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Experimental reversion of the optimal quantum cloning and flipping processes

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The quantum cloner machine maps an unknown arbitrary input qubit into two optimal clones and one optimal flipped qubit. By combining linear and nonlinear optical methods we experimentally implement a scheme that, after the cloning transformation, restores the original input qubit in one of the output channels, by using local measurements, classical communication, and feedforward. This nonlocal method demonstrates how the information on the input qubit can be restored after the cloning process. The realization of the reversion process is expected to find useful applications in the field of modern multipartite quantum cryptography.

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Quantum information theory exploits the most significant properties of quantum phenomena to develop completely different schemes of the information transfer and processing, thus enabling different forms of communication and enhancing computational power. In this framework today an increasing effort is devoted to the understanding of the very concept of *quantum information* and the fundamental quantum laws underlying the processing of information [1,2]. The inner structure of quantum mechanics forbids the realization of a perfect quantum cloning machine mapping a qubit $|\phi\rangle$ into $|\phi\rangle\,|\phi\rangle$ [3] and of the spin flipping operation, i.e., the NOT gate [4]. Even if the cloning and flipping operations are unrealizable in their exact forms, they can be approximated "optimally," i.e., with the minimum added noise, by the corresponding universal quantum machines, the universal optimal quantum cloning machine (UOQCM) [5], and the universal-NOT (UNOT) gate [4,6]. In any physical situation the cloning and flipping transformations are deeply interconnected [7,8]: these two processes are always realized contextually and their optimal fidelities are directly related. Actually, the information on the input qubit has not been degraded during the cloning process but only broadcasted on a larger quantum system. Furthermore it has been found that the stored information about $|\phi\rangle$ can be even recovered nonlocally by two different partners [9]. Recently it has been proposed to exploit this nonlocal encoding-decoding process in order to achieve different quantum information tasks: a multiparty agreement quantum communication in which all parties involved in the information distribution communicate in a very specific controlled way [10,11]. This represents a physical implementation of the secure quantum key distribution with the classical combination as a way to hide reversibly quantum information in multiqubit systems. Furthermore, such a scheme can be applied to enhance the transmission fidelity over a lossy (but noiseless) channel and the performance of a quantum memory with erasure [12].

In the present paper we experimentally address the problem of how the information of any input qubit $|\phi\rangle$ can be retrieved after a cloning process by fully counteracting the effect of the quantum noise necessarily implied by the "no cloning" theorem. By this physical effect, light is shed over the inner structure of the overall cloning process, its entanglement features and its nonlocal reversibility. More precisely, here the quantum information content of $|\phi\rangle$ is spread over the three qubits entangled state $|\Sigma(\phi)\rangle_{SAB}$ generated through the quantum cloner and then is fully retrieved on an output channel by implementing a local operation and classical communications (LOCC) method [9]. The coherent spreading of the initial information over $|\Sigma(\phi)\rangle_{SAB}$ is obtained by a simultaneous implementation of the $1\rightarrow 2$ UOQCM and the $1\rightarrow 1$ UNOT gate via a quantum injected optical parametric amplifier (QI OPA) [8,13] or an all linear optics setup [14]. In the previous experimental realizations these two processes have been investigated by the same device over different optical modes [8,13–15]. The mutual coherence of the UOQCM and UNOT gate has never been either observed or exploited. Each process was analyzed independently measuring the information associated to single-qubit observable. As shown in Fig. 1, at the input of UOQCM the three modes S, A, B support, respectively, the qubit $|\phi\rangle$ and the two ancillas. The quantum reversion process is completed by an application to the output of UOQCM of a LOCC procedure [16], indeed a modified teleportation protocol consisting of a Bell measurement on the modes S and

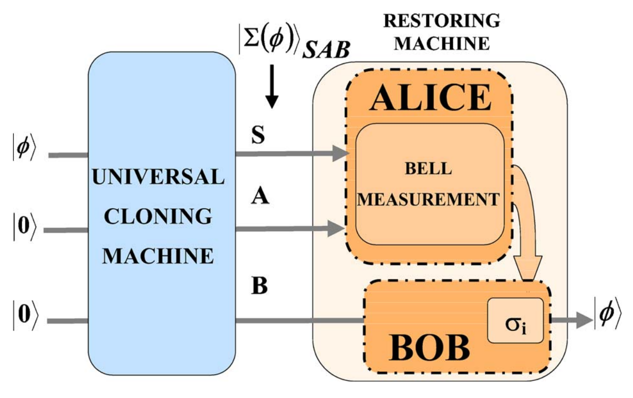

FIG. 1. (Color online) General scheme for the cloning machine and the restoring machine. The restoration of the initial input qubit $|\phi\rangle$ is obtained by means of a Bell measurement on the clone qubits S and A at Alice's site, classical communication, and feedforward operation σ_i on the qubit B at Bob's site.

