Bell experiments with random destination sources

Fabio Sciarrino,[1,2,*] Giuseppe Vallone,[3,1,*] Adán Cabello,[4] and Paolo Mataloni[1,2,*]
[1]*Dipartimento di Fisica "La Sapienza," Università di Roma, Roma I-00185, Italy*
[2]*Istituto Nazionale di Ottica, Consiglio Nazionale delle Ricerche (INO-CNR), Largo E. Fermi 6, I-50125 Florence, Italy*
[3]*Centro Studi e Ricerche "Enrico Fermi," Via Panisperna 89/A, Compendio del Viminale, Roma I-00184, Italy*
[4]*Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain*
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It is generally assumed that sources randomly sending two particles to one or two different observers, random destination sources (RDSs), cannot be used for genuine quantum nonlocality tests because of the postselection loophole. We demonstrate that Bell experiments not affected by the postselection loophole may be performed with (i) an RDS and local postselection using perfect detectors, (ii) an RDS, local postselection, and fair sampling assumption with any detection efficiency, and (iii) an RDS and a threshold detection efficiency required to avoid the detection loophole. These results allow the adoption of RDS setups which are simpler and more efficient for long-distance free-space Bell tests, and extend the range of physical systems which can be used for loophole-free Bell tests.

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I. INTRODUCTION

An experimental loophole-free violation of a Bell inequality is of fundamental importance not only for ruling out the possibility of describing nature with local hidden-variable theories [1], but also for proving entanglement-assisted reduction of classical communication complexity [2], device-independent eternally secure communication [3], and random number generation with randomness certified by fundamental physical principles [4]. This explains the interest in loophole-free Bell tests over long distances. There are three types of loopholes: the locality loophole [5], the detection loophole [6], and the postselection loophole [7–9]. The locality loophole occurs when the distance between the local measurements is too small to prevent causal influences between one observer's measurement choice and the other observer's result. To avoid this possibility these two events must be spacelike separated. The detection loophole occurs when the overall detection efficiency is below a minimum value, so although the events in which both observers have results might violate the Bell inequality there is still the possibility to make a local hidden-variable model which reproduces all the experimental results. To avoid this possibility the overall detection efficiency must be larger than a threshold value. Finally, the postselection loophole occurs when the setup does not always prepare the desired state, so the experimenter postselects those events with the required properties. It has been shown that in certain configurations the rejection of undesired events can be exploited by a local model to imitate the predictions of quantum mechanics [7]. However, it has been recently pointed out that the postselection loophole is not due to the rejection of undesired events itself, but rather to the geometry of the setup. The loophole can be fixed with a suitable geometry without dropping the postselection [8,10]. This leads to the question of when Bell experiments with postselection are legitimate. The answer is interesting since sources with postselection can be simpler and more efficient.

In this paper we analyze schemes which are believed to be affected by the postselection loophole [9,11,12]. In particular, we focus on random destination sources (RDSs) emitting two photons such that sometimes one photon ends in Alice's detector and the other in Bob's, but sometimes both end in the same party's detector. The same reasoning can be used with massive particles, as discussed later. Hereafter we will consider as benchmark the source shown in Fig. 1 [13]: two photons with horizontal and vertical polarization are generated via collinear spontaneous parametric down-conversion (SPDC) and a beam splitter (BS) splits the photons over the modes A and B. Three different cases may occur: Both photons emerge on mode A ($|HV\rangle_A$), both on mode B ($|HV\rangle_B$), or the two photons are divided into different modes $[(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)/\sqrt{2}]$. This setup is a natural candidate for long-distance free-space experiments requiring quantum entanglement and, specifically, for satellite-based quantum communications [14,15] since it satisfies the requirements of high efficiency (due to the adoption of periodically-poled crystals), stability, compactness (the beam splitter could be manufactured onto a single chip with the SPDC source), and emission over a single spatial mode.

We will show that Bell experiments not affected by the postselection loophole can be performed in these cases: (i) an RDS and local postselection with perfect detectors; (ii) an RDS, local postselection, and fair sampling (FS) assumption for any value of detection efficiency; and (iii) an RDS and a threshold detection efficiency required to avoid the detection loophole.

