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Experimental Bell-inequality violation without the postselection loophole

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We report on an experimental violation of the Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) inequality using energy-time-entangled photons. The experiment is not free of the locality and detection loopholes but is the first violation of the Bell-CHSH inequality using energy-time entangled photons which is free of the postselection loophole described by Aerts *et al.* [Phys. Rev. Lett. **83**, 2872 (1999)].

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Introduction. The violation of Bell inequalities [1,2] has both profound implications for our understanding of the universe and striking applications like eavesdropping-proof communication [3], reduction of communication complexity [4], and randomness certification [5]. Testing Bell inequalities requires reliable methods for entangling, distributing, and measuring particles in spatially separated regions. So far, no experiment [6–10] has shown a conclusive violation of a Bell inequality. Local hidden variable (HV) models exist which reproduce the results of any performed experiment. These local HV models exploit the locality and the detection loopholes [11,12].

In 1989, Franson [13] introduced a setup which allows us to create and measure two energy-time-entangled photons over large distances. Franson's setup used the essential uncertainty in the time of emission of a pair of photons to make indistinguishable two alternative paths that the photons can take. As a result, photons detected in coincidence become entangled in path. This type of entanglement is usually called "energy-time" or "time-bin" entanglement, depending on the method used to have uncertainty in the time of emission [14]. Franson's setup is widely used for testing violations of Bell inequalities [15–26].

However, in the Franson's scheme, a new loophole, the "postselection" loophole, is present. Indeed, Aerts *et al.* [27] showed that, even in the ideal case of two-particle preparation, enough spatial separation between the local measurements (which closes the locality loophole) and perfect detection efficiency (which closes the detection loophole), there are local HV models that reproduce the quantum predictions for the violation of the Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) inequality [2] using Franson's setup. This is because, in Franson's setup, the fact that photons are detected in coincidence or not could depend on the local measurement settings. This can be exploited to build local HV models which simulate the quantum predictions for this experiment [27,28].

One way to deal with this "postselection loophole" that affects *all* performed Bell tests using energy-time- or time-bin-entangled photons [14–26] is to add an extra assumption: the fact that a photon is detected at a specific time is independent of the local experiment performed on that photon [27,29]. Therefore, even assuming ideal devices, Franson's setup can only rule out local LHV theories when this extra property is assumed. However, without it, the results of all previous tests of the Bell-CHSH inequality using Franson's setup can be reproduced with a local HV model like those given in Refs. [27,28].

Three different strategies have been proposed to avoid the postselection loophole in Bell tests with energy-time entanglement. Aerts *et al.* [27] showed that general local HV models are not possible using Franson's setup when very fast local switching is used and a specific three-setting Bell inequality (instead of the standard two-setting Bell-CHSH inequality) is violated. This approach has two problems: It requires a very difficult to achieve fast switching and a violation which is very close to the maximum violation predicted by QM (i.e., it requires nearly perfect visibility).

Brendel *et al.* [14] proposed replacing the two beam splitters which are closer to the source in Franson's setup with switchers synchronized with the source. However, these active switchers do not exist for photonic sources, thus in actual experiments they are are replaced by passive beam splitters (see, e.g., Ref. [14]), so the resulting setup suffers from the same problem of the original Franson's setup. Recently, it has been pointed out that active switchers could be feasible if photons are replaced by molecules [30].

More recently, some of us have proposed a modification of Franson's scheme in which both the short path of the first (second) photon and the long path of the second (first) photon end in the same observer [28]. In this scheme, the rejection of events is local (i.e., it does not require communication between the distant observers, as in Franson's scheme), and the selection of events is independent of the local settings. This property is the one that ensures that this scheme does not suffer from the postselection loophole that affect *all* previous Bell-CHSH experiments with energy-time- or time-bin-entangled photons. This scheme has inspired a new source of electronic entanglement [31] and has been theoretically extended to multiparty Bell experiments with energy-time-entangled photons [32].

In this Rapid Communication we present a postselection loophole-free violation of the the Bell-CHSH inequality using energy-time entangled photons. The experiment is based on the scheme proposed in [28].

