Experimental achievement of the entanglement-assisted capacity for the depolarizing channel

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We experimentally demonstrate the achievement of the entanglement-assisted capacity for classical information transmission over a depolarizing channel. The implementation is based on the generation and local manipulation of two-qubit Bell states, which are finally measured at the receiver by realizing projective measurements in the Bell basis. The depolarizing channel is realized by introducing quantum noise in a controlled way on one of the two qubits. This work represents an investigation into the amount of information that can be shared in the presence of noise.

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I. INTRODUCTION

Noise is unavoidably present in any realistic implementation of a communication channel. It is therefore of great importance to design strategies that allow the optimizion of the flow of information transmitted in the presence of noise. In the most general scenario information can be transmitted by quantum states, and several notions of efficiency can be defined according to the considered task and the resource available along the transmission channel. For example, a quantum communication channel can be designed to transmit classical [1], private classical [2], or quantum information [3], and it can be employed on its own or with the addition of other resources, such as entanglement. The case of an entanglement-assisted quantum communication channel occurs when classical information is transmitted and entanglement is *a priori* available between sender and receiver [4]. The capacity is then given by the maximum amount of information that can be transmitted over the channel by optimizing the input signals and the output decoding procedure. In this paper we consider the latter scenario and provide an experimental demonstration of the performance achievable by an entanglement-assisted depolarizing channel for qubits. The experimental realization presented in this work relies on a quantum optical implementation, but it lays the ground for new perspectives of applications to a great variety of communication scenarios. At variance with previous experimental realizations dealing with the case of a noiseless channel [5] or classical noise [6], our work gives an experimental investigation into the information capacity for a controlled noisy quantum communication channel.

An entanglement-assisted communication scenario is given by a (generally noisy) quantum channel along which quantum states can be transmitted by assuming that an unlimited amount of noiseless entanglement is *a priori* available between the sender and the receiver [4]. The entanglement-assisted classical capacity (EACC) is then given by the maximum amount of classical mutual information that can be transmitted over such a channel. In this work we consider the case of a depolarizing qubit channel [7], where the action on a given quantum state of a two-dimensional system ρ can be described as

$$\Gamma_{\{p\}}[\rho] = \sum_{i=0}^{3} p_i \sigma_i \rho \sigma_i. \tag{1}$$

Here σ_0 is the identity operator, $\{\sigma_i\}$ $(i = 1,2,3)$ are the three Pauli operators $\sigma_x,\sigma_y,\sigma_z$, respectively, and $p_0 = 1 - p$ (with $p \in [0,1]$), while $p_i = p/3$ for $i = 1,2,3$.

In this case the entanglement-assisted classical capacity takes the simple form [4]

$$C = 2 + (1 - p)\log_2(1 - p) + p\log_2(p/3). \tag{2}$$

The scheme considered in this work is composed of two qubits, initially prepared in the singlet state $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, where the states $|0\rangle$ and $|1\rangle$ are a basis for each qubit. The singlet state then represents the noiseless entangled state that is *a priori* shared by the sender and the receiver. Classical information is then encoded by the sender by performing, with equal probabilities, either the identity or one of the three Pauli operators on his qubit, which is then transmitted along the depolarizing channel to the receiver. The receiver finally performs a Bell measurement in order to retrieve the information encoded in the two-qubit system. This scenario allows to achieve the capacity (2) [8].

In this work we implement an experimental scheme corresponding to the above scenario and demonstrate that the capacity (2) can actually be achieved, allowing the optimal transmission of classical information through the channel and therefore the use of the channel at the best of its possible performances. We then measure the classical information transmitted through the channel in terms of the classical mutual information [9], which can be expressed as

$$I = \sum_x p_1(x) \sum_y p(y|x) \log_2 \frac{p(y|x)}{p_2(y)}, \tag{3}$$

where x and y are the input/output variables, with corresponding probability distributions $p_1(x)$ and $p_2(y)$, while $p(y|x)$ represents the conditional probability of receiving y given transmission of x. In our experiment the variables x and y correspond to the four Bell states, $p_1(x) = 1/4$, and $p(y|x)$ is the conditional probability of detecting the Bell state y given that the Bell state x is transmitted.