Let us consider an experiment with RDSs producing two photons in different locations, Alice (A) and Bob (B), with probability p, two photons on Alice's side with probability $\frac{1-p}{2}$, and two photons on Bob's side with probability $\frac{1-p}{2}$. Detection efficiencies on Alice and Bob's sides are η_A and η_B, respectively. Alice and Bob also have photon number discriminators. The probability p will depend on the particular configuration. For instance, in the case of the source of Fig. 1, $p = 2T(1-T)$, where T is the transmittance of the beam splitter.

*http://quantumoptics.phys.uniroma1.it

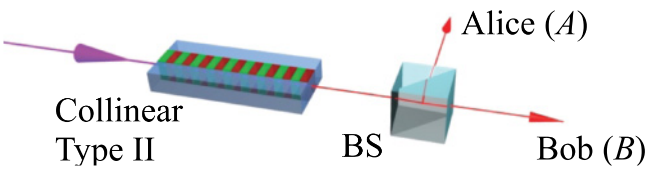

FIG. 1. (Color online) Collinear source of two photon states. The two particles are produced by type II phase matching and randomly broadcast to two observers A and B via a beam splitter (BS).

II. PERFECT DETECTORS AND LOCAL POSTSELECTION

Let us consider $\eta_A = \eta_B = 1$. Alice and Bob, depending on the measured number of photons, locally decide to postselect only those events in which entanglement has been successfully distributed (see column $\eta = 1$ of Table I). Does such local discarding of events introduce any loophole?

The answer is no, since the selection and rejection of events is independent of the local measurement settings (otherwise a local hidden-variable model could exploit the selection to violate the inequality). Indeed, any Bell experiment with local postselection, in the sense that it does not require Alice and Bob to communicate, is free of the postselection loophole. Local postselection is not a necessary requirement to be free of the postselection loophole (see [7] for a counterexample), but is a sufficient condition to be free of this loophole.

This is a crucial point which deserves a detailed examination. First, consider a selected event. The two photons have been detected at different locations, corresponding one to Alice's detector D_A and the other to Bob's detector D_B. If the detection in B is outside the forward light cone of the measurement setting in A (this is precisely the locality condition), no mechanism could turn a rejected event into a selected one (see Fig. 2). Next, consider a rejected event, for example when two photons have been detected at D_A. Both Alice (since she registers a double detection) and Bob (since he does not register any detection) locally discard the event. Again, due to the locality condition, the double detection at Alice's side cannot be caused by Bob's measurement setting, and the absence of Bob's detection cannot be influenced by Alice's measurement setting. The same happens when two photons go toward Bob's side.

TABLE I. Different types of events (I, II, and III), depending on the number of photons sent to observers A and B. Different nonlocality tests are considered according to the detection efficiency η, the adopted inequality [Clauser-Horne (CH), Clauser-Horne-Shimony-Holt (CHSH)], and whether fair sampling (FS) is assumed. The numbers refer to the photons generated or detected by each observer; no, discarded events; ×, events not discarded but not contributing to the inequality; and √, events which contribute to the inequality.

Generation			Detection					
			$\eta = 1$	$\eta \neq 1$				
Events	A	B	CHSH (no FS)	A	B	CHSH (FS)	CH (no FS)	CHSH (no FS)
I	1	1	√	1	1	√	√	√
				1	0	no	√	√
				0	1	no	√	√
				0	0	no	×	√
II	2	0	no	2	0	no	×	√
				1	0	no	√	√
				0	0	no	×	√
III	0	2	no	0	2	no	×	√
				0	1	no	√	√
				0	0	no	×	√

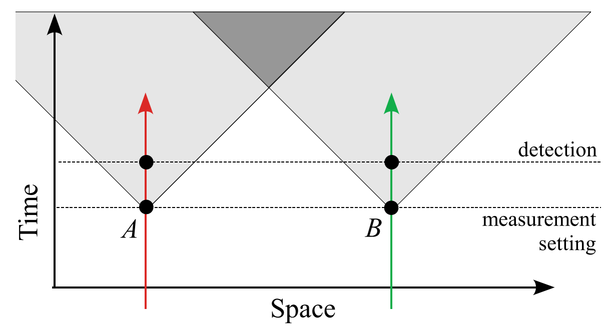

FIG. 2. (Color online) Space-time diagram for loophole-free Bell experiments.

Hence, when Alice or Bob locally discard the events, there is no physical mechanism preserving locality which can turn a selected (or rejected) event into a rejected (or selected) event. The selection of events is independent of the local settings. For the selected events only the result depends on the local settings. This is exactly the condition under which the Bell inequalities are valid. Therefore, an experimental violation of them based on local postselection with a random destination source provides a conclusive test of local realism when perfect detectors are used.