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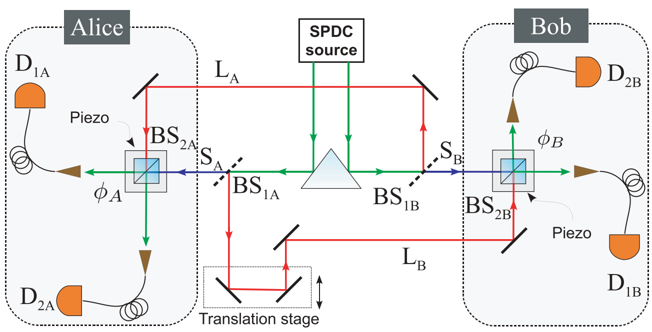

FIG. 1. (Color online) Experimental setup. A β-barium borate crystal splits incoming photons into pairs of photons of lower energy in a spontaneous parametric down-conversion (SPDC) process. The time of emission of each pair is unpredictable. A step-by-step translation stage allows us to create the indistinguishability condition ($L_A - S_A = L_B - S_B$). Detectors' dead time is 1 ns and, due to continuous wave pumping, the probability of two photon pair events is negligible. Alice and Bob's spacially separated experiments are indicated by boxes. The distance between Alice and Bob's experiments is approximately 1 m. The local phases ϕ_A and ϕ_B can be finely set by independent piezoelectric adjustment of beam splitters BS_{2A} and BS_{2B}, respectively.

Experimental setup. The experimental setup used for testing the Bell-CHSH inequality with energy-time-entangled photons is schematically shown in Fig. 1. A single-longitudinal-mode laser operating at 266 nm and with an average power of 70 mW is focused into a 5-mm-long β-barium borate (BBO) crystal cut for type I parametric down-conversion luminescence [33,34]. The generated energy-time-correlated photons are collimated by a 15-cm-focal-length lens put at focal distance from the crystal and then sent through two symmetrical interferometers. These interferometers are unbalanced, thus one can refer to their arms as short (S) and long (L). The optical paths of the down-converted photons are such that coincidences counts between detectors D_A and D_B can be seen only when they propagate through the short-short or long-long two-photon paths. These detectors are composed of interference filters with small bandwidths (5 nm FWHM), single-mode optical fibers, and pigtailed avalanche photo-counting modules that are connected to a circuit used to record the singles and the coincidences counts. The phases of local measurements (ϕ_A and ϕ_B) are set by moving piezoelectric driven stages on which the beam splitters are placed as it is shown in Fig. 1. The output D_{1A} (D_{2A}) corresponds to the projection onto $|\phi_A\rangle = \frac{1}{\sqrt{2}}(|L\rangle + e^{i\phi_A}|S\rangle)$ $[|\phi_A^{\perp}\rangle = \frac{1}{\sqrt{2}}(|L\rangle - e^{i\phi_A}|S\rangle)]$. A similar relation holds for the D_B detection.

The four probabilities of coincidence detection after the interferometers can be easily calculated [35]. They depend of the initial two-photon state, ρ, and in the case of down-converted photons belonging to the Bell state $\rho = |\Phi^+\rangle\langle\Phi^+|$, where $|\Phi^+\rangle \equiv \frac{1}{\sqrt{2}}(|SS\rangle + |LL\rangle)$, these probabilities will be given by

$$P_{ij}(\phi_A,\phi_B) = \tfrac{1}{4}[1 - (-1)^{\delta_{ij}} V_{ij}\cos(\phi_A + \phi_B)], \quad (1)$$

where $P_{ij}(\phi_A,\phi_B)$ is the probability of coincidence detection between D_{iA} and D_{jB}, and V_{ij} is the corresponding visibility of the interference.

A Bell inequality can be tested by considering the coincidence counts recorded by two detectors placed behind of the interferometers. Nevertheless, in this case the experiment requires an extra fair sampling assumption (e.g., that detected photons are a statistically representative subsensemble of the photons detected in the case where four detectors are present) and reduces the credibility of the conclusion. This extra assumption is not necessary in a test of the Bell-CHSH inequality, where the coincidence counts between all the four output ports of these interferometers are considered.

In our experiment, we recorded the coincidence counts between the detector D_{1A} (D_{2A}) and the detectors D_{1B} and D_{2B} illustrated in Fig. 1. We used these coincidences to test the Bell-CHSH inequality and to reconstruct the density operator of the energy-time-entangled two-photon state used in the test of the Bell-CHSH inequality.

Our experiment is not free of the locality loophole, since the distance between Alice and Bob's local experiments is around 1 m, which is not enough to prevent reciprocal influences between the two local phase settings. Furthermore, our experiment, like any other photonic Bell experiment, is also not free of the detection loophole, since the overall detection efficiency is less than 15%. The main aim of the experiment is to provide a proof of principle that, with enough separation and more efficient detectors, a conclusive violation of the Bell-CHSH inequality can be observed with energy-time-entangled photons.

