Realization of the optimal phase-covariant quantum cloning machine

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In several quantum information (QI) phenomena of large technological importance the information is carried by the phase of the quantum superposition states, or qubits. The phase-covariant cloning machine (PQCM) addresses precisely the problem of optimally copying these qubits with the largest attainable "fidelity." We present a general scheme which realizes the $1\rightarrow 3$ phase covariant cloning process by a combination of three different QI processes: the universal cloning, the NOT gate, and the projection over the symmetric subspace of the output qubits. The experimental implementation of a PQCM for polarization encoded qubits, the first ever realized with photons, is reported.

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In recent years a great deal of effort has been devoted to the realization of the optimal approximations to the quantum cloning and flipping operations over an unknown qubit $|\phi\rangle$. It is well known that these two processes are unrealizable in their exact forms [1,2], but they can be optimally approximated by the corresponding universal machines, i.e., by the universal optimal quantum cloning machine (UOQCM) and the universal-NOT (U NOT) gate [3]. The UOQCM has been experimentally realized following several schemes, i.e., by exploiting the process of stimulated emission in a quantum-injected optical parametric amplifier (QI-OPA) [4–7], by a quantum network [8], and by acting with projective operators over the symmetric subspaces of many qubits [9,10]. Since also the perfect cloning of subsets containing nonorthogonal states is forbidden, recently state-dependent cloning machines have been investigated that are optimal with respect to any given ensemble [11,12]. It has been found in general that for group-covariant cloning, i.e., where the set of input states is the orbit of a given state under the action of a group of unitary transformations, the smaller is the group the higher is the optimal fidelity averaged over the input states [11]. The simplest and most relevant case is represented by the cloning covariant under the Abelian group U(1) of phase rotations, the so-called "phase-covariant" cloning. There the information is encoded in the phase ϕ_i of the input qubit belonging to any equatorial plane i of the corresponding Bloch sphere, e.g., the general state may be expressed as $|\phi_i\rangle = 2^{-1/2}[|\psi_i\rangle + \exp(i\phi_i)|\psi_i^{\perp}\rangle]$ and $\{|\psi_i\rangle, |\psi_i^{\perp}\rangle\}$ is a convenient normalized basis, $\langle\psi_i|\psi_i^{\perp}\rangle = 0$ [12]. Precisely, in the general case the $N\rightarrow M$ phase covariant cloning map C_{NM} satisfies the following covariance relation, $C_{NM}(T_{\phi i}^{\otimes N}\rho_N T_{\phi i}^{\dagger\otimes N}) = T_{\phi i}^{\otimes M} C_{NM}(\rho_N) T_{\phi i}^{\dagger\otimes M}$, where $T_{\phi i} = \exp[-(i/2)\phi_i\sigma_i]$. There the σ_i Pauli operator identifies the set of input states which are cloned, e.g., σ_Y corresponding to states belonging to the x-z plane. The general $N\rightarrow M$ phase-covariant quantum cloning machine (PQCM) has been investigated theoretically, and the optimal fidelities $\mathcal{F}_{\text{cov}}^{N\rightarrow M}$ have been found [11]. Note that this cloning process is deeply connected with the quantum phase ϕ estimation and then with the limit precision of time measurements in clocks [13–17]. Furthermore, its relevance in modern information technology has been recently recognized, as the related no-go theorem can ensure the security of cryptographic protocols such as the Bennett-Brassard 1984 (BB84) protocol [11].