III. FAIR SAMPLING ASSUMPTION AND LOCAL POSTSELECTION

Let us now consider the case of imperfect detection efficiency of the measurement apparatus. Some of the events are lost and Alice and Bob only keep coincidences and assume fair sampling (i.e., that the coincidences are a statistically fair sample of the pairs in which one photon has gone to Alice and the other to Bob). Under this assumption, Bell experiments based on postselection are able to show violations of Bell's inequalities. Indeed, in the case of $p = 1$, the fair sampling assumption allows Alice and Bob to discard the event where just one particle is detected. When $p \neq 1$, we have already shown that the event can also be discarded due to double particle detection on the same observer. Hence, when fair sampling is assumed, there is no difference between Bell experiments with or without local postselection.

IV. THRESHOLD DETECTOR EFFICIENCY FOR LOOPHOLE-FREE TEST WITH RDSs

What happens when the fair sampling assumption is not considered? We will show that by considering all the possible events (i.e., without postselection or fair sampling) a loophole-free violation with RDSs can be obtained with a suitable threshold detection efficiency. For the Clauser-Horne (CH) [16] and the Clauser-Horne-Shimony-Holt (CHSH) Bell inequalities [17], this threshold can be obtained as follows.

In the perfect detection scenario (i), all the events in which two particles are sent to the same observer (events II and III in Table I) are discarded. Here, due to inefficiency, some of events II and III will contribute to the data and cannot be locally discarded (see the second column in Table I).

Let us consider two observers, Alice and Bob, with dichotomic observables $a_i = \pm 1$ and $b_i = \pm 1$, respectively. Any theory assuming realism and locality must satisfy the CH inequality,

$$I_{\rm CH} = p(a_1,b_1) + p(a_2,b_1) + p(a_1,b_2) - p(a_2,b_2) - p(a_1) - p(b_1) \leqslant 0, \tag{1}$$

where $p(a_i,b_j)$ is the probability that Alice obtains $a_i = 1$ and Bob obtains $b_j = 1$, while $p(a_1)$ and $p(b_1)$ are the probabilities that Alice and Bob obtain $a_1 = 1$ and $b_1 = 1$, respectively. We adopt one detector in each side corresponding to the $+1$ outcome. We set $a_i = +1$ ($b_i = +1$) when Alice (Bob) detects only one photon, while $a_i = -1$ ($b_i = -1$) when Alice (Bob) detects zero or two photons. In this way, the inequality is insensitive to any normalization implying that the vacuum contribution of standard SPDC sources does not contribute to $I_{\rm CH}$ (since the vacuum contribution corresponds to the outcomes $a_i = b_j = -1$). Then, event-ready sources or detectors [18] are not necessary to violate the inequality. Moreover, the events in which two particles are detected by Alice and no particle by Bob (or vice versa), indicated by $\times$ in Table I, do not contribute to Eq. (1). Indeed (1) involves only detection events in which at least one observable is $+1$.

Let us define Q as the value of I corresponding to the case when both particles are detected, M_A (M_B) as the value of I when only particle A (B) is detected from the (1 1) events, T_A (T_B) as the value of I when only particle A (B) is detected from (2 0) and (0 2) events, D_A (D_B) as the value of I when two particles are detected at side A (B), and X as the value when no particle is detected. Then, the average value of I will be

$$\begin{aligned}\langle I\rangle = {}& \eta_A^2\frac{1-p}{2}(D_A - 2T_A + X) + \eta_A[pM_A \\ & + (1-p)T_A - X] + \eta_B^2\frac{1-p}{2}(D_B - 2T_B + X) \\ & + \eta_B[pM_B + (1-p)T_B - X] + \eta_A\eta_B\, p(Q - M_A \\ & - M_B + X) + X.\end{aligned} \tag{2}$$

It is easy to show that, for the singlet entangled state and choosing the observables a_i and b_i that maximally violate the inequality, we obtain the following values for the CH inequality: $Q = \frac{1}{\sqrt{2}} - \frac{1}{2}$, $M_A = M_B = -\frac{1}{2}$, and $X = 0$. When the two particles are detected by Alice (Bob), we have $a_i = -1$ ($b_i = -1$), which implies $D_A = D_B = 0$. In order to calculate T_A (T_B), it is necessary to know the particular two-photon state sent to Alice (Bob). In most RDSs, when two photons are sent to the same observer they have orthogonal polarizations. This implies that when only one photon is detected, we have $T_A = T_B = -\frac{1}{2}$. The local realistic bound is violated when $\langle I\rangle > 0$:

$$\frac{1-p}{2}(\eta_A^2 + \eta_B^2) + p\,\eta_A\eta_B\left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right) - \frac{1}{2}(\eta_A + \eta_B) > 0. \tag{3}$$

Note that the (0 0), (2 0), and (0 2) events do not contribute to any term in Eq. (1).

