Implementation of optimal phase-covariant cloning machines

Fabio Sciarrino[1,2] and Francesco De Martini[2]
[1]*Centro di Studi e Ricerche "Enrico Fermi," Via Panisperna 89/A, Compendio del Viminale, Roma 00184, Italy*
[2]*Dipartimento di Fisica and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Universitá "La Sapienza," Roma 00185, Italy*
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The optimal phase-covariant quantum cloning machine (PQCM) broadcasts the information associated to an input qubit into a multiqubit system, exploiting a partial *a priori* knowledge of the input state. This additional *a priori* information leads to a higher fidelity than for the universal cloning. The present article first analyzes different innovative schemes to implement the $1\rightarrow 3$ PQCM. The method is then generalized to any $1\rightarrow M$ machine for an odd value of M by a theoretical approach based on the general angular momentum formalism. Finally different experimental schemes based either on linear or nonlinear methods and valid for single photon polarization encoded qubits are discussed.

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The problem of manipulating and controlling the flux of quantum information between many quantum systems has in general been tackled and solved by the theory of quantum cloning and broadcasting [1–3]. From a practical point of view, this feature renders the theory of cloning a fundamental tool for the analysis of the security of quantum cryptographic protocols, for the distribution of quantum information to many partners, and for the transmission of information contained in a system into correlations between many systems. In spite of the fact that, for fundamental reasons, the quantum cloning and flipping operations over an unknown qubit $|\phi\rangle$ are unrealizable in their exact forms [4,5], they can be optimally approximated by the corresponding universal quantum machines, i.e., the universal optimal quantum cloning machine (UQCM) and the universal-NOT (U-NOT) gate [6]. The $N\rightarrow M$ UQCM transforms N input qubits in the state $|\phi\rangle$ into M output qubits, each one in the same mixed state ρ_{out}. The quality of the copies is quantified by the fidelity parameter $\mathcal{F}_{univ}^{N\rightarrow M}=\langle\phi|\rho_{out}|\phi\rangle=\frac{N+1+\beta}{N+2}$ with $\beta=\frac{N}{M}\leqslant 1$. The optimal quantum cloning machine has been experimentally realized following different approaches: by exploiting the process of stimulated emission [7–9], by means of a quantum network [10], and by adopting projective operators into the symmetric subspaces of many qubits [11–13].

Not only the "universal" cloning of any unknown qubit is forbidden, but also the cloning of subsets containing nonorthogonal states. This no-go theorem ensures the security of cryptographic protocols as Bennett-Brassard (1984) (BB84) [14]. Recently *state-dependent,* nonuniversal, optimal cloning machines have been investigated where the cloner is optimal with respect to a given ensemble [15]. This partial *a priori* knowledge of the state allows one to reach a higher fidelity than for the universal cloning. The simplest and most relevant case is represented by the cloning covariant under the Abelian group U(1) of phase rotations, the so-called "phase-covariant" cloning. There the information is encoded in the phase ϕ_i of the input qubit belonging to any equatorial plane i of the corresponding Bloch sphere. In this context the general state may be expressed as $|\phi_i\rangle=[|\psi_i\rangle+\exp(i\phi_i)|\psi_i^{\perp}\rangle]$ and $\{|\psi_i\rangle,|\psi_i^{\perp}\rangle\}$ is a convenient normalized basis, $\langle\psi_i|\psi_i^{\perp}\rangle=0$ [15]. Precisely, in the general case the $N\rightarrow M$ phase covariant cloning map C_{NM} satisfies the following covariance relation, $C_{NM}(T_{\phi i}^{\otimes N}\rho_N T_{\phi i}^{\dagger\otimes N})=T_{\phi i}^{\otimes M}C_{NM}(\rho_N)T_{\phi i}^{\dagger\otimes M}$, where ρ_N is the density matrix of the N input qubits and $T_{\phi i}=\exp[-\frac{i}{2}\phi_i\sigma_i]$. There the σ_i Pauli operator identifies the set of input states which are cloned, e.g., σ_Y corresponding to states belonging to the x-z plane of the Bloch sphere. The values of the optimal fidelities $\mathcal{F}_{cov}^{N\rightarrow M}$ for this machine have been found in Ref. [16]. Restricting the analysis to a single input qubit to be cloned $N=1$ into $M>1$ copies, as we do in the present paper, the "cloning fidelity" is found: $\mathcal{F}_{cov}^{1\rightarrow M}=\frac{1}{2}\left(1+\frac{M+1}{2M}\right)$ for M assuming odd values, or $\mathcal{F}_{cov}^{1\rightarrow M}=\frac{1}{2}\left(1+\frac{\sqrt{M(M+2)}}{2M}\right)$ for M even. In particular, we have $\mathcal{F}_{cov}^{1\rightarrow 2}=0.854$ and $\mathcal{F}_{cov}^{1\rightarrow 3}=0.833$ to be compared with the corresponding figures valid for universal cloning: $\mathcal{F}_{univ}^{1\rightarrow 2}=0.833$ and $\mathcal{F}_{univ}^{1\rightarrow 3}=0.778$.