Rather surprisingly, no PQCM device has been implemented experimentally so far in the domain of quantum optics [18,19]. In the present work we report the realization of a $1\rightarrow 3$ PQCM by the implementation of a $1\rightarrow 2$ UOQCM, followed by a spin flipper σ_i and the projection of the output qubits over the symmetric subspace: Fig. 1 (inset) [5,9]. Let the state of the input qubit be expressed by $|\phi\rangle_S = 2^{-1/2}[|R\rangle_S + \exp(i\phi_Y)|L\rangle_S] = \alpha|0\rangle_S + \beta|1\rangle_S$, with $\langle R|L\rangle = 0, |\alpha|^2 + |\beta|^2 = 1$ and α, β real parameters. Here we consider, in particular, the ϕ_Y -covariant cloning and $\sigma_i = \sigma_Y$ realizes the NOT gate for the qubits belonging to the x-z plane. The output state of the $1\rightarrow 2$ UOQCM device reads $|\Sigma\rangle_{SAB} = \sqrt{\frac{2}{3}}|\phi\rangle_S|\phi\rangle_A|\phi^{\perp}\rangle_B - (1/\sqrt{6})(|\phi\rangle_S|\phi^{\perp}\rangle_A + |\phi^{\perp}\rangle_S|\phi\rangle_A)|\phi\rangle_B$, where the qubits S and A are the optimal cloned qubits while the qubit B is the optimally flipped one. According to the scheme represented by the inset of Fig. 1, the idea is now to exactly flip the qubit B for a given subset of the Bloch sphere.

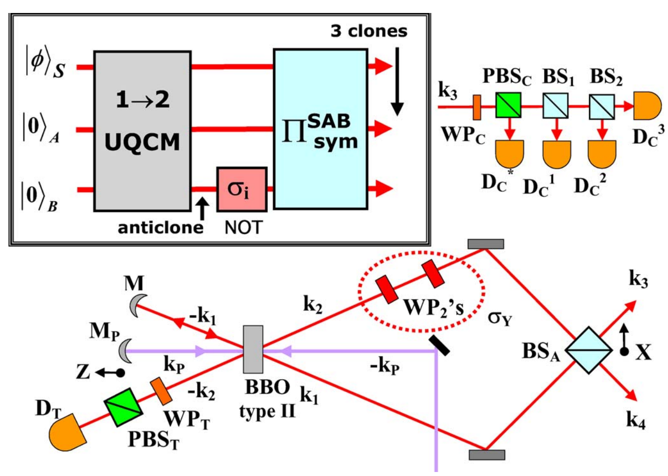

FIG. 1. (Color online) Experimental layout of the phase covariant cloning machine (PQCM) for the input qubits belonging to the x-z equatorial plane of the Bloch sphere. Insets: (left) Schematic network realizing the PQCM, (right) Measurement setup for the PQCM output states.

This local flipping transformation of $|\phi\rangle_B$ leads to

$$|\Upsilon\rangle_{SAB} = (\mathbb{I}_S \otimes \mathbb{I}_A \otimes \sigma_Y)|\Sigma\rangle_{SAB} = \sqrt{\tfrac{2}{3}}|\phi\rangle_S|\phi\rangle_A|\phi\rangle_B - 1/\sqrt{6}(|\phi\rangle_S|\phi^\perp\rangle_A + |\phi^\perp\rangle_S|\phi\rangle_A)|\phi^\perp\rangle_B.$$

By this non-universal cloning process three *asymmetric* copies have been obtained: two clones (qubits S and A) with fidelity 5/6, and a third one (qubit B) with fidelity 2/3. We may now project S,A, and B over the symmetric subspace and obtain three symmetric clones with a higher average fidelity. The symmetrization operator $\Pi_{\rm sym}^{SAB}$ reads as $\Pi_{\rm sym}^{SAB}=|\Pi_1\rangle\langle\Pi_1|+|\Pi_2\rangle\langle\Pi_2|+|\Pi_3\rangle\langle\Pi_3|+|\Pi_4\rangle\langle\Pi_4|$, where $|\Pi_1\rangle=|\phi\rangle_S|\phi\rangle_A|\phi\rangle_B$, $|\Pi_2\rangle=|\phi^\perp\rangle_S|\phi^\perp\rangle_A|\phi^\perp\rangle_B$, $|\Pi_3\rangle=(1/\sqrt{3})\times(|\phi\rangle_S|\phi^\perp\rangle_A|\phi^\perp\rangle_B+|\phi^\perp\rangle_S|\phi\rangle_A|\phi^\perp\rangle_B+|\phi^\perp\rangle_S|\phi^\perp\rangle_A|\phi\rangle_B)$, and $|\Pi_4\rangle=(1/\sqrt{3})(|\phi\rangle_S|\phi\rangle_A|\phi^\perp\rangle_B+|\phi^\perp\rangle_S|\phi\rangle_A|\phi\rangle_B+|\phi\rangle_S|\phi^\perp\rangle_B|\phi\rangle_A)$. The symmetric subspace has dimension 4 since three qubits are involved. The probability of success of the projection is equal to $\frac{8}{9}$. The normalized output state $|\xi\rangle_{SAB}=\Pi_{\rm sym}^{SAB}|\Upsilon\rangle_{SAB}$ is

