Anomalous Lack of Decoherence of the Macroscopic Quantum Superpositions Based on Phase-Covariant Quantum Cloning

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We show that all macroscopic quantum superpositions (MQS) based on phase-covariant quantum cloning are characterized by an anomalous high resilence to the decoherence processes. The analysis supports the results of recent MQS experiments and leads to conceive a useful conjecture regarding the realization of complex decoherence-free structures for quantum information, such as the quantum computer.

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Since the early decades of the last century the counterintuitive properties associated with the superposition state of macroscopic objects and the problem concerning the "classicality" of macroscopic states were the object of an intense debate epitomized in 1935 by the celebrated Schrödinger's paradox [1,2]. However, the actual feasibility of any macroscopic quantum superposition (MQS) adopting photons, atoms, and electrons in SQUIDS [3–6] was always found to be challenged by the very short persistence of its quantum coherence, i.e., by its overwhelmingly fast "decoherence." The latter property was interpreted as a consequence of the entanglement between the macroscopic system with the environment [7–9]. Recently, decoherence has received renewed attention in the framework of quantum information where it plays a detrimental role since it conflicts with the realization of any device bearing any relevant complexity, e.g., a quantum computer [10]. In particular, effort was aimed at the implementation of MQS involving coherent states of light, which exhibit elegant Wigner function representations [11]. Nevertheless, in all previous realizations the MQS was found so fragile that even the loss of a single particle spoiled any direct observation of its quantum properties.

The present work considers in general a novel type of MQS, one that is based on the amplification, i.e., or "quantum cloning", of a "microscopic" quantum state, e.g., a single-particle qubit: $|\phi\rangle = 2^{-(1/2)}(|\phi_1\rangle + e^{i\varphi}|\phi_2\rangle)$. Formally, the amplification is provided by a unitary cloning transformation $\hat{U}$, i.e., a quantum map which, applied to the microscopic state leads to the MQS macroscopic state: $|\Phi^{\phi}\rangle = \hat{U}|\phi\rangle$. In general, the amplification can be provided by a laser amplifier or by any quantum-injected nonlinear (NL) optical parametric amplification (OPA) process directly seeded by $|\phi\rangle$. The *atom laser* is also a candidate for this process, and then, within the exciting *matter-wave* context our model can open far reaching fields of novel scientific and technological endeavour. It can be shown that $\hat{U}$ can be "information preserving", albeit slightly noisy, and able to transfer in the macroscopic domain the quantum superposition character of the single-particle qubit [12–14]. Furthermore, unlike most of the other MQS schemes, $\hat{U}$, being a fixed intrinsic dynamical property of the amplifier is not affected in principle by events of scattering of particles out of the system, i.e., by loss, a process which is generally the dominant source of decoherence. For the same reasons $\hat{U}$ is largely insensitive to temperature effects. Let us investigate this interesting process by the quantum-injected OPA, often referred to as QI-OPA. By this device, indeed a high-gain phase ϕ-covariant cloning machine seeded by an entangled EPR photon pair, a macroscopic state consisting of a large number of photons $\mathcal{N} \approx 10^5$ was generated [14–18]. A sketchy draft of the apparatus is shown in the left part of Fig. 1. A polarization ($\vec{\pi}$) entangled couple of single photons (A, B) is parametrically generated by a standard Einstein-Podolsky-Rosen-Bohm (EPR) configuration in a NL crystal of BBO (beta-barium-borate) cut for type II phase matching and excited by a low intensity ultraviolet (UV) laser beam [12]. One of the photons, say A with state $|\phi^{\perp}\rangle_A$, measured by a detector (Det), provides the trigger signal for the overall experiment. The photon B with state $|\phi\rangle_B$ nonlocally correlated to A, is injected, via a dichroic mirror (DM) in another BBO NL-crystal excited