In the present framework it is worthwhile to enlighten the deep connection between the cloning processes and the theory of quantum measurement [17]. Indeed, the concept of universal quantum cloning is related to the problem of optimal quantum state estimation since, for $M\rightarrow\infty$, and $\beta\rightarrow 0$ the cloning fidelity converges toward the fidelity of state estimation of an arbitrary unknown qubit: $\mathcal{F}_{univ}^{N\rightarrow M}\rightarrow\mathcal{F}_{estim}^{N}=\frac{N+1}{N+2}$ [18]. In a similar way, the phase-covariant cloning is connected with the estimation of an equatorial qubit, that is, with the problem to find the optimal strategy to estimate the value of the phase ϕ [19,20]. The optimal strategy has been found in [20]: it consists of a positive operator-valued measure (POVM) measurement corresponding to a von Neumann measurement onto the N input qubits characterized by a set of $N+1$ orthogonal projectors and achieves a fidelity $\mathcal{F}_{phase}^{N}$. In general for $M\rightarrow\infty$, $\mathcal{F}_{cov}^{N\rightarrow M}\rightarrow\mathcal{F}_{phase}^{N}$. In particular, we have $\mathcal{F}_{cov}^{1\rightarrow M}=\mathcal{F}_{phase}^{1}+\frac{1}{4M}$ with $\mathcal{F}_{phase}^{1}=3/4$ for M assuming odd values.

Recently the experimental realization of the $1\rightarrow 3$ phase-covariant quantum cloning machine (PQCM) has been reported by adopting the methods of quantum optics [21]. The present article introduces in Sec. I different alternative approaches to implement the $1\rightarrow 3$ device for any quantum information technique. To this purpose, the projection of the input qubit and appropriate ancillary resources over the sym-

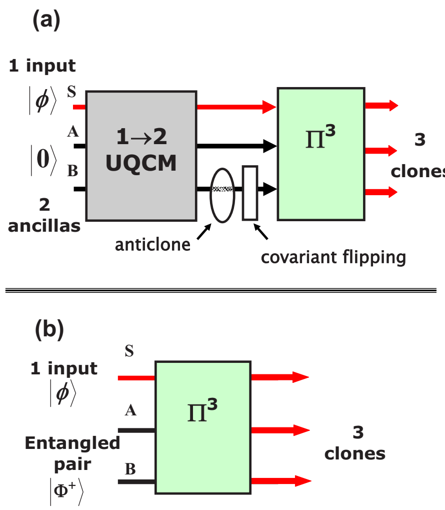

FIG. 1. (Color online) Scheme for the realization of the $1\rightarrow 3$ PQCM. (a) UQCM, phase-covariant flipping, and projection of the output state over the symmetric subspace adopting Π^3. (b) Symmetrization process acting on the input qubit and one entangled pair of qubits.

metric subspace is exploited. In Sec. II such methods are extended to any $1\rightarrow M$ PQCM machine for odd values of M. There the corresponding theoretical analysis based on the well-established $|J,J_z\rangle$ angular momentum formalism of a general J-spin system will be given. Finally, in Sec. III different experimental schemes that can be adopted for a single-photon polarization encoded qubit based either on linear and/or nonlinear methods will be presented.