$$|\xi\rangle_{SAB} = \frac{1}{2}\sqrt{3}[|\phi\rangle_S|\phi\rangle_A|\phi\rangle_B - 3^{-1}(|\phi\rangle_S|\phi^\perp\rangle_A|\phi^\perp\rangle_B + |\phi^\perp\rangle_S|\phi\rangle_A|\phi^\perp\rangle_B + |\phi^\perp\rangle_S|\phi^\perp\rangle_A|\phi\rangle_B)]. \quad (1)$$

Let us now estimate the output reduced density matrices of the qubits S, A, and B: $\rho_S=\rho_A=\rho_B=\frac{5}{6}|\phi\rangle\langle\phi|+\frac{1}{6}|\phi^\perp\rangle\langle\phi^\perp|$. This leads to the fidelity $\mathcal{F}_{\rm cov}^{1\to3}=5/6$ equal to the optimal one obtained in the general case [11,12].

By applying a different unitary operator σ_i to the qubit B we can implement the phase-covariant cloning for different equatorial planes. Interestingly, note that by this symmetrization technique a depolarizing channel $E_{\rm dep}(\rho)=\frac{1}{4}(\rho+\sigma_X\rho\sigma_X+\sigma_Y\rho\sigma_Y+\sigma_Z\rho\sigma_Z)$ on channel B transforms immediately the nonuniversal phase covariant cloning into the universal $1\to3$ UOQCM with the overall fidelity $\mathcal{F}_{\rm univ}^{1\to3}=7/9$. This represent a relevant new proposal to implement the $1\to3$ UOQCM cloning scheme [5,9,20].

Let us return to the $1\to3$ PQCM. In the experiment reported here the qubit to be cloned is encoded into the polarization ($\vec{\pi}$) state $|\phi\rangle_{\rm in}=\alpha|H\rangle+\beta|V\rangle$ of a single photon, where $|H\rangle\equiv|0\rangle=2^{-\frac{1}{2}}(|R\rangle+|L\rangle)$ and $|V\rangle\equiv|1\rangle=-i2^{-\frac{1}{2}}(|R\rangle-|L\rangle)$ stand for horizontal and vertical polarizations. We consider the optimal quantum cloning for *x*-*z* equatorial qubits by taking linear polarization states as input, that is, the ones adopted in the BB84 cryptographic protocol. The UOQCM has been realized by adopting a quantum-injected optical parametric amplifier (QI-OPA) [5], while the σ_Y operation and the $\Pi_{\rm sym}^{SAB}$ have been implemented with linear optics and post-selection techniques. Let us consider the experimental apparatus in more detail: Fig. 1.

The QI-OPA consisted of a nonlinear (NL) BBO (β-barium-borate), cut for type-II phase matching and excited by a sequence of UV mode-locked laser pulses having wavelength (WL) λ_p. The relevant modes of the NL three-wave interaction driven by the UV pulses associated with mode k_p were the two spatial modes with wave vector $k_i, i=1, 2$, each one supporting the two horizontal and vertical polarizations of the interacting photons. The QI-OPA was λ degenerate, i.e., the interacting photons had the same WL's $\lambda=2\lambda_p=795$ nm. The NL crystal orientation was set so as to realize the insensitivity of the amplification quantum efficiency to any input state $|\phi\rangle_{\rm in}$, i.e., the universality (U) of the "cloning machine" and of the U NOT gate [5]. This key property is assured by the squeezing Hamiltonian $\hat{H}_{\rm int}=i\chi\hbar(\hat{a}_{1\phi}^\dagger\hat{a}_{2\phi\perp}^\dagger-\hat{a}_{1\phi\perp}^\dagger\hat{a}_{2\phi}^\dagger)+{\rm H.c.}$, where the field operator $\hat{a}_{ij}^\dagger$ refers to the state of polarization j ($j=\phi,\phi^\perp$), realized on the two interacting spatial modes k_i ($i=1$, 2).

