Entanglement Test on a Microscopic-Macroscopic System

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A macrostate consisting of $\mathbf{N} \approx 3.5 \times 10^4$ photons in a quantum superposition and entangled with a far apart single-photon state (microstate) is generated. Precisely, an entangled photon pair is created by a nonlinear optical process; then one photon of the pair is injected into an optical parametric amplifier operating for any input polarization state, i.e., into a phase-covariant cloning machine. Such transformation establishes a connection between the single photon and the multiparticle fields. We then demonstrate the nonseparability of the bipartite system by adopting a local filtering technique within a positive operator valued measurement.

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In recent years two fundamental aspects of quantum mechanics have attracted a great deal of interest: namely, the investigation of the irreducible nonlocal properties of nature implied by quantum entanglement and the physical realization of the "Schrödinger cat" [1,2]. The last concept, by applying the nonlocality property to a combination of a microscopic and macroscopic systems, enlightens the concept of the quantum state, the dynamics of large systems and ventures into a most intriguing philosophical problem, i.e., the emergence of quantum mechanics in real life. In recent years quantum entanglement has been demonstrated within a two photon system [3], within a single photon and atomic ensemble [4,5] and within atomic ensembles [6–8]. While, according to the 1935 proposal, the nonlocal correlations were conceived to connect the dynamics of two "microscopic" objects, i.e., two spins within the well-known EPR-Bohm scheme [3], in the present work the entanglement is established between a microscopic and a "macroscopic", i.e., multiparticle quantum object, via cloning amplification: Fig. 1. The amplification is achieved by adopting a high-gain nonlinear (NL) parametric amplifier acting on a single-photon input carrier of quantum information, i.e., a qubit state: $|\phi\rangle$. This process, referred to as "quantum-injected optical parametric amplification" (QI-OPA) [9,10] turned out to be particularly fruitful in the recent past to gain insight into several little explored albeit fundamental, modern aspects of quantum information, as *optimal* quantum cloning machines [9,11,12], *optimal* quantum U-NOT gate [13], quantum no-signaling [14]. Here, by exploiting the amplification process, we convert by a unitary transformation a single-photon qubit into a single macroqubit involving a large number of photons, typically 5×10^4. At variance with the previous works [14,15], here we demonstrate for the first time the entanglement between the microscopic qubit and the macroscopic one obtained by the amplification process. This result is achieved performing a local dichotomic measurement on the multiphoton field. Let us venture in a more detailed account of our endeavor.

An entangled pair of photons in the singlet state $|\Psi^-\rangle_{A,B} = 2^{-(1/2)}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$ was produced through a spontaneous parametric down-conversion (SPDC) by the NL crystal 1 (C1) pumped by a pulsed UV pump beam: Fig. 2. There $|H\rangle$ and $|V\rangle$ stands, respectively, for a single photon with horizontal and vertical polarization while the labels A, B refer to particles associated, respectively, with the spatial modes $\mathbf{k}_A$ and $\mathbf{k}_B$. Precisely, A, B represent the two spacelike separated Hilbert spaces coupled by the entanglement. The photon belonging to $\mathbf{k}_B$, together with a strong ultraviolet (UV) pump laser beam, was fed into an optical parametric amplifier consisting of a NL crystal 2 (C2) pumped by the beam $\mathbf{k}'_P$. The crystal 2, cut for collinear operation, emitted over the two modes of linear polarization, respectively, horizontal and vertical associated with $\mathbf{k}_B$. The interaction Hamiltonian of the parametric amplification $\hat{H} = i\chi\hbar\hat{a}^\dagger_H\hat{a}^\dagger_V + \text{H.c.}$ acts on the single spatial mode $\mathbf{k}_B$ where $\hat{a}^\dagger_\pi$ is the one photon creation operator associated with the polarization $\vec{\pi}$. The main feature of this Hamiltonian is its property of "phase-covariance" for "equatorial" qubits $|\phi\rangle$, i.e., representing equatorial states of polarization, $\vec{\pi} = 2^{-1/2}(\vec{\pi}_H + e^{i\phi}\vec{\pi}_V)$, $\vec{\pi}_{\phi\perp} = \vec{\pi}^\perp_\phi$, in a Poincaré sphere representation having $\vec{\pi}_H$ and $\vec{\pi}_V$ as the opposite "poles" [15]. The equatorial qubits are expressed in terms of a