I. REALIZATION OF THE 1→3 PHASE-COVARIANT CLONING MACHINE

In this section we describe two different techniques to implement the $1\rightarrow 3$ PQCM. (a) The first method combines the implementation of a $1\rightarrow 2$ UQCM, together with a spin flipper σ_i and the projection of the output qubits over the symmetric subspace: Fig. 1(a). (b) The second one exploits the symmetrization of the input qubit to clone with an ancillary entangled pair: Fig. 1(b).

We describe first the approach (a) introduced in [21]. The input qubit is expressed as $|\phi\rangle_S = 2^{-1/2}[|R\rangle_S + \exp(i\phi_Y)|L\rangle_S] = \alpha|0\rangle_S + \beta|1\rangle_S$, with $\langle R|L\rangle = 0$, $|\alpha|^2+|\beta|^2=1$, and α, β real parameters. Here we consider, in particular, the ϕ_Y covariant cloning and $\sigma_i = \sigma_Y$ realizes the NOT gate for the qubits belonging to the x-z plane. The output state of the $1\rightarrow 2$ UQCM device reads

$$|\Sigma\rangle_{SAB} = \sqrt{\frac{2}{3}}|\phi\rangle_S|\phi\rangle_A|\phi^\perp\rangle_B - \frac{1}{\sqrt{6}}(|\phi\rangle_S|\phi^\perp\rangle_A + |\phi^\perp\rangle_S|\phi\rangle_A)|\phi\rangle_B, \quad (1)$$

where the qubits S and A are the optimal cloned qubits while the qubit B is the optimally flipped one. According to the scheme represented by Fig. 1(a), the idea is to exactly flip the qubit B for a given subset of the Bloch sphere. This local flipping transformation of $|\phi\rangle_B$ leads to

$$|\Upsilon\rangle_{SAB} = (I_S \otimes I_A \otimes \sigma_Y)|\Sigma\rangle_{SAB} = \sqrt{\tfrac{2}{3}}|\phi\rangle_S|\phi\rangle_A|\phi\rangle_B - \tfrac{1}{\sqrt{6}}(|\phi\rangle_S|\phi^\perp\rangle_A + |\phi^\perp\rangle_S|\phi\rangle_A)|\phi^\perp\rangle_B.$$

By this nonuniversal cloning process three *asymmetric* copies have been obtained: two clones (qubits S and A) with fidelity 5/6, and a third one (qubit B) with fidelity 2/3. We may now project S, A, and B over the symmetric subspace and obtain three symmetric clones with a higher average fidelity. The symmetrization operator Π^3_{SAB} reads as $\Pi^3_{SAB} = |\Pi_1\rangle\langle\Pi_1| + |\Pi_2\rangle\langle\Pi_2| + |\Pi_3\rangle\langle\Pi_3| + |\Pi_4\rangle\langle\Pi_4|$, where $|\Pi_1\rangle = |\phi\rangle_S|\phi\rangle_A|\phi\rangle_B$, $|\Pi_2\rangle = |\phi^\perp\rangle_S|\phi^\perp\rangle_A|\phi^\perp\rangle_B$, $|\Pi_3\rangle = \frac{1}{\sqrt{3}}(|\phi\rangle|\phi^\perp\rangle|\phi^\perp\rangle + |\phi^\perp\rangle|\phi\rangle|\phi^\perp\rangle + |\phi^\perp\rangle|\phi^\perp\rangle|\phi\rangle)$, and $|\Pi_4\rangle = \frac{1}{\sqrt{3}}(|\phi\rangle|\phi\rangle|\phi^\perp\rangle + |\phi^\perp\rangle|\phi\rangle|\phi\rangle + |\phi\rangle|\phi^\perp\rangle|\phi\rangle)$. The symmetric subspace has dimension 4 since three qubits are involved. The probability of success of the projection is equal to $\frac{8}{9}$. The normalized output state $|\xi\rangle_{SAB} = \Pi^3_{SAB}|\Upsilon\rangle_{SAB}$ is