The UV pump beam with WL λ_p, back reflected by the spherical mirror M_p with 100% reflectivity and μ-adjustable position **Z**, excited the NL crystal in both directions $-k_p$ and k_p, i.e., correspondingly oriented towards the right-hand side and the left-hand side of Fig. 1. The input qubit S codified in the polarization ($\vec{\pi}$) state of a single photon over the spatial mode k_1 with WL λ was created by adopting the following procedure. A spontaneous parametric down conversion (SPDC) process excited by the $-k_p$ UV mode created singlet states of photon polarization ($\vec{\pi}$). The photon of each SPDC pair emitted over the mode $-k_1$ was back reflected by a spherical mirror M into the NL crystal and provided the $N=1$ quantum injection into the OPA excited by the UV beam associated with the back-reflected mode k_p. The twin SPDC photon emitted over mode $-k_2$, selected by the "state analyzer" consisting of the combination (wave plate + polarizing beam splitter: $\mathrm{WP}_T+\mathrm{PBS}_T$) and detected by D_T, provided the "trigger" of the overall conditional experiment. Because of the EPR nonlocality of the emitted singlet, the $\vec{\pi}$ selection made on $-k_2$ implied deterministically the selection of the input state $|\phi\rangle_{\rm in}$ on the injection mode k_1. By adopting a $\lambda/2$ wave-plate (WP_T) with different orientations of the optical axis, the following $|\phi\rangle_{\rm in}$ states were injected: $|H\rangle$ and $2^{-1/2}(|H\rangle+|V\rangle)$. A more detailed description of the QI-OPA setup can be found in Ref. [5].

Let us consider the injected photon in the mode k_1 to have any linear polarization $\vec{\pi}=\phi$. We express this $\vec{\pi}$ state as $\hat{a}_{1\phi}^\dagger|0,0\rangle_{k_1}=|1,0\rangle_{k_1}$, where $|m,n\rangle_{k_1}$ represents a product state with m photons of the mode k_1 with polarization ϕ, and n photons with polarization $\phi^\perp$. Assume the input mode k_2 to be in the *vacuum state* $|0,0\rangle_{k_2}$. The initial $\vec{\pi}$ state of modes k_i reads $|\phi\rangle_{\rm in}=|1,0\rangle_{k_1}|0,0\rangle_{k_2}$ and evolves according to the unitary operator $\hat{\mathbf{U}}\equiv\exp[-i(\hat{H}_{\rm int}t/\hbar)]$. The first-order contribution of the output state of the QI-OPA is $\sqrt{\frac{2}{3}}|2,0\rangle_{k1}|0,1\rangle_{k2}-\sqrt{\frac{1}{3}}|1,1\rangle_{k1}|1,0\rangle_{k2}$. The above linearization procedure is justified here by the small experimental value of the gain $g\equiv\chi t\approx0.1$. In this context, the state $|2,0\rangle_{k_1}$, expressing two photons of the ϕ mode k_1 in the $\vec{\pi}$ state ϕ, corresponds to the state $|\phi\phi\rangle$, while the vector $|0,1\rangle_{k_2}$ represents the single photon state on mode k_2 with polarization $\phi^\perp$, i.e., the *flipped* version of the input qubit. In summary, the qubits S and A are realized by two single photons propagating along mode k_1 while the qubit B corresponds to the $\vec{\pi}$ state of the photon on mode k_2.