In the symmetric case ($\eta_A = \eta_B = \eta$), the minimum detection efficiency is

$$\eta > \eta_{\rm crit} \equiv \frac{2}{2 + p(\sqrt{2} - 1)}. \tag{4}$$

For $p = 1$, we recover $\eta > \frac{2}{1+\sqrt{2}} \simeq 0.83$ [19]. For $p = 0.5$, we obtain $\eta > 0.90$. The RDS imposes a stricter constraint on the experimental setting, but a loophole-free nonlocality test can still be achieved.

For the fully asymmetric case ($\eta_A = 1$), we have $\eta_B > \eta_{\rm crit}$ with

$$\eta_{\rm crit} = \frac{-1 + p(1+\sqrt{2}) - \sqrt{4p(1-p) + (1 - p - \sqrt{2}p)^2}}{2(p-1)}. \tag{5}$$

In the limit $p \to 1$ we recover $\eta_B > \frac{1}{\sqrt{2}} \simeq 0.71$ [20]. In Fig. 3 we show the critical values of the efficiency in the symmetric and totally asymmetric cases. For the general case, Eq. (3), Fig. 4 shows the values of η_A and η_B allowing a loophole-free Bell test for different values of p.

It is worth noting that an equivalent result is obtained using the CHSH inequality,

$$I_{\rm CHSH} = \langle a_1b_1\rangle + \langle a_2b_1\rangle + \langle a_1b_2\rangle - \langle a_2b_2\rangle - 2 \leqslant 0, \tag{6}$$

by using the arguments given in [11]. When one observer detects no particle or two, that observer sets a_i (b_i) $= +1$. When Alice detects two photons and Bob no photon [the (2 0) events], we have $\langle a_1b_1\rangle = \langle a_2b_1\rangle = \langle a_1b_2\rangle = \langle a_2b_2\rangle = 1$ [and similarly for the (0 2) events]. The same happens for the (0 0) events (where neither Alice nor Bob detects a particle). If the source produces the singlet state when the two particles are sent to different observers, inequality (6) holds.

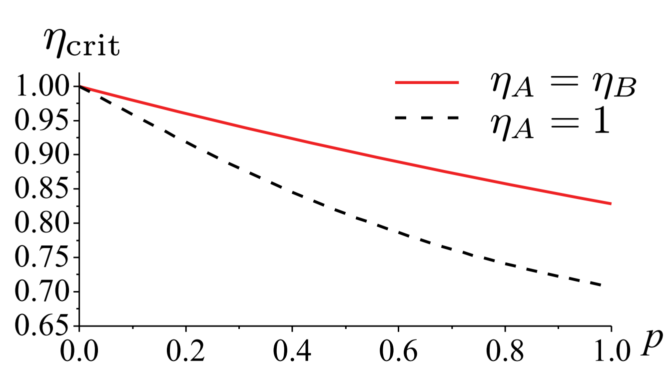

FIG. 3. (Color online) Critical efficiency for a loophole-free Bell test in the symmetric ($\eta_A = \eta_B > \eta_{\rm crit}$) or in the asymmetric ($\eta_A = 1, \eta_B > \eta_{\rm crit}$) case.

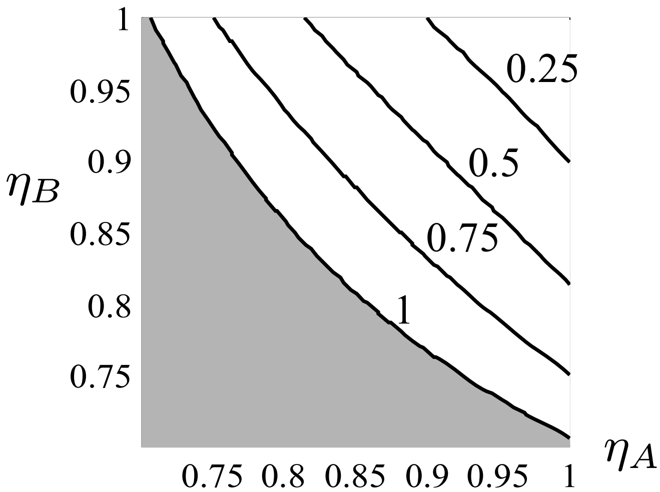

FIG. 4. Allowed values of η_A and η_B for a loophole-free Bell test for different values of p (1,0.75,0.5,0.25). For each value of p the allowed zone is in the upper-right part of the corresponding curve.