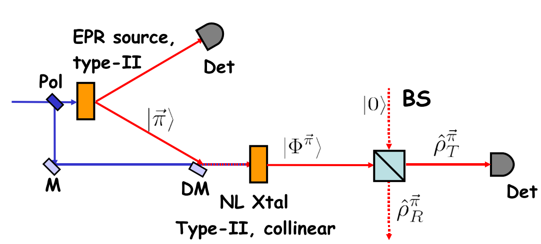

FIG. 1 (color online). Amplified states of the collinear optical parametric amplifier injected by a single-photon qubit generated in a type-II EPR source. Beam-splitter (BS) loss model for the amplified macroscopic states generated by the collinear QI-OPA.

by a large intensity UV beam. This second NL crystal, cut for collinear three-wave NL interaction provides the large amplification of the injected seed photon $|\phi\rangle_B$ with a NL parametric "gain" that attained in the experiment the large value $g \simeq 5$. Precisely its effect is formally represented by the map $|\Phi^{\phi}\rangle_B = \hat{U}|\phi\rangle_B$ with generation of the macroscopic state $|\Phi^{\phi}\rangle_B$, as said. A quantum analysis of the QI-OPA interaction will be given below. As we shall see, the interesting aspect of this process is that any quantum superposition property of the injected microscopic state $|\phi\rangle_B$ is deterministically mirrored by $\hat{U}$ into the same property for the macroscopic state $|\Phi^{\phi}\rangle_B$. Then the large fringe "visibility" easily attainable for a microscopic state quantum interference can be reproduced in the multiparticle MQS regime. In addition, the property of nonlocal microscopic-microscopic entanglement ($|\phi^{\perp}\rangle_A$, $|\phi\rangle_B$) correlating the two EPR single photons is transferred to an entangled microscopic-macroscopic system ($|\phi^{\perp}\rangle_A$, $|\Phi^{\phi}\rangle_B$) then realizing an experimental scheme reproducing exactly, for the first time, the original 1935 Schrödinger's proposal [2]. The experimental demonstration of the EPR nonseparability of a microscopic-macroscopic entangled "singlet" has been reported recently [15]. The same procedure can be straightforwardly extended to a macroscopic-macroscopic scheme involving two spacelike separated, entangled macroscopic states ($|\Phi^{\phi}\rangle_A$, $|\Phi^{\phi}\rangle_B$): indeed a novel conceptual achievement that goes even beyond the Schrödinger's paradigm [19].

The analysis reported in the present Letter accounts for the striking reduction of decoherence of the QI-OPA device and is further extended to consider the apparatus dubbed the orthogonality filter (OF), which is adopted to enhance the interference visibility implied by the EPR entanglement. The decoherence-free QI-OPA analyzed by our theory can open new perspectives for investigating physical systems in a regime close to the quantum-classical boundary. For instance, it has been recently proposed to exploit that technique to perform quantum experiments with human-eye detectors whose "threshold" response is simulated closely by the OF [20,21].

Let us analyze the resilience to decoherence of this new class of MQS's on the basis of the concept of *distance* $D(\hat{\rho}, \hat{\sigma})$ in Hilbert spaces between two density operators $\hat{\rho}$ and $\hat{\sigma}$ by further adopting the useful concepts of *pointer states*, *environment induced superselection*, and *information flow* [8]. Precisely, in order to characterize two macroscopic states $|\phi_1\rangle$ and $|\phi_2\rangle$ and the corresponding MQS's: $|\phi^{\pm}\rangle = \frac{\mathcal{N}_{\pm}}{\sqrt{2}}(|\phi_1\rangle \pm |\phi_2\rangle)$, we adopt the following criteria based on the "Bures metric": $D(\hat{\rho}, \hat{\sigma}) = \sqrt{1 - \mathcal{F}(\hat{\rho}, \hat{\sigma})}$ where $\mathcal{F}(\hat{\rho}, \hat{\sigma}) = \mathrm{Tr}(\sqrt{\hat{\rho}^{1/2}\hat{\sigma}\hat{\rho}^{1/2}})$ is a "quantum fidelity" [22,23].