$$|\xi\rangle_{SAB} = \frac{1}{2}\sqrt{3}[|\phi\rangle_S|\phi\rangle_A|\phi\rangle_B - 3^{-1}(|\phi\rangle_S|\phi^\perp\rangle_A|\phi^\perp\rangle_B + |\phi^\perp\rangle_S|\phi\rangle_A|\phi^\perp\rangle_B + |\phi^\perp\rangle_S|\phi^\perp\rangle_A|\phi\rangle_B)]. \quad (2)$$

Let us now estimate the output reduced density matrices of the qubits S, A, and B: $\rho_S = \rho_A = \rho_B = \frac{5}{6}|\phi\rangle\langle\phi| + \frac{1}{6}|\phi^\perp\rangle\langle\phi^\perp|$. This leads to the fidelity $\mathcal{F}^{1\rightarrow 3}_{cov} = 5/6$ equal to the optimal one obtained in the general case [15,16]. By applying a different unitary operator σ_i to the qubit B we can implement the phase-covariant cloning for the corresponding different equatorial planes of the Bloch sphere, orthogonal to the i axis.

Let us now consider the second approach (b), which represents an innovative simplification of the previous scheme. The PQCM device can be realized by applying the symmetrization projection Π^3_{SAB} to the input qubit and to an ancillary entangled pair $|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$. The output state reads

$$\Pi^3_{SAB}(|\phi\rangle_S \otimes |\Phi^+\rangle_{AB}) = |\xi\rangle_{SAB}. \quad (3)$$

Again the qubits S, A, and B are found to be the optimal phase-covariant clones of the input one. By modifying the ancillary entangled state, the set of states cloned is changed. Indeed the state $|\Psi^+\rangle_{AB}$ leads to the PQCM machine for the y-z plane, while $|\Phi^-\rangle_{AB}$ for the x-y plane. Such result is at variance with the one found for the universal cloning process [12]. The $1\rightarrow 3$ UQCM of Ref. [12] can be achieved by applying the projector Π^3_{SAB} to the qubit $|\phi\rangle_S$ and to two ancillas qubits, each one in a fully mixed state $\frac{I}{2}$.

II. GENERAL APPROACH: 1→M DEVICE

In the present section we generalize the previous scheme for an arbitrary odd number of clones. The $1\rightarrow M = 2P-1$ PQCM can realized through two approaches: the first one (a) exploits the universal cloning machine, covariant flipping, and final symmetrization while the second one (b) is based on appropriate symmetrization of the input qubit with ancillary entangled pairs of qubits.

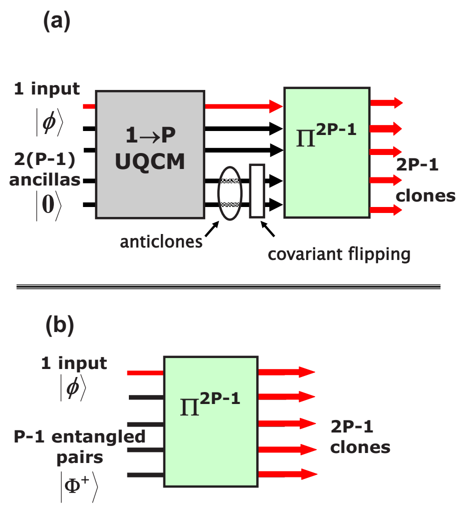

FIG. 2. (Color online) General scheme for the realization of the $1\rightarrow(2P-1)$ PQCM. (a) $1\rightarrow P$ UQCM, phase-covariant flipping, and projection of the output state over the symmetric subspace applying the projector Π^{2P-1}. (b) Symmetrization process acting on the input qubit and $(P-1)$ entangled pairs of qubits.