II. GENERAL SCHEME

We describe here the general experimental scheme to implement the scenario described above. The two-photon Bell states are engineered by exploiting a polarization entanglement

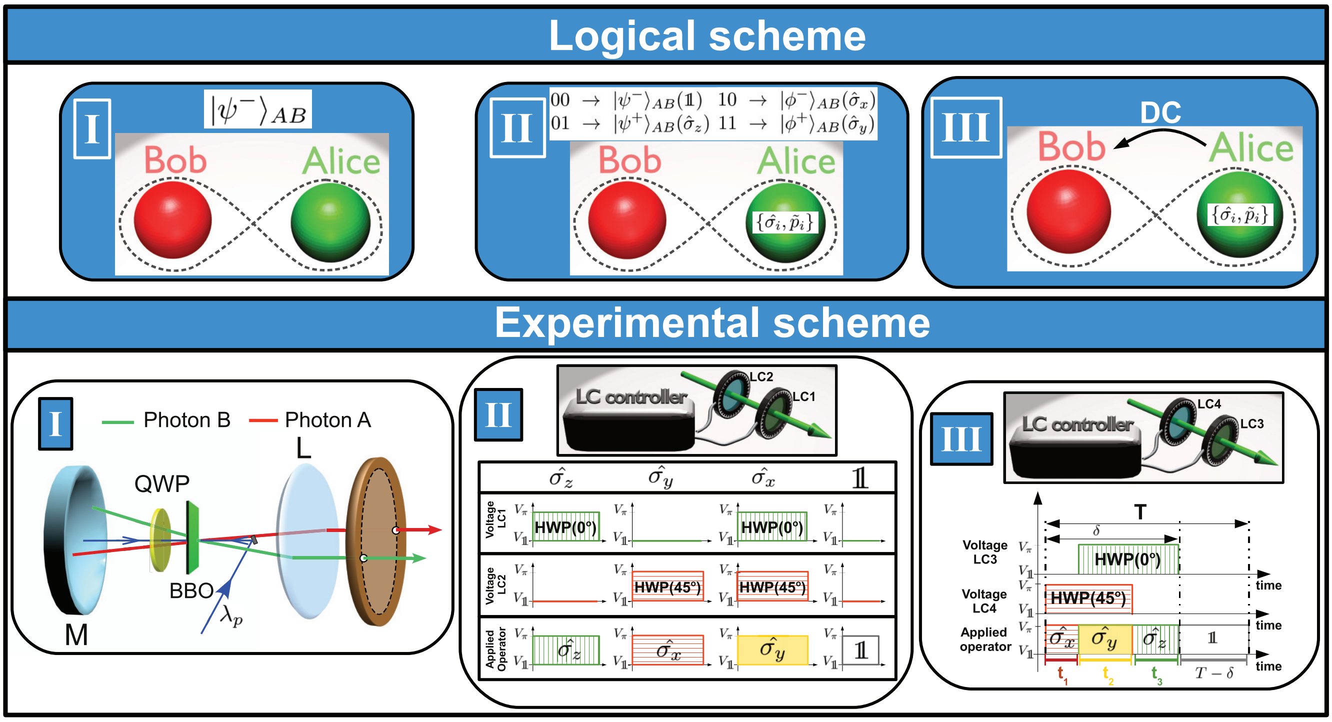

FIG. 1. (Color online) (I) Spontaneous parametric down conversion source of polarization entanglement. A β-barium borate (BBO) nonlinear crystal is illuminated by a vertically polarized continuous wave (cw) UV laser ($\lambda_p = 364$ nm), and the two photons are emitted at degenerate wavelength $\lambda = 728$ nm with horizontal polarization. Polarization entanglement is generated by the double passage (back and forth, after reflection on the spherical mirror M) of the UV beam. The backward emission generates the so-called V cone: The spontaneous parametric down conversion horizontally polarized photons passing twice through the quarter-wave plate (QWP) are transformed into vertically polarized photons. The forward emission generates the H cone. Because of the temporal and spatial superposition, the indistinguishability of the two perpendicularly polarized spontaneous parametric down conversion cones creates polarization entanglement $(|H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B)/\sqrt{2}$. The relative phase between the states $|HH\rangle_{AB}$ and $|VV\rangle_{AB}$ can be varied by translation of spherical mirror M. A lens L located at a focal distance from the crystal transforms the conical emission into a cylindrical one. (II) Alice encodes two bits of classical information by applying single-qubit Pauli transformations on one entangled particle. This can be realized by applying a suitable voltage to the liquid crystal modulator LC1 and LC2 acting as waveplates and implementing the transformations $\{\hat{\sigma}_i, \tilde{p}_i\}$. (III) Photon A is transmitted through a depolarizing channel (DC), here implemented by liquid crystal modulators LC3 and LC4. The LC's activation time corresponding to the three Pauli operators $\hat{\sigma}_x$, $\hat{\sigma}_y$, $\hat{\sigma}_z$ is the same, i.e., $t_1 = t_2 = t_3$.