Criterion I.—The *distinguishability* between $|\phi_1\rangle$ and $|\phi_2\rangle$ can be quantified as $D(|\phi_1\rangle, |\phi_2\rangle)$. *Criterion II.*—The *visibility*, i.e., "degree of orthogonality" of the MQS's $|\phi^{\pm}\rangle$ is expressed again by $D(|\phi^{+}\rangle, |\phi^{-}\rangle)$. Indeed, the value of the MQS visibility depends exclusively on the relative phase of the component states $|\phi_1\rangle$ and $|\phi_2\rangle$. Assume two orthogonal superpositions $|\phi^{\pm}\rangle$: $D(|\phi^{+}\rangle, |\phi^{-}\rangle) = 1$. In the presence of decoherence the relative phase between $|\phi_1\rangle$ and $|\phi_2\rangle$ progressively randomizes and the superpositions $|\phi^{+}\rangle$ and $|\phi^{-}\rangle$ approach an identical fully mixed state leading to: $D(|\phi^{+}\rangle, |\phi^{-}\rangle) = 0$. For more details on the criteria, refer to [24]. The physical interpretation of $D(|\phi^{+}\rangle, |\phi^{-}\rangle)$ as visibility of a superposition $|\phi^{\pm}\rangle$ is legitimate insofar as the component states of the corresponding superposition, $|\phi_1\rangle$ and $|\phi_2\rangle$ may be defined, at least approximately, as *pointer states* or *einselected states* [8]. These states, within the set of the eigenstates characterizing any quantum system, are defined as the ones least affected by the external noise and highly resilient to decoherence. In other words, the pointer states are "quasiclassical" states which realize the minimum flow of information from (or to) the system to (or from) the environment. They are involved in all criteria of classicality, such as the ones based on "purity" and "predictability" of the macroscopic states [8].

Let us refer to Fig. 1. A single-photon state, conditionally generated with polarization $\vec{\pi}_{\phi} = \frac{1}{\sqrt{2}}(\vec{\pi}_H + e^{\imath\phi}\vec{\pi}_V)$ by "spontaneous parametric down-conversion" in a first NL crystal, is injected into an OPA device. The interaction Hamiltonian of this process is $\hat{\mathcal{H}}_{\mathrm{coll}} = \imath\hbar\chi\hat{a}^{\dagger}_H\hat{a}^{\dagger}_V + \mathrm{H.c.}$ in the $\{\vec{\pi}_H, \vec{\pi}_V\}$ polarization basis, and $\hat{\mathcal{H}}_{\mathrm{coll}} = \frac{\imath\hbar\chi}{2}e^{-\imath\phi}(\hat{a}^{\dagger 2}_{\phi} - e^{\imath 2\phi}\hat{a}^{\dagger 2}_{\phi_{\perp}})$ for any "equatorial" basis $\{\vec{\pi}_{\phi}, \vec{\pi}_{\phi\perp}\}$ on the Poincaré sphere. Two relevant equatorial basis are $\{\vec{\pi}_{+}, \vec{\pi}_{-}\}$ and $\{\vec{\pi}_R, \vec{\pi}_L\}$ corresponding, respectively, to $\phi = 0$ and $\phi = \pi/2$. We remind that the *phase-covariant* cloning process amplifies identically all equatorial qubits. The symbols H and V refer to horizontal and vertical field polarizations, i.e., the extreme "poles" of the Poincaré sphere. By direct calculation, the amplified states for an injected qubit $\pi = \{H, V\}$ is $|\Phi^{\pi}\rangle = C^{-2}\sum_{i=0}^{\infty}\Gamma^{i}\sqrt{i+1}|(i+1)\pi, i\pi_{\perp}\rangle$, where $C = \cosh g$, $\Gamma = \tanh g$ and the ket $|n\pi, m\pi_{\perp}\rangle$ represents the number state with n photons with π polarization and m photons with $\pi_{\perp}$ polarization. The amplified state for an injected *equatorial* qubit is

$$|\Phi^{\phi}\rangle = \sum_{i,j=0}^{\infty}\gamma_{ij}|(2i+1)\phi, (2j)\phi_{\perp}\rangle, \tag{1}$$

where $\gamma_{ij} = 2^{-(i+j)}C^{-2}(e^{-\imath\varphi}\Gamma)^{i}(-e^{\imath\varphi}\Gamma)^{j}\frac{\sqrt{(2i+1)!}\sqrt{(2j)!}}{i!j!}$.