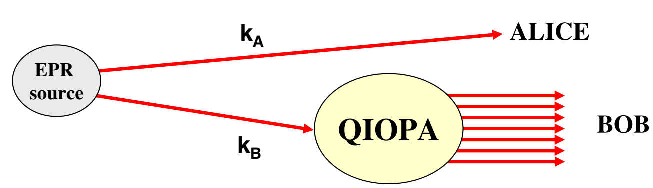

FIG. 1 (color online). Schematic diagram showing the single-photon quantum-injected optical parametric amplification.

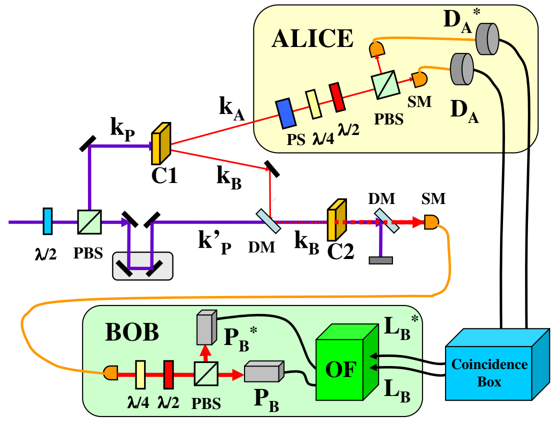

FIG. 2 (color online). Optical configuration of the QI-OPA apparatus. The excitation source was a Ti:sapphire Coherent MIRA mode-locked laser amplified by a Ti:sapphire regenerative REGA device operating with repetition rate 250 kHz. The output beam, frequency-doubled by second-harmonic generation, provided the OPA excitation field beam at the UV wavelength (wl) $\lambda_P = 397.5$ nm with power: 750–800 mW. A type II BBO crystal (crystal 1: C1) generates pair of photons with wavelength $\lambda = 2\lambda_p = 795$ nm. C1 generates an average photon number per mode equal to about 0.35, while the overall detection efficiency of the trigger mode was estimated to be $\simeq 5\%$ with detection rates of about 5 kHz. The NL BBO crystal 2: C2, realizing the optical parametric amplification, is cut for collinear type II phase matching. Both crystals C1 and C2 are 1.5 mm thick. The fields are coupled to single-mode fibers. The overall detection efficiency on mode k_B has been estimated to be $\sim 2\%$.

single phase $\phi \in (0, 2\pi)$ in the basis $\{|H\rangle, |V\rangle\}$. The overall output state amplified by the OPA apparatus is expressed, in any polarization equatorial basis $\{\vec{\pi}_\phi, \vec{\pi}_{\phi\perp}\}$, by the micro-macro entangled state [16]:

$$|\Sigma\rangle_{A,B} = 2^{-1/2}(|\Phi^\phi\rangle_B |1\phi^\perp\rangle_A - |\Phi^{\phi\perp}\rangle_B |1\phi\rangle_A), \quad (1)$$

where the mutually orthogonal multiparticle "macrostates" are

$$|\Phi^\phi\rangle_B = \sum_{i,j=0}^{\infty} \gamma_{ij} \frac{\sqrt{(1+2i)!(2j)!}}{i!j!} |(2i+1)\phi; (2j)\phi^\perp\rangle_B, \quad (2)$$

$$|\Phi^{\phi\perp}\rangle_B = \sum_{i,j=0}^{\infty} \gamma_{ij} \frac{\sqrt{(1+2i)!(2j)!}}{i!j!} |(2j)\phi; (2i+1)\phi^\perp\rangle_B, \quad (3)$$