A, a classical communication channel, and of a final unitary operation on the qubit B [17]. This strategy is at variance with an unitary process acting collectively on the three qubits, a hardly achievable task with nowadays technology. Let us point out another aspect of this procedure: the ability to perfectly recover the initial state $|\phi\rangle$ even if it has undergone the QI OPA amplification, which is affected by the unavoidable squeezed-vacuum noise. In other words, the stochastic decohering effect of noise are completely undone by the protocol itself.

The cloning machine (CM), which realizes simultaneously the $1\rightarrow 2$ UOQCM and the $1\rightarrow 1$ UNOT gate, starts from the input qubit $|\phi\rangle_S=\alpha|0\rangle_S+\beta|1\rangle_S$ and the ancilla qubits A and B in the state $|0\rangle$. After the cloning process the overall output state $|\Sigma(\phi)\rangle_{SAB}$ reads

$$|\Sigma(\phi)\rangle_{SAB}=\sqrt{\frac{2}{3}}|\phi\rangle_S|\phi\rangle_A|\phi^{\perp}\rangle_B-\frac{1}{\sqrt{6}}(|\phi\rangle_S|\phi^{\perp}\rangle_A+|\phi^{\perp}\rangle_S|\phi\rangle_A)|\phi\rangle_B \quad (1)$$

with $|\phi^{\perp}\rangle_S=\alpha^{*}|1\rangle_S-\beta^{*}|0\rangle_S$. The qubits S and A end up in the state $\rho_S=\rho_A=\frac{5}{6}|\phi\rangle\langle\phi|+\frac{1}{6}|\phi^{\perp}\rangle\langle\phi^{\perp}|$ and are the optimal clones of the input qubit, while the qubit B, the optimally flipped of $|\phi\rangle$, is found in the output state $\rho_B=\frac{1}{3}|\phi\rangle\langle\phi|+\frac{2}{3}|\phi^{\perp}\rangle\langle\phi^{\perp}|$. The state (1) can be interpreted as the quantum superposition of two three-qubit entangled states, which depends on the complex parameters α and β:

$$|\Sigma(\phi)\rangle_{SAB}=\alpha|\Sigma(0)\rangle_{SAB}+\beta|\Sigma(1)\rangle_{SAB}, \quad (2)$$

where $|\Sigma(0)\rangle=(\frac{2}{3})^{-1/2}|0\rangle_S|0\rangle_A|1\rangle_B-6^{-1/2}(|0\rangle_S|1\rangle_A|0\rangle_B+|1\rangle_S|0\rangle_A|0\rangle_B)$ and $|\Sigma(1)\rangle=-(\frac{2}{3})^{-1/2}|1\rangle_S|1\rangle_A|0\rangle_B+6^{-1/2}(|1\rangle_S|0\rangle_A|1\rangle_B+|0\rangle_S|1\rangle_A|1\rangle_B)$.

The procedure adopted to reverse the cloning and flipping processes consists of a LOCC approach similar to the quantum teleportation protocol (QST), as said [18]. Differently from QST, the reversion procedure here acts on the complex three-qubit entangled state of Eq. (1). To understand how the restoring of the initial qubit is obtained, the state (1) can be reexpressed introducing the Bell states of the qubits S and A: $|\Phi^{\pm}\rangle_{SA}=2^{-1/2}(|0\rangle_S|0\rangle_A\pm|1\rangle_S|1\rangle_A)$ and $|\Psi^{\pm}\rangle_{SA}=2^{-1/2}(|0\rangle_S|1\rangle_A\pm|1\rangle_S|0\rangle_A)$. The cloner output state can hence be recast as

$$|\Sigma(\phi)\rangle_{SAB}=\frac{1}{\sqrt{3}}[-|\Phi^{+}\rangle_{SA}i\sigma_Y|\phi\rangle_B+|\Phi^{-}\rangle_{SA}\sigma_X|\phi\rangle_B+|\Psi^{+}\rangle_{SA}\sigma_Z|\phi\rangle_B]. \quad (3)$$

We note that only the symmetric Bell states of S and A appear since the two clones belong to the symmetric subspace.