Consider the MQS of the macroscopic states $|\Phi^{+}\rangle$, $|\Phi^{-}\rangle$: $|\Psi^{\pm}\rangle = \frac{\mathcal{N}_{\pm}}{\sqrt{2}}(|\Phi^{+}\rangle \pm i|\Phi^{-}\rangle)$. Because of the linearity of the OPA process [13], it can be easily found: $|\Psi^{\pm}\rangle = |\Phi^{R/L}\rangle$. Our interest is aimed at the resilience properties of the QI-OPA-generated MQS (q-MQS) after the propagation over a lossy channel. This one is modeled by a linear beam splitter (BS) with transmittivity T and reflectivity

$R = 1 - T$ acting on a state $\hat{\rho}$ associated with a single BS input mode: Fig. 1 [25]. The calculation of the output density matrix $\hat{\rho}_T$ consists of the insertion of the BS unitary as a function of T followed by the partial tracing of the emerging field on the loss variables of the reflected mode.

We have evaluated numerically the distinguishability of $\{|\Phi^{+,-}\rangle\}$ through the distance $D(|\Phi^+\rangle, |\Phi^-\rangle)$ between the states: Fig. 2(a). It is found that this property of $\{|\Phi^{+,-}\rangle\}$ coincides with the MQS visibility of $|\Psi^\pm\rangle$, in virtue of the ϕ covariance of the process: $D(|\Psi^+\rangle, |\Psi^-\rangle) = D(|\Phi^R\rangle, |\Phi^L\rangle) = D(|\Phi^+\rangle, |\Phi^-\rangle) \equiv D_q$. The investigation on the Glauber's states leading to the α-MQS's case [26]: $|\Phi_{\alpha\pm}\rangle = (|\alpha\rangle \pm |-\alpha\rangle)[2(1 \pm e^{-4|\alpha|^2})]^{-(1/2)}$ in terms of the "pointer states" $|\pm\alpha\rangle$ leads to the closed form result: $D(|\Phi_{\alpha+}\rangle, |\Phi_{\alpha-}\rangle) = \sqrt{1 - \sqrt{1 - e^{-4R|\alpha|^2}}} \neq D(|\alpha\rangle, |-\alpha\rangle) = \sqrt{1 - e^{-2(1-R)|\alpha|^2}}$. $D(|\alpha\rangle, |-\alpha\rangle)$ and $D(|\Phi_{\alpha+}\rangle, |\Phi_{\alpha-}\rangle)$ are plotted in Fig. 2(a) (dash-dot-dot, blue lines) as function of the average lost photons: $x \equiv R\langle n\rangle$. Note that $D(|\alpha\rangle, |-\alpha\rangle) \approx 1$ keeps constant, as expected for pointer states, throughout the values of x, up to the maximum value $x = \langle n\rangle$, corresponding to full particle loss, viz to $R \sim 1$. On the contrary, the function $D(|\Phi_{\alpha+}\rangle, |\Phi_{\alpha-}\rangle)$ drops from 1 to 0.095 with zero slope upon loss of only one photon: $x = 1$, in agreement with all experimental observations. The visibility of the q-MQS $\{|\Psi^{+,-}\rangle\}$ as well as for the MQS components $|\Phi^+\rangle$ and $|\Phi^-\rangle$ was evaluated numerically analyzing the corresponding Bures distance $D_q(x)$ as a function of x. The results for different values of the gain are reported in Fig. 2(a). Note that for small values of x the decay of $D_q(x)$ is far slower than for the coherent α-MQS case. Furthermore, interestingly enough, after a common inflexion point at $D_q \sim 0.6$ the *slope* of all functions $D_q(x)$ corresponding to different values of $\langle n\rangle$ increases fast towards the *infinite* value, for increasing $x \rightarrow \langle n\rangle$ and: $R \rightarrow 1$. All this means that the q-MQS visibility can be very large even if the average number x of lost particles is close to the initial total number $\langle n\rangle$, i.e. $R \sim 1$. This behavior is opposite to the α-MQS case where the function $D(R|\alpha|^2)$ approaches zero with a *zero slope*, as said. Note in Fig. 2(b) a faster decay of $D_q(|\Phi^H\rangle, |\Phi^V\rangle)$ due to the fact that $|\Phi^H\rangle$ and $|\Phi^V\rangle$ *do not* belong to the equatorial plane of the Poincaré sphere, i.e., to the privileged ϕ-covariant Hilbert subspace. This behavior supports all our previous experimental results, as the recent ones related to the microscopic-macroscopic nonseparability [15] and demonstrates the high resilience to decoherence of the q-MQS solution here considered.

