1) \) with \( \bar{n} = \sin^2 \frac{\varphi}{2} g \), and \( g \) the gain of the amplifier, while the average number of photons emitted over the polarization mode \( \hat{\pi}_+ \) is equal to \( \langle n_+ \rangle = \sqrt{\bar{n}} \sin \frac{\varphi}{2} (2\bar{n} + 1) \). The previous expressions lead to a phase-dependent intensity with a visibility \( V = \frac{(n_+)^2 - (n_-)^2}{(n_+)^2 + (n_-)^2} \rightarrow 0.5 \) for \( g \rightarrow \infty \). The resilience to losses of such multiphoton fields [25] renders them suitable for the implementation of quantum information applications in which noisy channels and low detection efficiency are involved. We consider the case in which the losses are unavoidable during the detection process and happen after the single-photon amplification (Fig. 1). After the propagation over a lossy channel, the state evolves from \( |\Phi\rangle \) into a mixed state \( \hat{p}_\eta \). For details on the explicit expressions of the coefficients of the density matrix \( \hat{p}_\eta \), see [25].

After the amplification stage and the transmission losses, the received field is analyzed through single-photon detectors \( (D'_1, D'_2) \) in the \( \{ \hat{\pi}_+, \hat{\pi}_- \} \) polarization basis. Our aim is to compare the achievable sensitivity with and without the optical amplifier \( (g = 0) \). To take into account experimental imperfections, we divide the losses \( \ell \) in two contributions: the first one includes all the losses between the sample and the optical amplifier \( (p) \), while the second parameter takes into account all the inefficiencies up to the detection stage \( (\eta) \): \( \ell = p \times \eta \). Our strategy cannot compensate for losses that occur before the amplifier \( (p) \), but can compensate for large (even very large, if the gain is high enough) losses after the amplification \( (\eta) \). A first insight on this property of the optical parametric amplifier has been given in Ref. [28] by analyzing the signal to noise of the amplification of a coherent state in lossy conditions. The sensitivity \( S_{\text{amp},l} \) obtained by measuring the difference \( \langle D \rangle = \langle n_+ \rangle - \langle n_- \rangle \) intensity signals provided by the detectors around the phase value \( \varphi = \frac{\pi}{2} \), is found to be

\[ S_{\text{amp},l} = \frac{\sqrt{N p \eta c}}{\eta \left[ p \bar{n}(4c + 2) + 2\bar{n}c + \eta (pc + 2\bar{n}) \right]^{1/2}}, \]

with \( c = 2\bar{n} + 1 \).

Let us first consider the case \( p = 0.5 \): Fig. 2(a) reports the logarithm of the enhancement of the squared sensitivity \( E = \log \left( \frac{S_{\text{amp},l}}{S_{\text{amp},w}} \right) \) versus \( g \) and \( \eta \). \( E \) represents the reduction factor in the number of photons sent onto the sample in order to obtain the same information on the phase \( \varphi \), by exploiting the amplification strategy with respect to the single-photon probe scheme. As it can be observed in Fig. 2(a), a large improvement can be obtained in the regime of high losses and large gain of the amplifier. The motivation of such behavior is the following: the present approach allows us to increase the number of detected photons by a factor \( 4\bar{n} \) with respect to the single-photon case keeping a visibility of the fringe patterns reduced only to 50\% (for \( g \rightarrow \infty \), \( p = 1 \)). The enhancement \( E \) is then slightly affected by the reduction in the visibility, due to the amplification noise [29], while it is significantly improved by the increase in the detected signal.

In Fig. 2(b) we report the trend of the enhancement as a function of losses \( 0.15 \leq p \leq 0.5 \) and \( 0.05 \leq \eta \leq 0.2 \) for a nonlinear gain of \( g = 4.5 \). Such a range corresponds to
pump beam ($k'_p$). The output fringe patterns have been recorded for different values of the gain $g$ and hence of the generated number of photons in the amplifier. In the extreme condition with $\eta = 3 \times 10^{-4}$ and $g = 4.5$, we observed an enhancement of a factor $\sim 210$, as shown in Fig. 3(a), in which is reported the trend of $E$ as a function of the amplifier gain, compared with the theoretical prediction.