Let us now describe the restoring machine (RM). For this purpose we introduce two partners: Alice ($\mathcal{A}$) and Bob ($\mathcal{B}$) (Fig. 1). Alice holds the qubits S and A while Bob holds the qubit B. Alice performs a Bell measurement on the qubits S and A, that is in the basis $\{|\Phi^{\pm}\rangle_{SA},|\Psi^{\pm}\rangle_{SA}\}$, and communicates the measurement result to Bob sending a classical trit through a classical channel. Depending on Alice's communication [19], Bob applies a suitable Pauli operator σ_i according to the following table:

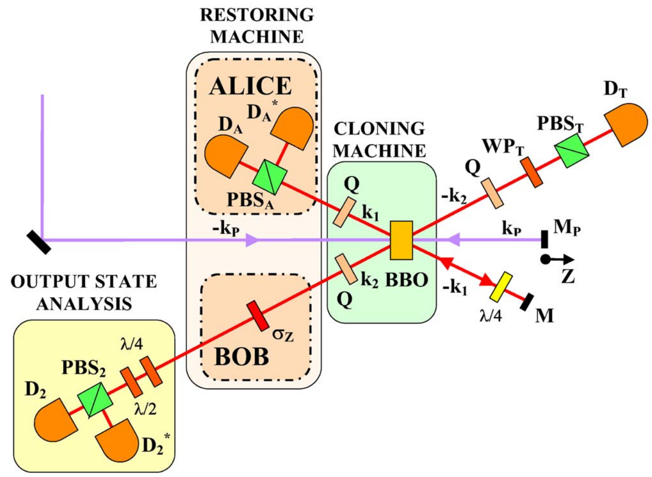

FIG. 2. (Color online) Schematic diagram of the *self-injected* optimal parametric amplifier. The Bell measurement is performed on the cloning mode k_1 (Alice's site). The photon on mode k_2 undergoes a σ_Z operation (Bob's site). The output qubit S on mode k_2 is analyzed adopting the single-qubit tomography.

Alice's result	Bob's operation
$\lvert\Phi^{+}\rangle_{SA}$	$i\sigma_Y$
$\lvert\Phi^{-}\rangle_{SA}$	σ_X
$\lvert\Psi^{+}\rangle_{SA}$	σ_Z

As can be easily obtained from the expression (3), at the end of the protocol the qubit B is found in the state $|\phi\rangle$: the initial state of the qubit has hence been restored deterministically. It is worth noting that the ancilla qubit B is necessary to restore the initial qubit state.

To implement the restoring machine we adopted polarization encoded qubits by exploiting the isomorphism between the qubit state $\alpha|0\rangle+\beta|1\rangle$ and the polarization state $|\phi\rangle_{in}=\alpha|H\rangle+\beta|V\rangle$ of a single photon. The cloning process has been realized adopting the quantum injected optical parametric amplifier (QI OPA) as said: Fig. 2 [8]. Alice's site is placed on the k_1 mode, while Bob's site is placed on the k_2 mode. Consider first the case of an input $\vec{\pi}$-encoded qubit $|\phi\rangle_{in}$ associated with a single photon with wavelength (wl) λ, injected on the input mode k_1 of the QI OPA; the other input mode k_2 being in the vacuum state. The photon was injected into a nonlinear (NL) BBO (β-barium-borate) 1.5 mm thick crystal slab, cut for type II phase matching and excited by a sequence of uv mode-locked laser pulses having a duration $\tau\approx 140\,f\,\mathrm{sec}$ and wl λ_p: Fig. 2. The relevant modes of the NL three-wave interaction driven by the uv pulses associated with mode k_p were the two spatial modes with wave vector (wv) k_i, $i=1,2$, each supporting the two horizontal (H) and vertical (V) *linear*-$\vec{\pi}$'s of the interacting photons. The QI OPA was λ-degenerate, i.e., the interacting photons had the same wl's $\lambda=2\lambda_p=795$ nm. The QI OPA apparatus was arranged in the self-injected configuration shown in Fig. 2 and described in Ref. [8]. The uv pump beam, back-reflected by a spherical mirror M_p with reflectivity $R>99.9\%$ and μ-adjustable position **Z**, excited the NL crystal in both directions $-k_p$ and k_p, i.e., correspondingly oriented towards the