The σ_Y flipping operation on the output mode k_2, implemented by means of two $\lambda/2$ wave plates (WP2's), transformed the QI-OPA output state into $|\Upsilon\rangle_{SAB}=\sqrt{\frac{2}{3}}|2,0\rangle_{k1}|1,0\rangle_{k2}-\sqrt{\frac{1}{3}}|1,1\rangle_{k1}|0,1\rangle_{k2}$. The physical imple-

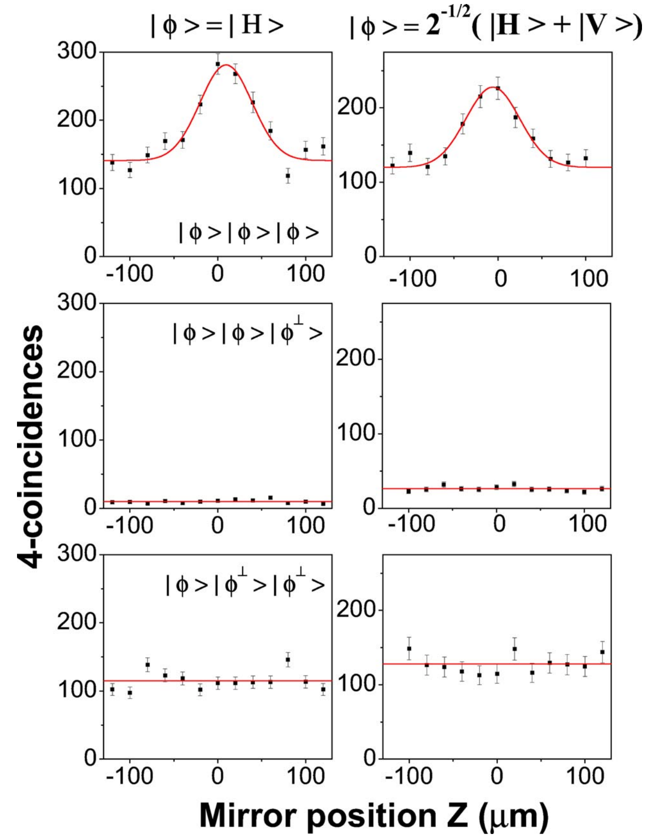

FIG. 2. (Color online) Experimental results of the PQCM for the input qubits $|\phi\rangle=|H\rangle$ (left column) and $|\phi\rangle=2^{-1/2}(|H\rangle+|V\rangle)$ (right column). For each state the coincidence counts for the components $|\phi\phi\phi\rangle, |\phi\phi\phi^{\perp}\rangle, |\phi\phi^{\perp}\phi^{\perp}\rangle$ are reported versus the mirror position Z. The measurement time of each 4-coincidence experimental datum was ~13 000 s. The solid line represents the best Gaussian fit.

mentation of the projector Π_{sym}^{SAB} on the three photons $\vec{\pi}$ states was carried out by linearly superimposing the modes k_1 and k_2 on the 50:50 beam-splitter BS_A and then by selecting the case in which the three photons emerged from BS_A on the same output mode k_3 (or, alternatively on k_4) [9]. The device BS_A was positioned onto a motorized translational stage: the position $X=0$ in Fig. 2 was conventionally assumed to correspond to the best overlap between the interacting photon wave packets which propagate along k_1 and k_2. By considering the case in which three photons emerge over the mode k_3, the output state is found to be $\sqrt{3}/2|3,0\rangle_{k3}+\frac{1}{2}|1,2\rangle_{k3}$. The previous expression leads to the output fidelity $\mathcal{F}_{\mathrm{cov}}^{1\to3}=\frac{5}{6}$.