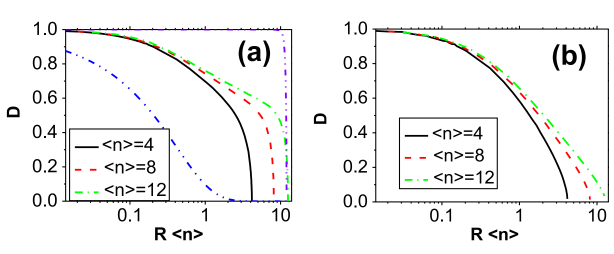

FIG. 2 (color online). (a) Numerical evaluation of the Bures distance $D_q(x)$ between two orthogonal equatorial macroqubits $|\Phi^{\phi,\phi_\perp}\rangle$ as function of the average lost particle $x = R\langle n\rangle$, plotted in a logarithmic scale. Black continuous line refers to $g = 0.8$ and to $\langle n\rangle \approx 4$, red dashed line to $g = 1.1$ and $\langle n\rangle \approx 8$, green dash-dotted line to $g = 1.3$ and $\langle n\rangle \approx 12$. The two dash-dot-dot lines correspond to the function $D_\alpha(x)$ for coherent state MQS. (b) Numerical evaluation of the Bures distance between $|\Phi^H\rangle$ and $|\Phi^V\rangle$ for the same values of the gain of (a).

The above considerations lead quite naturally to an important conjecture. If the structure of a generic quantum system allows any MQS $|\Theta^\pm\rangle = \frac{\mathcal{N}_\pm}{\sqrt{2}}(|\Xi^+\rangle \pm i|\Xi^-\rangle)$ and the component interfering macroscopic states $|\Xi^\pm\rangle$ to belong to a common covariant Hilbert subspace such as $D(|\Theta^+\rangle, |\Theta^-\rangle) = D(|\Xi^+\rangle, |\Xi^-\rangle)$, that MQS as well as the component macroscopic states are decoherence-free, i.e., bear at least approximately the property of the "pointer state". In other words this covariant subspace is the only location in the world assuring the detectable survival of all MQS. If the present theorem is true, we could have established a first, useful criterion for realizing any complex decoherence-free quantum information structure, e.g., a quantum computer.

The distinguishability between the pairs of states $\{|\Phi^\phi\rangle, |\Phi^{\phi_\perp}\rangle\}$ after losses was greatly improved by the use of the OF device, allowing the demonstration of the microscopic-macroscopic entanglement [15]. The *local* POVM-like OF technique [27] selects the events for which the difference between the photon numbers associated with two orthogonal polarizations $|m - n| > k$, i.e., larger than an adjustable threshold, k [16]. By this method a sharper discrimination between the output states $|\Phi^\phi\rangle$ and $|\Phi^{\phi_\perp}\rangle$ is achieved. This technique presents close analogies with the human-eye detection scheme presented in [20,21]. Indeed, as shown in Fig. 3(b) and 3(c), the two schemes select similar zone of the bidimensional Fock-Space proper of the QI-OPA output states. We analyzed the discriminating effects of the OF device on the orthogonal macroqubits after losses $\hat{\rho}_T^+$ and $\hat{\rho}_T^-$. In Fig. 3(a) the results of the numerical analysis carried out for $g = 0.8$ and different values of k are reported. Note the increase of the value of $D(x)$, i.e., of the q-MQS visibility, by increasing k. Of course, all this is achieved at the cost of a lower success probability. All these results are in agreement with a recent MQS experiment involving a number of photons in excess of 5×10^4, indeed the only experiment so far exploiting this technique [15]. There a q-MQS visibility $>50\%$ was measured upon a loss of more than $R \sim 90\%$ of the QI-OPA-generated particles, at *room temperature*.