Any local HV model should satisfy the Bell-CHSH inequality

$$S \leqslant 2, \quad (2)$$

where

$$S \equiv E(\phi_A,\phi_B) + E(\phi_A',\phi_B) + E(\phi_A,\phi_B') - E(\phi_A',\phi_B') \quad (3)$$

and

$$E(\phi_A,\phi_B) \equiv P_{11}(\phi_A,\phi_B) + P_{22}(\phi_A,\phi_B) - P_{12}(\phi_A,\phi_B) - P_{21}(\phi_A,\phi_B). \quad (4)$$

If all the four possible interference curves have the same average visibility V, the initial two-photon state is $\rho = |\Phi^+\rangle\langle\Phi^+|$, and the phase settings are

$$\phi_A = \frac{\pi}{4}, \quad \phi_B = 0, \quad \phi_A' = -\frac{\pi}{4}, \quad \phi_B' = \frac{\pi}{2}, \quad (5)$$

then, the expected value for S is $S = 2\sqrt{2}V$. Therefore, we will have an experimental violation of S whenever the

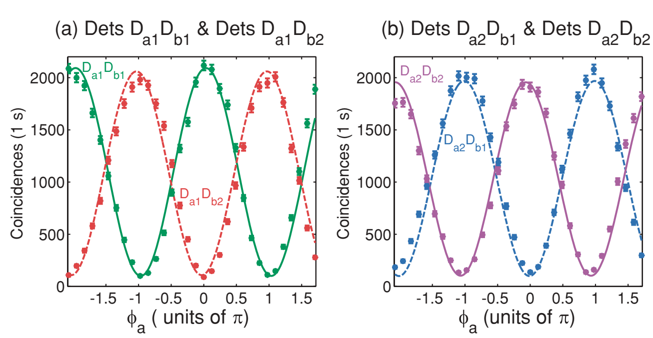

FIG. 2. (Color online) The four coincidence curves obtained while $\phi_B = 0$.

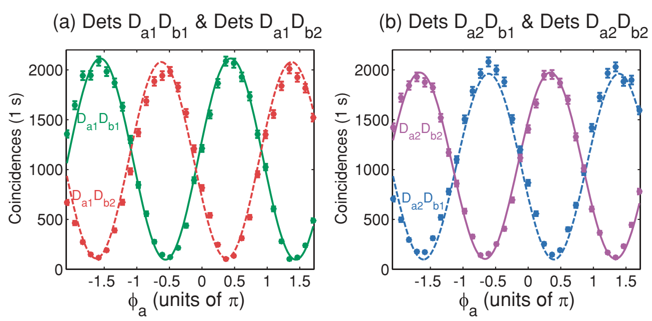

FIG. 3. (Color online) The four coincidence curves obtained while $\phi_B = \frac{\pi}{2}$.

average visibility of the two-photon interferences is larger than $1/\sqrt{2} \approx 0.71$.

The correlation functions E that appear in the Bell-CHSH inequality can be determined directly from the coincidence counts recorded by the four detectors. The experimental coincidence counts recorded between the four detectors, when $\phi_B = 0$ and $\phi_B = \frac{\pi}{2}$, are shown in Figs. 2 and 3, respectively. From these curves we calculated the values of the probability correlation functions E and, therefore, S. We calculated S in two different ways: directly from the experimental points obtained and from the values of the fit of the experimental data. Both are shown in Table I.

The mean visibility observed in the curves of Figs. 2 and 3 is $V = 0.90 \pm 0.015$, and therefore the expected value of S is $S_{\text{exp}} = 2\sqrt{2}V = 2.54 \pm 0.04$. The experimental results obtained for S (see Table I) are slightly lower than S_{exp} due to small phase shifts that exists between the eight curves recorded. However, they correspond to a violation of the Bell-CHSH inequality by 20 standard deviations.

The energy-time two-photon state was tested by quantum tomography. The reconstructed density operator was obtained considering an analogy between the projective phase measurements and the measurements used for doing the standard quantum tomography of polarization two-photon states [36]. The data acquired and the analogy between these measurements are shown in Table II.

The density operator ρ_{exp} obtained after numerical optimization is shown in Fig. 4. It has a fidelity [37] of 0.93 ± 0.02 with the Bell state $\rho = |\Phi^+\rangle\langle\Phi^+|$. For this reconstruction, we did not consider accidental substraction. In this case, the predicted maximum value of S is $\text{tr}(S\rho_{\text{exp}}) = 2.488$. The actual measured values of S (see Table I) are very close to this value. By considering accidental subtraction, the fidelity of the reconstructed state was 0.97 ± 0.02.

TABLE I. Experimental violation of the Bell-CHSH inequality obtained (a) from the experimental points recorded and (b) based on the experimental fits. In both cases we did not subtract accidental coincidences. SD refers to Standard Deviations.