with $\gamma_{ij} \equiv C^{-2}(-\frac{\Gamma}{2})^i \frac{\Gamma^j}{2}$, $C \equiv \cosh g$, $\Gamma \equiv \tanh g$, being g the NL gain [13]. There, $|p\phi; q\phi^\perp\rangle_B$ stands for a Fock state with p photons with polarization $\vec{\pi}_\phi$ and q photons with $\vec{\pi}_{\phi\perp}$ over the mode $\mathbf{k}_B$. Most important, any injected single-particle qubit $(\alpha|\phi\rangle_B + \beta|\phi^\perp\rangle_B)$ is transformed by the *information-preserving* QI-OPA operation into a corresponding macroqubit $(\alpha|\Phi^\phi\rangle_B + \beta|\Phi^{\phi\perp}\rangle_B)$, i.e., a macroscopic quantum superposition [10]. The quantum states of Eqs. (2) and (3) deserve some comments. The multiparticle states $|\Phi^\phi\rangle_B$, $|\Phi^{\phi\perp}\rangle_B$ are orthonormal and exhibit observables bearing macroscopically distinct average values. Precisely, for the polarization mode $\vec{\pi}_\phi$ the average number of photons is $\bar{m} = \sinh^2 g$ for $|\Phi^{\phi\perp}\rangle_B$, and $(3\bar{m} + 1)$ for $|\Phi^\phi\rangle_B$. For the π-mode $\vec{\pi}_{\phi\perp}$ these values are interchanged among the two macrostates. On the other hand, as shown by [10], by changing the representation basis from $\{\vec{\pi}_\phi, \vec{\pi}_{\phi\perp}\}$ to $\{\vec{\pi}_H, \vec{\pi}_V\}$, the same macrostates, $|\Phi^\phi\rangle_B$ or $|\Phi^{\phi\perp}\rangle_B$ are found to be quantum superpositions of two orthogonal states $|\Phi^H\rangle_B$, $|\Phi^V\rangle_B$ which differ by a single quantum. This unexpected and quite peculiar combination, i.e., a large difference of a measured observable when the states are expressed in one basis and a small Hilbert-Schmidt distance of the same states when expressed in another basis turned out to be a useful and lucky property since it rendered the coherence patterns of our system very robust toward coupling with environment, e.g., losses. The decoherence of our system was investigated experimentally and theoretically in the laboratory: cf.: [12,15,17].

As shown in Fig. 2, the single-particle field on mode $\mathbf{k}_A$ was analyzed in polarization through a Babinet-Soleil phase-shifter (PS), i.e., a variable birefringent optical retarder, two wave plates $\{\frac{\lambda}{4}, \frac{\lambda}{2}\}$ and polarizing beam splitter (PBS). It was finally detected by two single-photon detectors D_A and D_A^* (ALICE box). The multiphoton QI-OPA amplified field associated with the mode $\mathbf{k}_B$ was sent, through a single-mode optical fiber (SM), to a measurement apparatus consisting of a set of wave plates $\{\frac{\lambda}{4}, \frac{\lambda}{2}\}$, a (PBS) and two photomultipliers (PM) P_B and P_B^* (BOB box). The output signals of the PM's were analyzed by an "orthogonality filter" (OF) that will be described shortly in this Letter.