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right-hand side and the left-hand side of Fig. 2. A spontaneous parametric down conversion (SPDC) process excited by the $-k_p$ uv mode created *singlet states* of photon polarization ($\vec{\pi}$). The photon of each SPDC pair emitted over $-k_1$ was back-reflected by a spherical mirror M into the NL crystal and provided the $N=1$ *quantum injection* into the OPA excited by the uv beam associated with the back-reflected mode k_p. The twin SPDC photon emitted over mode $-k_2$, selected by the devices (wave-plate+polarizing beam splitter: WP_T $+\mathrm{PBS}_T$) and detected by D_T, provided the "trigger" of the overall conditional experiment. Because of the EPR nonlocality of the emitted singlet, the $\vec{\pi}$-selection made on $-k_2$ implied deterministically the selection of the input state $|\phi\rangle_{in}$ on the injection mode k_1. By adopting $\lambda/2$ or $\lambda/4$ WP with different orientations of the optical axis, the following $|\phi\rangle_{in}$ states were injected: $|H\rangle$, $2^{-1/2}(|H\rangle+|V\rangle)=|+\rangle$, and $2^{-1/2}(|H\rangle$ $+i|V\rangle)=|R\rangle$. The three fixed quartz plates (Q) inserted on the modes k_1, k_2, and $-k_2$ provided the compensation for the unwanted walk-off effects due to the birefringence of the NL crystal. An additional walk-off compensation into the BBO crystal was provided by the $\lambda/4$ WP exchanging on mode $-k_1$ the $|H\rangle$ and $|V\rangle$ $\vec{\pi}$ components of the injected photon. All adopted photodetectors (D) were equal SPCM-AQR14 Si-avalanche single photon units. One interference filter with bandwidth $\Delta\lambda=6$ nm was placed in front of each D.

The reversion machine has been realized adopting linear optics and single-photon detectors (Fig. 2). A complete Bell measurement can be realized adopting a deterministic CNOT gate [20], while the four Bell state identifications cannot be obtained by simple linear optics elements [21]. In the present experiment we have restricted our analysis to the detection of the state $|\Psi^+\rangle_{SA}$ (3) realized by means of the polarizing beam splitter PBS_A and the detector D_A and D_A^* (the state $|\Psi^-\rangle_{SA}$ is absent since the qubits S and A belong to the symmetric subspace). Bob performed the σ_Z operation by means of a $\lambda/2$ wave plate.

At the first experimental step we have identified the position $\mathbf{Z}=\mathbf{0}$ corresponding to the overlap between the injected photon and the uv pump pulse [Fig. 3(a)]. Let us consider the situation in which we inject the state $|\phi\rangle$ and we detect the Bell state $|\Psi^+\rangle_{SA}$. When there is a perfect matching between the uv pump and the injected photon, the polarization state of the photon over mode k_2 should be $\rho_B=|\phi\rangle\langle\phi|$, while for $\mathbf{Z}$ $\gg\mathbf{0}$ the cloning and restoring machines are turned off and $\rho_B=\frac{1}{2}|\phi\rangle\langle\phi|+\frac{1}{2}|\phi^{\perp}\rangle\langle\phi^{\perp}|$. We have injected the state $|R\rangle_S$ and analyzed the output state over the mode k_2 with a $\lambda/4$ wave plate and a polarizing beam splitter PBS_2. The output state of the qubit B was analyzed adopting a couple of detectors $\{D_2,D_2^*\}$; the states $|R\rangle_B$ and $|L\rangle_B=2^{-1/2}(|H\rangle_B-i|V\rangle_B)$ were, respectively, detected by D_2 and D_2^*. The detection of a photon from the detector D_T ensured the injection of the state $|\phi\rangle_S$ over the mode k_1. A coincidence between D_A and D_A^* identified the state $|\Psi^+\rangle_{SA}$. The coincidence counts (D_T,D_A,D_A^*,D_2) and (D_T,D_A,D_A^*,D_2^*) are reported in Fig. 3(a) versus the position $\mathbf{Z}$ of the uv mirror M_P. The peak in the coincidence counts of (D_T,D_A,D_A^*,D_2) and the dip in the coincidence counts of (D_T,D_A,D_A^*,D_2^*) in correspondence of the matching between the injected qubit and the uv pump beam are a signature of the realization of the state $|\phi\rangle_B$.