The output state on mode k_3 was analyzed by the setup shown in the inset of Fig. 1: the field on mode k_4 was disregarded, for simplicity. The polarization state on mode k_3 was analyzed by the combination of the $\lambda/2$ WP WP_C and of the polarizer beam splitter PBS_C. For each input $\vec{\pi}$ state $|\phi\rangle_S$, two different measurements were performed. In a first experiment WP_C was set in order to make PBS_C to transmit $|\phi\rangle$ and reflect $|\phi^{\perp}\rangle$. The cloned state $|\phi\phi\phi\rangle$ was detected by a coincidence between the detectors $[D_C^1, D_C^2, D_C^3]$ while the state $|\phi\phi\phi^{\perp}\rangle$, in the ideal case not present, was detected by a coincidence recorded either by the D set $[D_C^1, D_C^2, D_C^*]$, or by $[D_C^1, D_C^3, D_C^*]$, or by $[D_C^2, D_C^3, D_C^*]$. In order to detect the contribution due to $|\phi\phi^{\perp}\phi^{\perp}\rangle$, WP_C was rotated in order to make PBS_C to transmit $|\phi^{\perp}\rangle$ and reflect $|\phi\rangle$ and by recording the coincidences by one of the sets $[D_C^1, D_C^2, D_C^*], [D_C^1, D_C^3, D_C^*], [D_C^2, D_C^3, D_C^*]$. The different overall quantum efficiencies have been taken into account in the processing of the experimental data. The precise sequence of the experimental procedures was suggested by the following considerations. Assume the cloning machine turned off, by setting the optical delay $|Z| \gg c\tau_{\mathrm{coh}}$, i.e., by spoiling the temporal overlap between the injected photon and the UV pump pulse. In this case since the states $|\phi\phi\rangle$ and $|\phi^{\perp}\phi^{\perp}\rangle$ are emitted with the same probability by the machine, the rate of coincidences due to $|\phi\phi\phi\rangle$ and $|\phi\phi^{\perp}\phi^{\perp}\rangle$ were expected to be equal. By turning on the PQCM, i.e., by setting $|Z| \ll c\tau_{\mathrm{coh}}$, the output state (1) was realized, showing a factor $R=3$ enhancement of the counting rate of $|\phi\phi\phi\rangle$ and *no* enhancement of $|\phi\phi^{\perp}\phi^{\perp}\rangle$. In Fig. 2 the coincidences data for the different state components are reported versus the delay Z for the two input qubits $|\phi\rangle_{\mathrm{in}}$ while BS_A was set on the position $X=0$. During the measurement the single rates and two rates were constant within the statistical fluctuations. We may check that the phase covariant cloning process affects only the $|\phi\phi\phi\rangle$ component, as expected. Let us label by the symbol h the output state components as follows: $\{h=1 \leftrightarrow |\phi\phi^{\perp}\phi^{\perp}\rangle, 2 \leftrightarrow |\phi\phi\phi^{\perp}\rangle, 3 \leftrightarrow |\phi\phi\phi\rangle\}$. For each index h, b_h is the average coincidence rate when the cloning machine is turned off, i.e., $|Z| \gg c\tau_{\mathrm{coh}}$. while the signal-to-noise (S/N) parameter R_h is the ratio between the peak values of the coincidence rates detected, respectively, for $Z \simeq 0$ and $|Z| \gg c\tau_{\mathrm{coh}}$. The optimal values obtained by the above analysis are $R_3=3, R_1=1, b_3=b_1$ and $b_2=0, R_2=0$. These last values, $h=2$ are considered since they are actually measured in the experiment: Fig. 2. The fidelity has been evaluated by means of the expression $\mathcal{F}_{\mathrm{cov}}^{1\to3}(\phi) = (3b_3R_3+2b_2R_2+b_1R_1) \times (3b_3R_3+3b_2R_2+3b_1R_1)^{-1}$ and by the experimental values of b_h, R_h. By adopting the experimental data reported in Fig. 2 it has been obtained $\mathcal{F}_{\mathrm{cov}}^{1\to3}(|H\rangle)=0.804\pm0.006$ and $\mathcal{F}_{\mathrm{cov}}^{1\to3}(|H\rangle+|V\rangle)=0.764\pm0.006$ for the input states $|\phi\rangle_{\mathrm{in}}=|H\rangle$ and $|\phi\rangle_{\mathrm{in}}=2^{-1/2}(|H\rangle+|V\rangle)$. These figures are lower than the theoretical one $\mathcal{F}_{\mathrm{cov}}^{1\to3}=0.833$ mostly because of the imperfect correction of the walk-off effect in the BBO crystal. As is well known, this correction is critical for the polarizations state $|H\rangle+|V\rangle$. Since the adopted π states $\{|H\rangle, |H\rangle+|V\rangle\}$ experience decoherence effects with lowest and highest intensity, respectively, the average fidelity can be realistically estimated as $[\mathcal{F}_{\mathrm{cov}}^{1\to3}]_{\mathrm{ave}}=\frac{1}{2}[\mathcal{F}_{\mathrm{cov}}^{1\to3}(|H\rangle)+\mathcal{F}_{\mathrm{cov}}^{1\to3}(|H\rangle+|V\rangle)] = 0.784\pm0.004$. We note that $[\mathcal{F}_{\mathrm{cov}}^{1\to3}]_{\mathrm{ave}}$ is clearly larger than the optimal phase estimation fidelity equal to 0.75. Such value could be obtained by estimating the input qubit and then by exploiting the obtained information to manufacture copies of the input qubit. The average measured value is also larger by about 1.5 standard deviations than the optimal value $\mathcal{F}_{\mathrm{univ}}^{1\to3}=7/9=0.777$ for the $1\to3$ universal cloning process. A more appropriate comparison can be made with a recent realization of the $1\to3$ UOQCM , which achieved an average value of the fidelity equal to 0.764 ± 0.004 [20].