We now discuss the optimality of the measurements performed on the multiphoton state, in order to extract the maximum information about the phase $\varphi$ codified in the optical field. This quantity is expressed by the quantum Fisher information [3,31], defined as $H(\varphi) = \text{Tr}(\dot{\rho}_\varphi \hat{L}_\varphi)$, where $\hat{L}_\varphi$ is the symmetric logarithmic derivative $\partial_\varphi \hat{\rho}_\varphi = \frac{1}{2} \hat{L}_\varphi \rho + \hat{\rho}_\varphi \hat{L}_\varphi$ and $\hat{\rho}_\varphi$ is the density matrix of the state in which the phase is codified. The quantum Cramer-Rao bound [3] quantifies the maximum precision achievable on the estimation of the phase $\varphi$ optimized over all possible measurements as $\Delta^2 \varphi \geq 1/H(\varphi)$. In the high lossy regime $\eta(\varphi) \ll 1$, the single-photon amplified states lead to a quantum Fisher information equal to $H_{\text{ampl}}(\varphi) = 2\eta \eta p (1 + p^{-1})^{-1}$, to be compared with the single-photon case, which gives $H_1(\varphi) = \eta p$. This result allows us to investigate the optimality of the counting measurement strategy. The sensitivity achieved with this scheme, given by Eq. (1), can be written in the high lossy regime as $S_{\text{ampl}}^2 = (\Delta^2 \varphi)^{-1} = 2\eta \eta p (1 + p^{-1})^{-1}$, thus saturating the Cramer-Rao bound and ensuring the optimality of this scheme in the high lossy regime.

As a more sophisticated strategy, it is possible to elaborate on an approach which leads to higher visibility of the detected fringe patterns at the cost of a reduced detection rate of the signal: the output radiation is measured in polarization with two linear detectors, for instance, photomultipliers. The intensity signals generated by the detectors proportional to the orthogonally polarized number of photons are compared shot by shot by the orthogonality-filter (OF) electronic device introduced in Ref. [24]. When the number of photons $n_{\varphi,\perp}$, detected in the $\vec{n}_{\varphi,\perp}$ polarization, exceeds $n_{\varphi,\perp}$, detected in the $\vec{n}_{\varphi,\perp}$ polarization, over a
certain adjustable threshold \( k \), i.e., \( m_\varphi - n_\varphi > k \), the 
(\( +1 \)) outcome is assigned to the event and the state 
\(|\Phi^+\rangle\) is detected. On the contrary, when the condition 
\( n_\varphi > m_\varphi > k \) is satisfied, the (\( -1 \)) outcome is assigned 
and the state \(|\Phi^\perp\rangle\) is detected. Finally, an inconclusive 
result (0) is obtained when the unbalance between detected 
pulses does not exceed the threshold \( k \). As the gain is 
increased, the number of transmitted photons \( \eta(n) \) 
comes sufficient to detect all the \( N \) repeated trials. In the 
high losses regime, at variance with the single-photon case, 
all pulses can be exploited to extract information about the 
phase \( \varphi \). The action of the OF is then to select those events 
that can be discriminated with higher fidelity, leading to 
an increase in the visibility, at the cost of discarding part of 
the data.

According to these considerations, the “detection” effi-
ciency of the scheme, i.e., the percentage of detected events, 
is given by the average signal \( \bar{\eta} = R_{\text{mean}}(k) \) filtered by the 
OF device. This parameter \( \bar{\eta} \) corresponds to the overall 
efficiency of the amplification-OF-based detection scheme. 
We calculated the phase measurement uncertainty through 
the standard definition \( \Delta \varphi_{\text{OF}} = \Delta R_{\text{OF}}(\varphi) \frac{\delta R_{\text{OF}}(\varphi)}{\delta \varphi} \). The minimum uncertainty is achieved for \( \varphi = \frac\pi2 \). The resulting 
sensitivity averaged over \( N \) trials is thus \( S_{\text{OF}} = V \sqrt{R_{\text{mean}}/N} \), where \( V \) is the visibility of the fringe pattern. 
This expression shows that the phase fluctuations no longer 
depend on the efficiency \( \eta \) of the channel, but only on the 
average percentage of detected pulses \( R_{\text{mean}} \).

We have experimentally tested the enhancement ob-
tained by the OF strategy. We report in Fig. 3(b) the 
experimental trend of the enhancement as a function of 
the signal rate compared with the expected theoretical 
trend \( (p = 0.14, \eta = 0.005) \). In the adopted apparatus 
the single-photon fringe pattern shows a visibility \( \sim 50\% \) 
due to the generation of more than a single-photon pair by 
the first nonlinear crystal adopted as the heralded single-
photon source. This seed visibility value is also responsible 
for a reduction of the amplified state visibility and has been 
taken into account in the comparison between the two 
strategies. By comparing the enhancement obtained 
through the counting and the OF-based detection methods, 
we can conclude that the first one allows us to achieve a 
higher enhancement.

In conclusion, the ability to generate suitable quantum 
light probes and quantum detectors is a crucial prerequisite 
for the operation of any quantum sensor. The optimal 
probes maximizing the sensitivity and performance of the 
sensors can be theoretically determined, but the resulting 
quantum states are often very complicated, difficult 
to generate, and extremely sensitive to losses and noise. We 
proposed and realized a simple conceptual strategy to 
by a losscy scenario that can be engineered with the 
existing quantum-optics technology. Our results show that 
a large sensitivity improvement can be achieved even in a 
high losses condition in which the dominant losses act 
after the interaction of the probe with the sample, hence 
including all the inefficiencies in the detection of the probe 
(spatial and spectral filtering, transmission, efficiency of the 
detectors).

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details on the experimental setup and a precise explanation of the performed measurements.