The efficiency of the transfer of classical or quantum information in the interactive dynamics involving the paradigmatic quantum-statistical combination (system + environment) is at the focus of the present investigation.

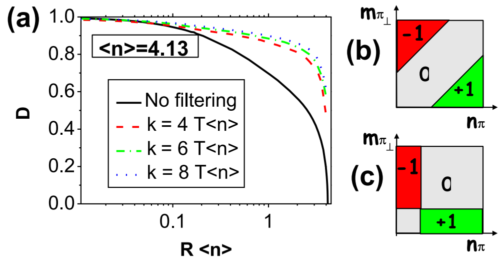

FIG. 3 (color online). (a) Numerical evaluation of the Bures distance between two orthogonal equatorial O-filtered macroqubits for different values of the threshold k ($g = 0.8$). (b) Bidimensional Fock-space representation of the action of the O-filter device. The various outcomes are assigned depending on the difference between the measured photon number in the two polarization n_π and $m_{\pi_\perp}$. The threshold k allows to vary the dimension of the selected zone. (c) Human-eye detection scheme presented in [20].

In order to gain insight into the general picture and to support the congruence of our final conclusions we find useful to relate here the various aspects of the cloning process with the current MQS physical models [8]. (i) The *system* in our entangled OPA scheme is represented by the assembly of $N + 1$ photon particles associated with the entangled singlet macroscopic state $2^{-(1/2)}(|\phi_\perp\rangle|\Phi^\phi\rangle - |\phi\rangle|\Phi_\perp^\phi\rangle)$ generated by the combined EPR—quantum cloning apparatus. (ii) The flow of (classical) ''noise information'' directed from the *environment* towards the *system* is generally provided by the unavoidable squeezed-vacuum noise affecting the building up of the macroscopic state $|\Phi^\phi\rangle$ within the OPA process. As already stressed, the *optimality* of the ϕ-covariant cloning generally implies, and literally means, that the flow of classical noise is the *minimum* allowed by the principles of quantum mechanics, i.e., by the *no-cloning theorem* [14,18]. (iii) The flow of quantum information directed from the system towards the environment is provided by the controlled decoherence in action provided by the artificial BS-scattering process and by the losses taking place in all photodetection processes. We have seen that by the use of the OF, or even in the absence of it, the interference phase-distrupting effects caused by the adopted artificial decoherence can be efficiently tamed and even cancelled for the ''equatorial'' macroscopic states and for their quantum superpositions. (iv) The selected ϕ-covariant cloning method considered here allows to define the equatorial plane as a privileged *minimum noise-minimum decoherence* Hilbert subspace of the quantum macroscopic states that, according to our decoherence model, exhibit simultaneously the maximum allowed distinguishability and visibility. (v) In any cloning apparatus a unitary $\hat{U}$ connects all physical properties belonging to the microscopic world to the corresponding ones belonging to the macrosopic classical world. Any lack of perceiving this close correspondence, for instance in connection with the realization of the 1935 Schrödinger's proposal must be only attributable to the intrinsic limitations of our perceiving senses, of our observational methods or of our measurement apparata. In other words, at least in our case, the two worlds are deterministically mirrored one into the other by the map $\hat{U}$ which is provided by quantum mechanics itself. This is the key to understand our results. (vi) The q-MQS based on the cloning process is not a ''thermodynamic'' system as its dynamics and decoherence do not depend on the temperature T. It rather belongs to, and indeed establishes a first and most insightful physical model of, the class of the *parametrically driven, open quantum-statistical systems* that have been recently invoked to provide and sustain extended long-range nonlocal coherence processes in complex biological photosyntetic systems [28].

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