source [10,11] [see Fig. 1(I)] which allows accurate generation of the states $|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, with qubit $|0\rangle$ ($|1\rangle$) corresponding to the horizontal H (vertical V) polarization of the photon. The other Bell states $|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ can be obtained by applying simple single-qubit local operations. Let us now assume that the initial state, shared by the sender Alice and the receiver Bob, is represented by the singlet $|\psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. In this case Alice encodes classical information by performing local transformations on photon A, which is then transmitted to Bob through the depolarizing channel, while photon B belongs to Bob.

In the experiment information was encoded by employing two liquid crystals (LC1 and LC2) acting simultaneously on photon A. This corresponds to apply the single-qubit local operation $\hat{\sigma}_i$. The LC's acted as phase retarders, with relative phase between the ordinary and extraordinary radiation components depending on the applied voltage V. Precisely, V_π and $V_{\mathbb{1}}$ [Fig. 1(II)] corresponded to the case of LC's operating as half-waveplate (HWP) and as the identity operator, respectively. The LC1 and LC2 optical axes were set at 0° and 45° with respect to the V polarization. With the voltage V_π applied, LC1 (LC2) acted as a σ_z (σ_x) on the single qubit. The simultaneous application of V_π on both LC1 and LC2 corresponds to the σ_y operation [as described in Fig. 1(II)], LC's were suitably activated by a remote control in order to perform the four Pauli operators with equal probabilities. In this way, depending on the transformation implemented by the LC's, Alice encodes two bits of classical information in the shared Bell state, i.e., $\mathbb{1} \rightarrow 00$, $\hat{\sigma}_z \rightarrow 01$, $\hat{\sigma}_x \rightarrow 10$, $\hat{\sigma}_y \rightarrow 11$.

The manipulated qubit was then sent through a depolarizing channel (DC). Several experimental techniques have been previously proposed and realized to engineer a DC [12–15]. In our case the noisy channel was implemented according to the scheme proposed in Ref. [16] [see Fig. 1(III)]. The necessary logical operations were implemented by careful adjustment of the voltage applied to two LC's and of the activation interval of each LC system. In the following we will discuss in more detail how the realized noisy channel works.

In order to evaluate the joint probabilities, the four possible outputs y need to be measured for each input state x. This final Bell measurement was realized with a set of projections onto the Bell states $\{|\psi^-\rangle\langle\psi^-|, |\psi^+\rangle\langle\psi^+|, |\phi^-\rangle\langle\phi^-|, |\phi^+\rangle\langle\phi^+|\}$ by using a suitable interferometric setup and by performing those local transformations which allow to select the desired Bell state. The present measurement scheme does not allow to perform the complete Bell state discrimination realized in other experiments based on the simultaneous entanglement of the photons in more degrees of freedom [5,17,18].

III. EXPERIMENTAL REALIZATION

Let us consider the expression of the classical mutual information given in Eq. (3). As mentioned above, after encoding, photon A is transmitted through the depolarizing channel. The experiment was performed by starting from the shared singlet state $|\psi^-\rangle$ and for each one of the other Bell states obtained after the encoding operation. The DC was performed by two liquid crystal retarders (LC3 and LC4) inserted within the path of one of the two photons. We were able to switch between $V_{\mathbb{1}}$ and V_π in a controlled way and independently for both LC3 and LC4. We could also adjust the temporal delay between the intervals corresponding to a V_π voltage applied to the two retarders. Let us define t_1, t_2, t_3 as the activation time of the operators σ_x, σ_y, or σ_z, respectively, while T is the period of the LC's activation cycle. The condition $t_1 = t_2 = t_3$ corresponds to the case of a depolarizing channel, with the three Pauli operators acting on the single qubit with the same probability $p_{\text{exp}} = \frac{\delta}{T} = \frac{t_1+t_2+t_3}{T}$. In the experiment, the parameter p_{exp} was varied by changing the interval δ for a fixed period T.