We now investigate the bipartite entanglement between the modes $\mathbf{k}_A$ and $\mathbf{k}_B$. We define the $\frac{1}{2}$-spin Pauli operators $\{\hat{\sigma}_i\}$ for a single-photon polarization state, where the label $i = (1, 2, 3)$ refers to the polarization bases: $i = 1 \Longleftrightarrow \{\vec{\pi}_H, \vec{\pi}_V\}$, $i = 2 \Longleftrightarrow \{\vec{\pi}_R, \vec{\pi}_L\}$, $i = 3 \Longleftrightarrow \{\vec{\pi}_+, \vec{\pi}_-\}$. Here $\vec{\pi}_R = 2^{-1/2}(\vec{\pi}_H - i\vec{\pi}_V)$, $\vec{\pi}_L = \vec{\pi}_R^\perp$ are the right- and left-handed circular polarizations and $\vec{\pi}_\pm = 2^{-1/2}(\vec{\pi}_H \pm \vec{\pi}_V)$. It is found $\hat{\sigma}_i = |\psi_i\rangle\langle\psi_i| - |\psi_i^\perp\rangle\langle\psi_i^\perp|$ where $\{|\psi_i\rangle, |\psi_i^\perp\rangle\}$ are the two orthogonal qubits corresponding to the $\vec{\pi}_i$ basis, e.g., $\{|\psi_1\rangle, |\psi_1^\perp\rangle\} = \{|H\rangle, |V\rangle\}$, etc. By the QI-OPA unitary process the single-photon $\hat{\sigma}_i$ operators evolve into the "macrospin" operators: $\hat{\Sigma}_i = \hat{U}\hat{\sigma}_i\hat{U}^\dagger = |\Phi^{\psi i}\rangle\langle\Phi^{\psi i}| - |\Phi^{\psi i\perp}\rangle\langle\Phi^{\psi i\perp}|$. Since the operators $\{\hat{\Sigma}_i\}$ are built from the unitary evolution of eigenstates of $\hat{\sigma}_i$, they satisfy the same commutation rules of the single-particle $\frac{1}{2}$-spin: $[\hat{\Sigma}_i, \hat{\Sigma}_j] = \varepsilon_{ijk} 2i\hat{\Sigma}_k$, where ε_{ijk} is the Levi-Civita tensor density. The generic state $(\alpha|\Phi^H\rangle_B + \beta|\Phi^V\rangle_B)$ is a macroqubit in the Hilbert space B spanned by $\{|\Phi^H\rangle_B, |\Phi^V\rangle_B\}$, as said. To test whether the overall output state is entangled, one should measure the correlation between the single-photon spin operator $\hat{\sigma}_i^A$ on the mode $\mathbf{k}_A$ and the macrospin operator $\hat{\Sigma}_i^B$ on the mode $\mathbf{k}_B$. We then adopt the criteria for two qubit bipartite systems based on the spin correlation. We define the "visibility" $V_i = |\langle \hat{\Sigma}_i^B \otimes \hat{\sigma}_i^A \rangle|$, a parameter which quantifies the correlation between the systems A and B. Precisely $V_i = |P(\psi_i, \Phi^{\psi i}) + P(\psi_i^\perp, \Phi^{\psi i \perp}) - P(\psi_i, \Phi^{\psi i \perp}) - P(\psi_i^\perp, \Phi^{\psi i})|$, where $P(\psi_i, \Phi^{\psi i})$ is the probability to detect the systems A and B in the states $|\psi_i\rangle_A$ and $|\Phi^{\psi i}\rangle_B$, respectively. The value $V_i = 1$ corresponds to perfect anticorrelation, while $V_i = 0$ expresses the absence of any correlation. The following upper bound criterion for a separable state holds [18]:

$$S = (V_1 + V_2 + V_3) \leq 1. \quad (4)$$

In order to measure the expectation value of $\hat{\Sigma}_i^B$ a discrimination among the pair of states $\{|\Phi^{\psi i}\rangle_B, |\Phi^{\psi i \perp}\rangle_B\}$ for the three different polarization bases 1, 2, 3 is required. Consider the macrostates $|\Phi^+\rangle_B$, $|\Phi^-\rangle_B$ expressed by Eqs. (2) and (3), for $\phi = 0$ and $\phi = \pi$. In principle, a perfect discrimination can be achieved by identifying whether the number of photons over the $\mathbf{k}_B$ mode with polarization $\vec{\pi}_+$ is even or odd, i.e., by measuring an appropriate "parity operator." This requires the detection of the macroscopic field by a perfect *photon-number resolving* detectors operating with an overall quantum efficiency $\eta \approx 1$, a device out of reach of the present technology.