	S	δS	SD
(a)	2.468	0.024	19.57
(b)	2.473	0.024	20.02

TABLE II. Summary of the phase measurements used to perform the reconstruction of the energy-time-entangled photon state. We compare the measurements with those used in the quantum tomography of two polarized qubits. $|H\rangle$, $|V\rangle$, $|D\rangle$, $|l\rangle$, and $|r\rangle$ represent polarized qubits with horizontal, vertical, diagonal, and left- and right-circular polarization, respectively. $|S\rangle$ and $|L\rangle$ represent short and long path states. The total coincidence, C_T, was recorded in 2 s.

Reconstructing ρ				
Meas.	Pol. proj.	Prop. Mode A	Prop. Mode B	C_T
1	$\|VV\rangle\langle VV\|$	$\|L\rangle$	$\|L\rangle$	3058
2	$\|HV\rangle\langle HV\|$	$\|S\rangle$	$\|L\rangle$	31
3	$\|HH\rangle\langle HH\|$	$\|S\rangle$	$\|S\rangle$	3416
4	$\|VH\rangle\langle VH\|$	$\|L\rangle$	$\|S\rangle$	35
5	$\|Vl\rangle\langle Vl\|$	$\|L\rangle$	$\|\phi_B = \frac{\pi}{2}\rangle$	1737
6	$\|Hl\rangle\langle Hl\|$	$\|S\rangle$	$\|\phi_B = \frac{\pi}{2}\rangle$	1799
7	$\|HD\rangle\langle HD\|$	$\|S\rangle$	$\|\phi_B = 0\rangle$	1708
8	$\|VD\rangle\langle VD\|$	$\|L\rangle$	$\|\phi_B = 0\rangle$	1797
9	$\|lD\rangle\langle lD\|$	$\|\phi_A = \frac{\pi}{2}\rangle$	$\|\phi_B = 0\rangle$	1795
10	$\|DD\rangle\langle DD\|$	$\|\phi_A = 0\rangle$	$\|\phi_B = 0\rangle$	3304
11	$\|Dl\rangle\langle DL\|$	$\|\phi_A = 0\rangle$	$\|\phi_B = \frac{\pi}{2}\rangle$	1727
12	$\|DV\rangle\langle DV\|$	$\|\phi_A = 0\rangle$	$\|L\rangle$	1762
13	$\|DH\rangle\langle DH\|$	$\|\phi_A = 0\rangle$	$\|S\rangle$	1801
14	$\|rH\rangle\langle rH\|$	$\|\phi_A = \frac{\pi}{2}\rangle$	$\|S\rangle$	1713
15	$\|rV\rangle\langle rV\|$	$\|\phi_A = \frac{\pi}{2}\rangle$	$\|L\rangle$	1744
16	$\|rl\rangle\langle rl\|$	$\|\phi_A = \frac{\pi}{2}\rangle$	$\|\phi_B = \frac{\pi}{2}\rangle$	97

Conclusions. We have presented a violation of the Bell-CHSH inequality using energy-time-entangled photons which is free of the postselection loophole reported by Aerts *et al.* [27] and which affects all previous Bell experiments using energy-time- or time-bin-entangled photons [14–26].

The experiment is not free of the locality and detection loopholes. Increasing the length of the interferometers and adding a fast switch between the local setting is required to scape from the locality loophole. The required values and the stability problems were discussed in Ref. [28]. Increasing the efficiency of the photodetectors or replacing low-energy photons by easily detectable particles, like massive particles, is required to escape from the detection loophole. However, the experiment provides a proof of principle that the scheme proposed in Ref. [28] is suitable for a testing the Bell-CHSH inequality with energy-time-entangled particles. Further developments might include nonphotonic versions of the setup and larger-scale implementations.

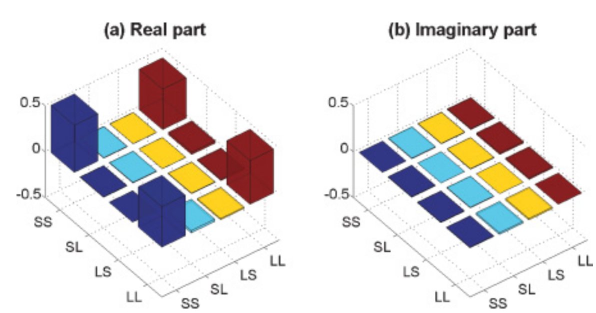

FIG. 4. (Color online) Tomographic reconstruction of the quantum state used for the Bell-CHSH test.

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