Finally, we demonstrate that in the asymmetric case $\eta_A \neq \eta_B$ an inequality with a lower bound with respect to Eq. (5) does exist for some values of p. Consider the I_{3322} inequality [21],

$$I_{3322} = p(a_1,b_1) + p(a_1,b_2) + p(a_1,b_3) + p(a_2,b_1) + p(a_2,b_2) + p(a_3,b_1) - p(a_2,b_3) - p(a_3,b_2) - 2p(a_1) - p(a_2) - p(b_1) \leqslant 0. \quad (7)$$

By setting $a_i = 1$ ($b_i = 1$) when Alice (Bob) detects zero or two photons, it is possible to show that the inequality is violated if

$$\frac{1-p}{2}(\eta_A^2 + 3\eta_B^2) + p\ \eta_A\eta_B\frac{9}{4} - \frac{1}{2}(\eta_A + 3\eta_B) > 0. \quad (8)$$

Equation (8) depends in a different way on η_A and η_B; hence we will consider separately the two conditions $\eta_A = 1$ and $\eta_B = 1$. We compare the efficiency threshold in Fig. 5. The plot is divided in three regions, (a), (b), and (c), depending on which inequality leads to the lowest efficiency threshold.

For $\eta_A = 1$, the lower bound on η_B is $\eta_B > 4p/(9p - 6 + \sqrt{36 - 60p + 33p^2})$, which is better than Eq. (5) for any $p > 0.863$ [(c) in Fig. 5]. For $p = 1$, we obtain the same results presented in [22]. By setting $a_i = -1$ ($b_i = -1$) when Alice (Bob) detects zero or two photons, we obtain the same result with $\eta_A \leftrightarrow \eta_B$. In this case, for $\eta_A = 1$, the lower bound on η_B is better than the CH condition for any $p < 0.099$ [see Fig. 5(a)]. In the central region (b), CH is still the optimal choice. Due to the T_A and T_B terms in Eq. (2), the efficiency bound depends on the specific form of the source. Here we have calculated the bound for the case of two photons sent to the same observers with orthogonal polarization.

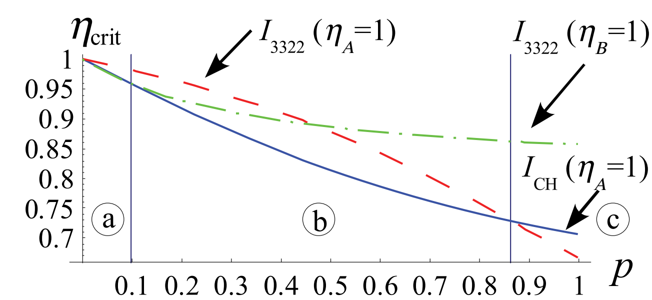

FIG. 5. (Color online) Critical detection efficiencies in the completely asymmetric case for the CH and the Collins-Gisin inequalities. I_{3322} with $\eta_A = 1$ (dashed line) and with $\eta_B = 1$ (dash-dotted line); CH (solid line). Labels (a), (b), and (c) identify the three ranges where different inequalities lead to the lower efficiency threshold.

Finally, RDSs are useful for loophole-free Bell tests beyond the most promising proposals to date (see [23] and references therein). This scenario is useful, for instance, when dealing with photon-atom entanglement. The idea is to combine RDSs of pairs of massive particles such as neutrons or molecules with interferometric setups like [8] into Bell experiments with postselection and take advantage of the fact that the detection efficiencies for these particles are above the thresholds obtained in this paper. So far, the scheme in [8] has been tested with photons [10] and electronic currents [24], but there seems to be no fundamental problem in performing similar experiments with molecules (see [25] for an example of an RDS with molecules) and accelerator-based sources of neutrons [26]. Our analysis could be extended to other types of nonlocality tests and/or generated states.

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