It is nevertheless possible to exploit, by a somewhat sophisticated electronic device dubbed "orthogonality filter" (OF), the macroscopic difference existing between the functional characteristics of the probability distributions of the photon numbers associated with the quantum states $\{|\Phi^\pm\rangle_B\}$. The measurement scheme works as follows: Figs. 2 and 3. The multiphoton field is detected by two PM's (P_B, P_B^*) which provide the electronic signals (I_+, I_-) corresponding to the field intensity on the mode $\mathbf{k}_B$ associated with the π-components ($\vec{\pi}_+$, $\vec{\pi}_-$), respectively. By (OF) the difference signals $\pm(I_+ - I_-)$ are compared with a threshold $\xi k > 0$. When the condition $(I_+ - I_-) > \xi k$ is satisfied, the detection of the state $|\Phi^+\rangle_B$ is inferred and a standard transistor-transistor-logic (TTL) electronic square-pulse L_B is realized at one of the two output ports of (OF). This corresponds to the measurement of the eigenvalue $+1$ of the operator $\hat{\Sigma}_3^B$. Likewise, when the condition $(I_- - I_+) > \xi k$ is satisfied, the detection of the state $|\Phi^-\rangle_B$ is inferred, a TTL pulse is realized at other output port of (OF) and the eigenvalue of $\hat{\Sigma}_3^B$ is -1. The PM output signals are discarded for $-\xi k < (I_+ - I_-) < \xi k$, i.e., on condition of low state discrimination. By increasing the value of the threshold k an increasingly better discrimination is obtained together with a decrease of detection efficiency. This "local distillation" procedure is conceptually justified by the following theorem: since entanglement cannot be created or enhanced by any "local" manipulation of the quantum state, the nonseparability condition demonstrated for a "distilled" quantum system, e.g., after application of the OF-filtering procedure, fully applies to the same system in absence of distillation [18]. This statement can be applied to the measurement of I_ϕ and $I_{\phi\perp}$ for any pair of quantum states $\{|\Phi^\phi\rangle_B, |\Phi^{\phi\perp}\rangle_B\}$. This method is but an application of a positive operator value measurement procedure (POVM) [19] by which a large discrimination between the two states $\{|\Phi^\pm\rangle_B\}$ is attained at the cost of a reduced probability of a successful detection.

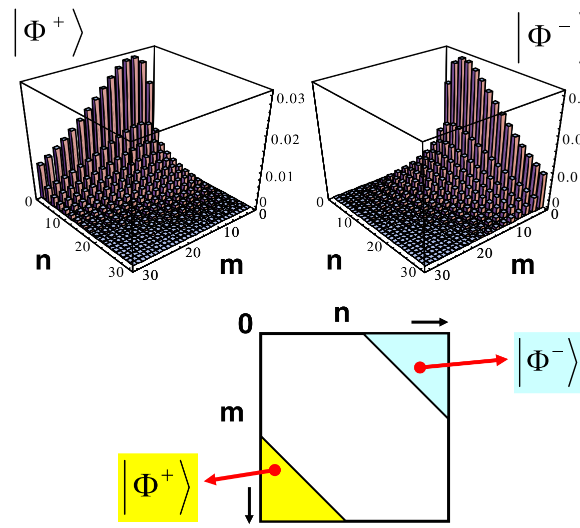

FIG. 3 (color online). Theoretical probability distributions $P^\pm(m, n)$ of the number of photons associated with the macrostates $|\Phi^\pm\rangle$ ($g = 1.6$). Probabilistic identification of the wave functions $|\Phi^\pm\rangle$ by OF-filtering the $P^\pm(m, n)$ distributions over the photon-number two-dimensional space $\{m, n\}$. The white section in the Cartesian plane (m, n) corresponds to the "inconclusive events" of our POVM OF-filtering technique.

The present experiment was carried out with a gain value $g = 4.4$ leading to a number of output photons $N \approx 3 \times 10^4$, after OF filtering. In this case the probability of photon transmission through the OF filter was $p \approx 10^{-4}$. A NL gain $g = 6$ was also achieved with no substantial changes of the apparatus. Indeed, an unlimited number of photons could be generated in principle by the QI-OPA technique, the only limitation being due to the fracture of the NL crystal 2 in the focal region of the laser pump. In order to verify the correlations existing between the single photon generated by the NL crystal 1 and the corresponding amplified macrostate, we have recorded the coincidences between the single-photon detector signal D_A (or D_A^*) and the TTL signal L_B (or L_B^*) both detected in the same π-basis $\{\vec{\pi}_+, \vec{\pi}_-\}$: Fig. 2. This measurement has been repeated by adopting the common basis $\{\vec{\pi}_R, \vec{\pi}_L\}$. Since the filtering technique can hardly be applied to the $\{\vec{\pi}_H, \vec{\pi}_V\}$ basis, because of the lack of a broader $SU(2)$ covariance of the amplifier, the quantity $V_1 > 0$ could be