In Fig. 2 we show the actual interferometric setup adopted to perform projective measurements $\{|\psi^-\rangle\langle\psi^-|, |\psi^+\rangle\langle\psi^+|, |\phi^-\rangle\langle\phi^-|, |\phi^+\rangle\langle\phi^+|\}$. The transformation $\mathcal{U}$, sketched in the same figure, represents the different local unitaries necessary to transform each Bell state $|\phi^+\rangle$, $|\phi^-\rangle$, $|\psi^+\rangle$ into the singlet state $|\psi^-\rangle$, i.e., the only one that allows measurement of coincidences between the two output modes of the BS in condition of complete indistinguishability. They are implemented by proper setting of the optical axes of a quarter-wave plate and a half-wave plate. In this way we could perform a complete Bell measurement by carrying out the four projections at different times.

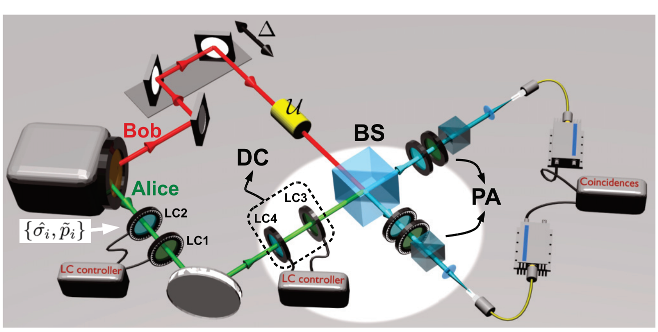

FIG. 2. (Color online) Experimental setup. Alice and Bob share a two-qubit singlet state. LC1 and LC2 implement the four Pauli operators σ_i with probability $\tilde{p}_i$, while LC3 and LC4 systems perform the depolarizing channel operation. By properly setting both temporal delay and spatial mode superposition, the two photons arrive in the BS in condition of complete indistinguishability. The noisy state is projected onto the four Bell states by exploiting the BS and the transformation $\mathcal{U}$. Each polarization analysis (PA) setup for each photon consists of a half-waveplate, a quarter-waveplate, and a polarizing BS inserted before the detectors.

For each value of the noise degree, the probabilities associated to the measurement of each Bell state, corresponding to the conditional probabilities $p(y|x)$ of Eq. (3), were obtained by considering the coincidence counts measured by the interferometer for each projection normalized over the results of the four measurements. After the transmission through the beamsplitter (BS), photons were coupled into single-mode fibers by using GRaded INdex (GRIN) lenses [19] and detected by single-photon detectors.

IV. EXPERIMENTAL RESULTS

Let us consider Eq. (3), with $x,y = 1,\ldots,4$ representing the four Bell states. In our case $1 \to |\psi^-\rangle$, $2 \to |\psi^+\rangle$, $3 \to |\phi^-\rangle$, $4 \to |\phi^+\rangle$. Since the probabilities of the four input Bell states are equal, we have $p_1(x) = \frac{1}{4}$. Therefore, Eq. (3) reads

$$I_{\text{meas}} = \frac{1}{4}\sum_{x,y} p(y|x)\log_2\frac{p(y|x)}{p_2(y)}. \tag{4}$$

The probabilities $p(y|x)$ in the above expression were measured by considering separately each Bell state x and by projecting it onto the four Bell states after the action of the DC. The experiment was carried out for each state obtained after the encoding operation and for several values of the noise degrees, i.e., of the parameter p_{exp}. The values of I_{meas} were then obtained from the measured probabilities as in Eq. (4) by summing the contributions of the four Bell states for each value of p.

The experimental data were analyzed by taking into account the imperfect purity of the actual input singlet state. This was evaluated for the singlet state $|\psi^-\rangle$ from the visibility $\mathcal{V} \approx 94\%$ of the coincidence count peak measured in condition of temporal and spatial indistinguishability (see the inset of Fig. 3). According to this value, we could express the actual input state entering the DC as

$$\rho_{\mathcal{V}} = \tfrac{1}{3}(1+2\mathcal{V})|\psi^-\rangle\langle\psi^-| + 2/3(1-\mathcal{V})\tfrac{\mathbb{1}}{4}. \tag{5}$$

When the visibility $\mathcal{V} = 1$ ($\mathcal{V} = -0.5$) only the contribution of the state $|\psi^-\rangle\langle\psi^-|$ ($\mathbb{1}/4$) is present and a peak (dip) in the coincidence counts is measured. A -50% visibility is expected by the complete polarization mixed state included in the second term of Eq. (5). In fact, it can be decomposed in the computational basis as $(|HH\rangle\langle HH| + |HV\rangle\langle HV| + |VH\rangle\langle VH| + |VV\rangle\langle VV|)/4$, with terms $|HH\rangle\langle HH|$, $|VV\rangle\langle VV|$ giving a -100% visibility, while the terms $|HV\rangle\langle HV|$, $|VH\rangle\langle VH|$ give a 0% contribution to visibility.