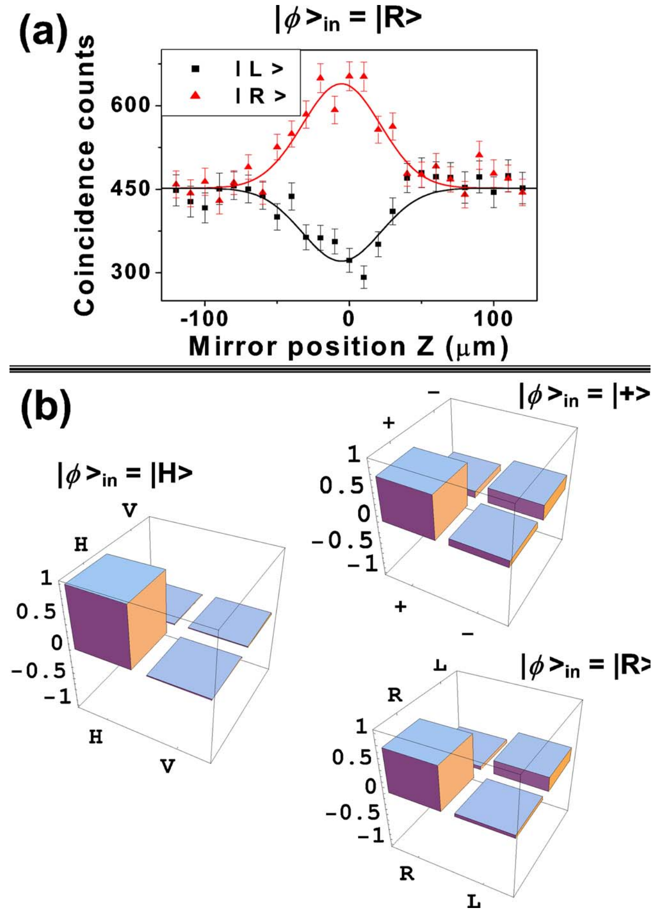

FIG. 3. (Color online) (a) Coincidence counts (D_T,D_A,D_A^*,D_2) and (D_T,D_A,D_A^*,D_2^*) versus the position $\mathbf{Z}$ of the uv mirror M_P. Each experimental point has been measured in a time of 2400 s. (b) Quantum state tomography of the output qubits B carried out by means of a $\lambda/4$, a $\lambda/2$, PBS_2, D_2, and D_2^* in the position $\mathbf{Z}=\mathbf{0}$. For each injected state $|\phi\rangle$ the experimentally reconstructed density matrix is represented in the $\{|\phi\rangle,|\phi^{\perp}\rangle\}$ basis. The time required for each matrix reconstruction was ~24 h.

To completely characterize the output state of the CM +RM process we have positioned the mirror M_P in the position $\mathbf{Z}=\mathbf{0}$ and we have carried out a single-qubit quantum state tomography [22] on the k_2 mode for three different states of the input qubit $|\phi\rangle=|H\rangle$, $|\phi\rangle=|+\rangle$, and $|\phi\rangle=|R\rangle$ [Fig. 3(b)] with $|\pm\rangle=2^{-1/2}(|H\rangle\pm|V\rangle)$. This analysis is performed through a $\lambda/4$, a $\lambda/2$, PBS_2, and the detector D_2 and D_2^*. The coincidence counts (D_T,D_A,D_A^*,D_2) and (D_T,D_A,D_A^*,D_2^*) are acquired for different settings of the wave-plate positions in order to measure the different Stokes parameters of the k_2 mode. The density matrices ρ_{out} in Fig. 3(b) are represented in the basis $\{|\phi\rangle,|\phi^{\perp}\rangle\}$. In the ideal case $\rho_B=|\phi\rangle\langle\phi|$, as said. To ensure a higher visibility of the overall process in the measurements, the different modes selection was assured by a narrower pinhole. The experimental results confirm our theory; the fidelity $\mathcal{F}_{\phi}=\langle\phi|\rho_{out}|\phi\rangle$ of the overall process are found to be $\mathcal{F}_H=0.98\pm0.01$, $\mathcal{F}_+=0.78\pm0.01$, and $\mathcal{F}_R$ $=0.76\pm0.01$. The average experimental fidelity of the overall process has been found to be $\mathcal{F}=0.84$. This value is found to be largely above the classical estimation bound achievable by measuring the input qubit, equal to 0.67.

In conclusions, we have demonstrated experimentally the reversibility of the cloning-flipping processes by showing how the output of these processes can be exploited to nonlocally restore the input qubit $|\phi\rangle$. This result is expected to represent a contribution in modern multipartite quantum cryptography protocols [23,11]. In particular, the adoption of W entangled states (2) for quantum secure communication [24,25] and secret sharing protocols belong to the most advanced issues raised recently in this field. Finally the present experimental realization is a step towards the adoption of the cloning process for optimal partial state estimation, as recently proposed by [12].

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