Let us consider the scheme of Fig. 2(a). The $1\rightarrow P$ UQCM broadcasts the information on the input qubit over $2P-1$ qubits. The overall output state after the UQCM map reads

$$|\Omega'\rangle=\sum_{k=0}^{P-1} b_k|\{(P-k)\phi;k\phi^{\perp}\}\rangle_C\otimes|\{k\phi;(P-1-k)\phi^{\perp}\}\rangle_{AC}, \tag{4}$$

where $b_k=(-1)^k\sqrt{\frac{2}{P+1}}\sqrt{\frac{(P-1)!(P-k)!}{P!(P-1-k)!}}$ and the notation $|\{p\phi;q\phi^{\perp}\}\rangle$ stands for a total symmetric combination of p qubits in the state $|\phi\rangle$ and of q qubits in the state $|\phi^{\perp}\rangle$ [6]. The labels C and AC identify, respectively, the cloning and anticloning subsystems. Hereafter, we assume the input qubit to be in the state $|\phi\rangle=|0\rangle$ without lack of generality. We associate to each qubit state a spin $\frac{1}{2}$ system. The previous expression can hence be expressed by exploiting the formalism of the angular momentum $|J,J_z\rangle$ of a general J-spin system. The overall state in the basis $|j;m_j\rangle_C\otimes|j;m_j\rangle_{AC}$ reads

$$|\Omega'\rangle=\sum_{k=0}^{P-1} b_k\left|\frac{P}{2};\frac{P}{2}-k\right\rangle_C\otimes\left|\frac{P-1}{2};\frac{-(P-1)}{2}+k\right\rangle_{AC}. \tag{5}$$

In the above representation, the overall output state of the cloner is written as the composition of two angular momenta: $\mathbf{J}_C,\mathbf{J}_{AC}$ defined, respectively, over the "cloning" and "anticloning" output channels.

As the following step, a covariant flipping process is applied to the subspace AC transforming $|\Omega'\rangle$ into

$$|\Omega''\rangle=I_C\otimes(\sigma_Y^{\otimes(P-1)})_{AC}|\Omega'\rangle=\sum_{k=0}^{P-1} b_k\left|\frac{P}{2};\frac{P}{2}-k\right\rangle_C\otimes\left|\frac{P-1}{2};\frac{(P-1)}{2}-k\right\rangle_{AC}. \tag{6}$$

Such an expression holds for any qubit belonging to the equatorial plane under consideration. Let us now express $|\Omega''\rangle$ adopting the overall angular momentum $\mathbf{J}_T=\mathbf{J}_C+\mathbf{J}_{AC}$ in the basis $|j_C;j_{AC};j_T;m_T\rangle$

$$|\Omega''\rangle=\sum_{j_T=1/2}^{2P-1}\sum_{m_T=-j_T}^{j_T} c(j_T,m_T)\left|\frac{P}{2};\frac{P-1}{2};j_T;m_T\right\rangle, \tag{7}$$

where $c(j_T,m_T)$ can be derived exploiting the Clebsch-Gordan coefficient $\langle j_1;j_2;m_1;m_2|j_1;j_2;j_T;m_T\rangle$ with $j_1=\frac{P}{2}$, $j_2=\frac{P-1}{2}$, $m_{1k}=\frac{P}{2}-k$, $m_{2k}=\frac{(P-1)}{2}-k$ [22].

To complete the protocol, the overall output state is symmetrized by applying the projector Π^M with $M=2P-1$ defined as $\Pi^M=\sum_{j=0}^{M}|\frac{P}{2};\frac{P-1}{2};\frac{M}{2};\frac{M}{2}-j\rangle\langle\frac{P}{2};\frac{P-1}{2};\frac{M}{2};\frac{M}{2}-j|$. The nonvanishing contributions to the projected state comes from terms with $j_T=\frac{2P-1}{2}$. After the action of Π^M we obtain the following normalized output state

$$|\Omega'''\rangle=\sum_{k=0}^{P-1} d_k\left|\frac{P}{2};\frac{P-1}{2};\frac{2P-1}{2};\frac{2P-1}{2}-2k\right\rangle$$

with

$$d_k=(-1)^k\sqrt{\frac{2}{P+1}}\binom{P-1}{k}\binom{2P-1}{2k}^{-1/2}$$

and the normalization factor reads

$$|\Pi^M|\Omega''\rangle|^2=\frac{2}{P+1}\sum_{k=0}^{P-1}\frac{\binom{P-1}{k}^2}{\binom{2P-1}{2k}}. \tag{8}$$