A direct test of the correct interplay of the parametric amplification and of the flipping-symmetrization process has been carried out by setting the pump mirror in the position of maximum amplification $Z \simeq 0$ and by changing the position X of the symmetrizing BS_A between the two extreme conditions $|X|=0$ and $|X| \gg c\tau_{coh}$, i.e., corresponding to maximum and zero Hong-Ou-Mandel quantum interference. Due to quantum interference, the coincidence rate was enhanced by a factor V^* moving from the position $|X| \gg c\tau_{coh}$ to the condition $X \approx 0$. The $|\phi\phi^\perp\phi^\perp\rangle$ enhancement was found to be $V^*_{exp}=1.70\pm0.10$, to be compared with the theoretical value $V^*=2$ while the enhancement of the term $|\phi\phi\phi\rangle$ was found to be $V^*_{exp}=2.18\pm0.10$, to be compared with the theoretical value $V^*=3$ for the injected state $|\phi\rangle_{in}=2^{-1/2}(|H\rangle+|V\rangle)$. These results, not reported in Fig. 2, are a further demonstration of the three-photon interference in the Hong-Ou-Mandel device.

Interestingly enough, the same overall state evolution can also be obtained, with no need of the final BS_A symmetrization, at the output of a QI-OPA with a type II crystal working in a collinear configuration [21]. In this case the interaction Hamiltonian $\hat{H}_{coll}=i\chi\hbar(\hat{a}^\dagger_H\hat{a}^\dagger_V)+$H.c.. acts on a single spatial mode k. A fundamental physical property of $\hat{H}_{coll}$ consists of its rotational invariance under U(1) transformations, that is, under any arbitrary rotation around the z axis. This device performs the optimal phase-covariant cloning on any injected single photon with $\vec{\pi}$-state $|\psi\rangle_{in}=2^{-1/2}(|H\rangle+e^{i\psi}|V\rangle)$.

In conclusion, we have implemented the optimal quantum triplicators for equatorial qubits. The present approach can be extended to the case of $1\rightarrow M$ PQCM for M odd by allowing for a generation of multiple photon couples in the (QI-OPA). These results are relevant in the modern science of quantum communication, as the PQCM is deeply connected to the optimal eavesdropping attack at BB84 protocol, which exploits the transmission of quantum states belonging to the x-z plane of the Bloch sphere. [22,11,13]. In addition, the phase-covariant cloning can be useful to optimally perform different quantum computation tasks by adopting qubits belonging to the equatorial subspace [23].

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