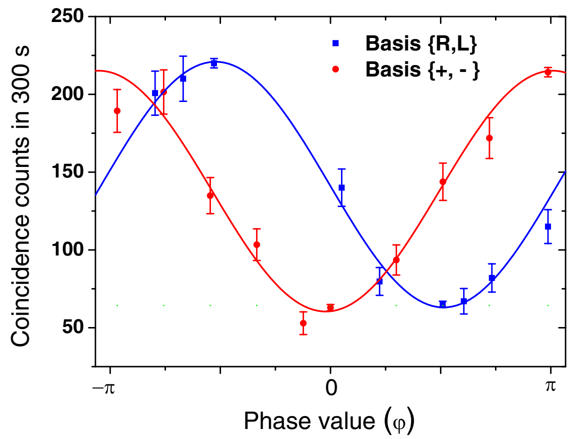

FIG. 4 (color online). Coincidence counts $[L_B, D_A]$ versus the phase ϕ of the injected qubit for the basis $\{\vec{\pi}_+, \vec{\pi}_-\}$ (circle data) and the basis $\{\vec{\pi}_R, \vec{\pi}_L\}$ (square data). The experimental points corresponding to the minima and maxima have been adopted to estimate V_2 and V_3. Accordingly, they have been determined by higher statistics and exhibit a smaller error flag.

measured adopting a photon-number resolving detector with quantum efficiency equal to 1: a device not made available by the present technology. The phase ϕ between the π-components $\vec{\pi}_H$ and $\vec{\pi}_V$ on mode $\mathbf{k}_A$ was determined by the Babinet-Soleil variable phase shifter (PS). Figure 4 shows the fringe patterns obtained by recording the rate of coincidences of the signals detected by the Alice's and Bob's measurement apparata, for different values of ϕ. These patterns were obtained by adopting the common analysis basis $\{\vec{\pi}_R, \vec{\pi}_L\}$ with a filtering probability $\simeq 10^{-4}$, corresponding to a threshold ξk about 8 times higher than the average photomultiplier signals I. In this case the average visibility has been found $V_2 = (54.0 \pm 0.7)\%$. A similar oscillation pattern has been obtained in the basis $\{\vec{\pi}_+, \vec{\pi}_-\}$ leading to: $V_3 = (55 \pm 1)\%$. Since always is $V_1 > 0$, our experimental result $S = V_2 + V_3 = (109.0 \pm 1.2)\%$ implies the violation of the separability criteria of Eq. (4) and then demonstrates the nonseparability of our micro-macro system belonging to the spacelike separated Hilbert spaces A and B. By evaluating the experimental value of the "concurrence" for our test, connected with the "entanglement of formation", it is obtained $C \geq 0.10 \pm 0.02 > 0$ [20,21]. This result again confirms the nonseparability of our bipartite system. The value of C could be increased by improving the matching of the injected photon with the pump field on crystal $C2$. A method similar to ours to test the nonseparability of a 2-atom bipartite system was adopted recently by [7].

In conclusion, we have demonstrated the entanglement of a micro-macro system, in which the macro state has been obtained by the amplification of a 1-photon qubit. The QI-OPA approach could be directly applicable to the field of quantum information and computation in virtue of the intrinsic *information-preserving* property of the QI-OPA dynamics. Indeed, this property implies the direct realization of the quantum map $(\alpha|\phi\rangle + \beta|\phi^\perp\rangle) \rightarrow (\alpha|\Phi^\phi\rangle + \beta|\Phi^{\phi\perp}\rangle)$ connecting any single-particle qubit to a corresponding macroqubit, by then allowing the extension to the multiparticle regime of most binary logic algorithms and techniques. For instance, let us consider a 2-qubit phase gate in which the control-target interaction is provided by a Kerr-type optical nonlinearity. The strength of this nonlinearity is far too small to provide a sizable interaction between the "control" and the "target" single-particle qubits. However, by replacing these ones by the corresponding macroqubits associated with N photons, the NL interaction strength can be enhanced by many orders of magnitude since the 3d-order NL polarization scales as $N^{3/2}$. This application is made possible by another general property of the QI-OPA scheme, i.e., the direct accessibility of the macrostates at the output of the QI-OPA. In summary, the amplification process applied to a micro system is a natural approach to enlighten the quantum-to-classical transition and to investigate the persistence of quantum phenomena into the "classical" domain by measurement procedures applied to systems of increasing size.

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