The fidelities of the phase-covariant cloning process can be inferred rearranging the output state as follows

$$|\Omega^M\rangle=\sum_{k=0}^{2P-1} d_k|\{(2P-1-2k)\phi;2k\phi^{\perp}\}\rangle. \tag{9}$$

All the $2P-1$ qubits belonging to such a state have an identical reduced density matrix equal to

$$\rho_{cov}=\mathcal{F}_{1\rightarrow M}|\phi\rangle\langle\phi|+(1-\mathcal{F}_{1\rightarrow M})|\phi^{\perp}\rangle\langle\phi^{\perp}| \tag{10}$$

with

$$\mathcal{F}_{1\rightarrow M}=\frac{1}{2}\left(1+\frac{M+1}{2M}\right).$$

The previous expression has been demonstrated numerically, for values of M up to 2000, and is found equal to the optimal one.

As an alternative approach, the $1\rightarrow M$ PQCM device can be obtained by applying the symmetrization projector Π^M over the input qubit and $(P-1)$ ancilla entangled pairs

$|\Phi^+\rangle_{AB}$: Fig. 2(b). Such a result can easily be obtained by manipulating the scheme of Fig. 2(a) as follows. The UQCM of Fig. 2(a) can be realized starting from the input qubit $|\phi\rangle$ and $(P-1)$ entangled pairs $|\Psi^-\rangle_{AB}$ as shown in Ref. [12]. The cloning map is achieved by symmetrization of the input qubit and $(P-1)$ ancilla qubits A, each one belonging to an entangled pair $|\Psi^-\rangle_{AB}$,

$$\Pi^P_{SA} \otimes I^{P-1}_B(|\phi\rangle_S|\Psi^-\rangle^{\otimes(P-1)}_{AB}). \tag{11}$$

The output state is equal to the one $|\Omega'\rangle$ of Eq. (5) up to a normalization factor. To implement the PQCM device, the covariant flipping σ_Y is then applied to the $(P-1)$ qubits belonging to the subset B,

$$(I^P_{SA} \otimes \sigma^{\otimes(P-1)}_{Y-B})[\Pi^P_{SA} \otimes I^{P-1}_B(|\phi\rangle_S|\Psi^-\rangle^{\otimes(P-1)}_{AB})]$$
$$= [\Pi^P_{SA} \otimes I^{P-1}_B(|\phi\rangle_S|\Phi^+\rangle^{\otimes(P-1)}_{AB})]. \tag{12}$$

As the final step the overall state is projected into the symmetric subspace through the projector Π^{2P-1}_{SAB},

$$|\Omega'''\rangle = \Pi^{2P-1}_{SAB}[\Pi^P_{SA} \otimes I^{P-1}_B(|\phi\rangle|\Phi^+\rangle^{\otimes(P-1)}_{AB})] \tag{13}$$

$$= \Pi^{2P-1}_{SAB}(|\phi\rangle_S|\Phi^+\rangle^{\otimes(P-1)}_{AB}). \tag{14}$$

In the previous expression we have exploited the concatenation property of the symmetrization projector $\Pi^{2P-1}_{SAB}(\Pi^P_{SA} \otimes I^{P-1}_B) = \Pi^{2P-1}_{SAB}$, which has been demonstrated experimentally in Ref. [23]. This concludes our simple proof of the scheme of Fig. 2(b).