The state in Eq. (5) can be interpreted as the result of the action of a preliminary DC channel on the pure state $|\psi^-\rangle\langle\psi^-|$, characterized by the noise parameter $p' = (1-\mathcal{V})/2$. The action of the sequence of two DC's may be then expressed as a global DC with the following global noise parameter:

$$p = \mathcal{V}p_{\text{exp}} + p'. \tag{6}$$

We report in Fig. 3 the experimental results for the transmitted information with the theoretical value of the capacity (2) with p given by Eq. (6). The error bars have been obtained by propagating the Poissonian uncertainties associated to the coincidence counts. Each experimental point is also affected

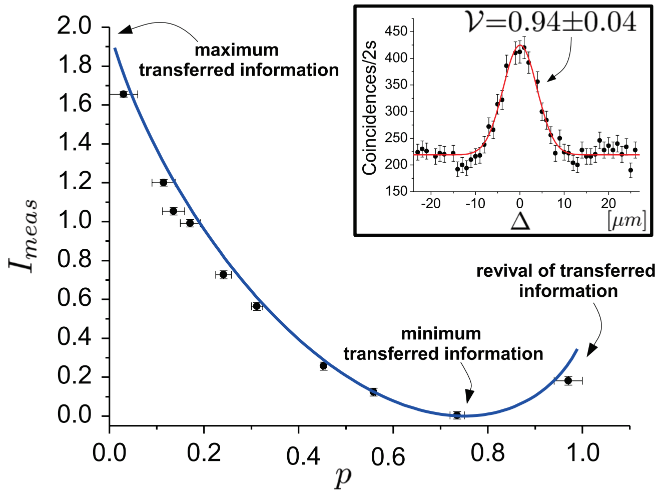

FIG. 3. (Color online) Measured values (black dots) of the mutual information and theoretical curve (blue line) for the entanglement-assisted capacity for several values of the parameter p which fully characterize the total amount of depolarizing noise introduced in the experiment. The error bars have been obtained by propagating the Poissonian uncertainties associated to the coincidence counts measured in 10 seconds. Inset: peak of the coincidence counts measured in 2 seconds as a function of the optical path delay Δ for a state $|\psi^-\rangle\langle\psi^-|$ entering the BS in the absence of controlled noise.

by the uncertainty of the parameter p deriving directly from the one of $\mathcal{V}$. The agreement between the experimental data analyzed as explained above and the theoretical behavior is very high.

V. DISCUSSION AND CONCLUSIONS

The theoretical curve in Fig. 3 allows to single out three working regimes corresponding to three different values of the noise parameter p:

(1) $p = 0$ (absence of noise). The maximum allowed transferred information consists of 2 bits.

(2) $p = 0.75$. The noisy state is given by the two-qubit maximally mixed state $\frac{\mathbb{1}}{4}$. In these conditions there is no correlation between the output and the input state, hence no information can be transmitted.

(3) $p > 0.75$. In spite of the large amount of noise, a partial revival of the mutual information is observable due to the nonuniform distribution of the conditional probabilities $p(y|x)$.

The capability of exploring the three regimes strongly depends on the purity of the entangled input state. It is remarkable that the purity, attainable from the visibility $\mathcal{V}$ in absence of any active noise applied by LC3 and LC4 (corresponding to an effective noise parameter $p = 0.03$), easily allows to achieve a quite high value of I_{meas}, $I_{\text{meas}} = 1.655 \pm 0.014$. As a further consideration, in these experimental conditions, the growing of mutual information occurring for large values of p (>0.75) may be clearly detected.

In this work we have given a proof-of-principle experimental demonstration of the entanglement-assisted capacity for classical information transmission over a depolarizing quantum communication channel, where classical information is encoded locally on a preshared maximally entangled state of two qubits and a controlled noise is then introduced on the transmitted qubit. Our experimental implementation of the protocol indicates a way to achieve the classical information capacity theoretically predicted for the depolarizing channel, therefore showing the optimal way in which the depolarizing channel can be used when classical information is to be transmitted and *a priori* entanglement is available.

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