III. REALIZATION BY QUANTUM OPTICS

In quantum optics the qubit can be implemented by exploiting the isomorphism between the qubit state $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ and the polarization state $\alpha|H\rangle + \beta|V\rangle$ of a single photon. In this context it has been proposed to realize the unitary transformation, $U_{N\rightarrow M}$, leading to the deterministic UQCM, by means of the "quantum injected" optical parametric amplification (QIOPA) in the entangled configuration. The experimental demonstrations of both optimal cloning and flipping processes by exploiting this technique have been reported in [8,12,13]. At the same time, a different scenario has been disclosed by the discovery that it is possible to implement contextually the $1\rightarrow 2$ universal quantum cloning machine (UQCM) and the $1\rightarrow 1$ universal NOT gate by modifying the quantum state teleportation protocol [11,12]. The last procedure is based on a symmetric projective operation realized by combining single-photon interferometry and postselection techniques, and it can be extended to the generic $N\rightarrow M$ cloning device.

The symmetrization of two polarization encoded qubits can be achieved by letting two independent qubits impinge onto the input arms of a beam splitter (BS) in an Hong-Ou-Mandel interferometer [24], and then by probabilistically postselecting the events in which the two photons emerge in the same spatial output mode. The basic principle at the heart of these realizations is the following: the two photons are initially superimposed at the BS interface in order to make them indistinguishable; then, a spatial symmetric wave function of the two photons is postselected by the measurement apparatus. Such a scheme can be extended in a controlled way to a higher number of photons, as shown in Ref. [23]. There a linear optics multiqubit symmetrization apparatus has been realized by a chain of interconnected Hong-Ou-Mandel interferometers.

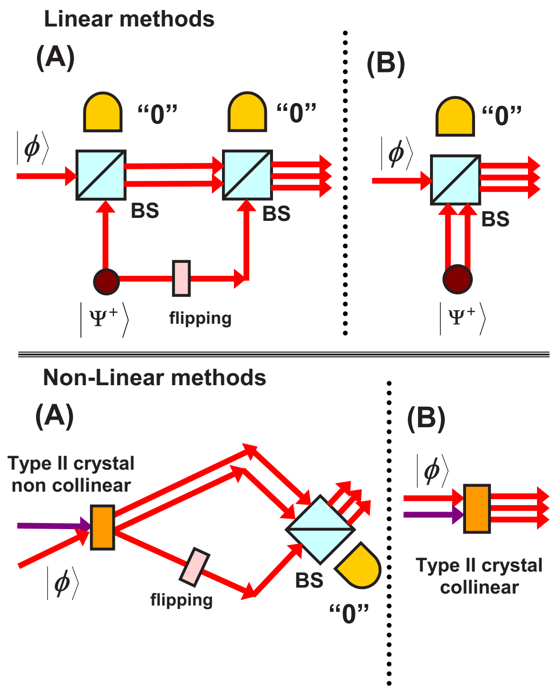

FIG. 3. (Color online) Linear methods: (a) schematic diagram of the linear optics multiqubit symmetrization apparatus realized by a chain of interconnected Hong-Ou-Mandel interferometers; (b) symmetrization of the input photon and the ancilla polarization entangled pairs. Nonlinear methods: (a) UQCM by optical parametric amplification, flipping by a couple of waveplates, and projection over the symmetric subspace; (b) collinear optical parametric amplification within a type II BBO (beta-barium borate) crystal.

Here we introduce a variety of schemes which can be realized through the methods of quantum optics outlined above. We restrict our attention to the $1\rightarrow 3$ PQCM; the diagram below can be easily extended to the general case $1\rightarrow M$ for odd values of M following the guidelines of the previous section. Let us consider first the linear optics approach. Figure 3 shows the experimental scheme implementing, respectively, the scheme of Fig. 1(a), (A), and Fig. 1(b), (B). The flipping operation σ_Y is realized by means of two $\lambda/2$ waveplates acting on the polarization state, while the symmetrization is implemented by overlapping the incoming photons on a beam splitter and postselecting the events in which they emerge over the same mode, as said. We note that such a scheme is similar to the one recently proposed by Zou *et al.* [25] to implement the $1\rightarrow 3$ PQCM for photonic qubits.

Finally let us observe that the same results can be obtained adopting nonlinear methods. We consider the $1\rightarrow 3$ PQCM, in particular the optimal quantum cloning for $x-z$ equatorial qubits by taking linear polarization states as input. The UQCM has been realized by adopting a quantum-injected optical parametric amplifier (QIOPA), while the σ_Y

operation and the Π^3 projection have been implemented with linear optics and postselection techniques, Fig. 3(a). The flipping operation on the output mode $\mathbf{k}_{AC}$ was realized by means of two $\lambda/2$ waveplates, while the physical implementation of the projector Π^3 on the three photons states was carried out by linearly superimposing the modes $\mathbf{k}_C$ and $\mathbf{k}_{AC}$ on the 50:50 beam splitter BS and then by selecting the case in which the three photons emerged from BS on the same output mode $\mathbf{k}_{PC}$ (or, alternatively, on $\mathbf{k}'_{PC}$).

Interestingly, the same overall state evolution can also be obtained, with no need of the final BS symmetrization, at the output of a QIOPA with a type II crystal working in a *collinear* configuration, (b) [26]. In this case the interaction Hamiltonian $\hat{H}_{coll}=i\chi\hbar(\hat{a}_H^{\dagger}\hat{a}_V^{\dagger})+\text{H.c.}$ acts on a single spatial mode k. A fundamental physical property of $\hat{H}_{coll}$ consists of its rotational invariance under U(1) transformations, that is, under any arbitrary rotation around the z axis. Indeed $\hat{H}_{coll}$ can be re-expressed as $\frac{1}{2}i\chi\hbar e^{-i\phi}(\hat{a}_{\phi}^{\dagger 2}-e^{i2\phi}\hat{a}_{\phi\perp}^{\dagger 2})+\text{H.c.}$ for $\phi \in (0,2\pi)$, where $\hat{a}_{\phi}^{\dagger}=2^{-1/2}(\hat{a}_H^{\dagger}+e^{i\phi}\hat{a}_V^{\dagger})$ and $\hat{a}_{\phi\perp}^{\dagger}=2^{-1/2}(-e^{-i\phi}\hat{a}_H^{\dagger}+\hat{a}_V^{\dagger})$. Let us consider an injected single photon with polarization state $|\phi\rangle_{in}=2^{-1/2}(|H\rangle+e^{i\phi}|V\rangle)=|1,0\rangle_k$, where $|m,n\rangle_k$ represents a product state with m photons of the mode k with polarization ϕ, and n photons with polarization $\phi^{\perp}$. The first contribution to the amplified state, $\sqrt{6}|3,0\rangle_k-\sqrt{2}e^{i2\phi}|1,2\rangle_k$ is identical to the output state obtained with the device introduced above up to a phase factor which does not affect the fidelity value.

CONCLUSIONS

We have introduced different schemes to implement the optimal $1\rightarrow M>1$ phase covariant cloning machine, by exploiting the projection of the input qubit and ancillary resources over the symmetric subspace. The present technique is probabilistic; however, such limitation does not spoil the main physical relevance since the optimal fidelity value of the phase-covariant cloning cannot be improved by any probabilistic procedure implementation [27]. We also note that these schemes do not hold for even values of M, a result consistent with the underlying feature of the PQCM. Indeed it has been noticed that different properties affect the $1\rightarrow 2P$ and $1\rightarrow(2P-1)$ PQCM maps [15]. Recently an optical scheme to realize the $1\rightarrow 2$ PQCM has been proposed [28] and realized experimentally [29].

The experimental realization of the different cloning protocols with the standard quantum optics techniques has also been discussed. There we answered the question recently raised by Scarani *et al.* [1] concerning the possibility of implementing any cloning transformation, different from the universal one, adopting the process of amplification through stimulated emission. The PQCM can be implemented directly either by linear optics elements, or by a nonlinear, quantum injected optical parametric amplification process in a collinear configuration. The generalization of such schemes to a higher number of input qubits $N>1$ has been found to be nonoptimal and hence deserves further investigation.

Finally we shall mention that the present cloning maps are economical, that is, do not require any extra physical resources than the clone qubits [30].

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