Contextual realization of the universal quantum cloning machine and of the universal-NOT gate by quantum-injected optical parametric amplification

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A simultaneous, contextual experimental demonstration of the two processes of *cloning* an input qubit $|\Psi\rangle$ and of *flipping* it into the orthogonal qubit $|\Psi^{\perp}\rangle$ is reported. The adopted experimental apparatus, a quantum-injected optical parametric amplifier is transformed simultaneously into a universal optimal quantum cloning machine and into a universal-NOT quantum-information gate. The two processes, indeed *forbidden* in their *exact* form for fundamental quantum limitations, were found to be *universal* and *optimal*, i.e., the measured *fidelity* of both processes $F<1$ was found close to the limit values evaluated by quantum theory. A contextual theoretical and experimental investigation of these processes, which may represent the basic difference between the classical and the quantum worlds, can reveal in a unifying manner the detailed structure of quantum information. It may also enlighten the yet little explored interconnections of fundamental axiomatic properties within the deep structure of quantum mechanics.

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I. INTRODUCTION

Classical information is represented by *bits* which can be either 0 or 1. Quantum information is represented by *quantum bits*, or "qubits," which are two-dimensional quantum systems. A qubit, unlike a classical bit can exist in a state $|\Psi\rangle$ that is a superposition of two orthogonal basis states $\{|\uparrow\rangle;|\downarrow\rangle\}$, i.e., $|\Psi\rangle=\tilde{\alpha}|\uparrow\rangle+\tilde{\beta}|\downarrow\rangle$. The fact that qubits can exist in these superposition states gives quantum information unusual properties. Specifically, information encoded in quantum system has to obey rules of quantum physics which impose strict bounds on possible manipulations with quantum information. The common denominator of these bounds is that all quantum-mechanical transformations have to be represented by *completely positive* (CP) *maps* [1], which in turn impose a constraint on the fidelity of quantum-mechanical measurements. That is, an unknown state of a qubit cannot be precisely determined (or reconstructed) from a measurement performed on a finite ensemble of identically prepared qubits [2–4]. In particular, the mean fidelity of the best possible (optimal) state estimation strategy based on the measurement of N identically prepared qubits is $F=(N+1)/(N+2)$. One of the obvious consequences of this bound on the fidelity of estimation is that unknown states of quantum systems cannot be cloned, i.e., copied perfectly [5], namely, the perfect cloning map of the form $|\Psi\rangle\Rightarrow|\Psi\rangle|\Psi\rangle$ is not permitted by the rules of quantum mechanics. Certainly if this would be possible, then one would be able to violate the bound on the fidelity of estimation. Moreover, this possibility would trigger more dramatic changes in the present picture of the physical world, e.g., it would be possible to utilize quantum nonlocality for superluminal signaling [6–8]. Another map which cannot be performed perfectly on an *unknown* state of a qubit is the *spin flip* or the universal NOT, i.e., the operation $|\Psi\rangle\Rightarrow|\Psi^{\perp}\rangle$, where the state $|\Psi^{\perp}\rangle$ is orthogonal to the original $|\Psi\rangle$ [3,9]. Spin flipping is indeed an *antiunitary*, i.e., time-reversal map $T=i\sigma_y K$ which realizes for any input qubit the *inversion* over the Bloch sphere, as shown in Fig. 1. Precisely, the *phase-conjugation* operator K is responsible for antiunitarity since $K|\Psi\rangle=|\Psi^*\rangle$, the complex conjugate of $|\Psi\rangle$ [10].

In spite of the fact that some quantum-mechanical transformations on unknown states of qubits cannot be performed perfectly, one still may ask what are the best possible approximations of these maps within the given structure of quantum theory, which is *linear* and where all maps are CP maps [1,11]. Namely, in the present context, what is the *optimal* universal cloning and the *optimal* universal-NOT (U-NOT) gate. The *universality* condition is required to ensure that all *unknown* inputs states, i.e., all points on the Bloch sphere, are transformed with the same *quantum efficiency* (QE), viz, *fidelity*. Investigation of these universal optimal

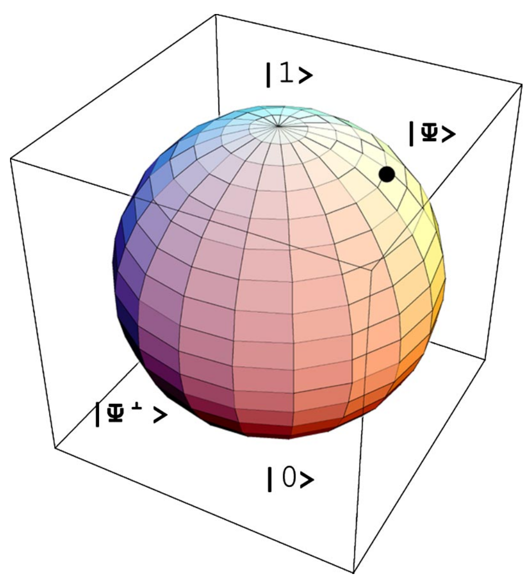

FIG. 1. (Color online) Bloch sphere, the state space of a quantum bit (*qubit*). *Pure states* are represented by points on the sphere while *statistical mixtures* are points inside the sphere. The universal-NOT gate operation corresponds to the inversion of the sphere, i.e., states $|\Psi\rangle$ and $|\Psi^{\perp}\rangle$ are antipodes.

transformations, which are also called *universal quantum machines* [12], is extremely important since it reveals bounds on optimal manipulations with quantum information. Consequently, in recent years theoretical investigation on the universal optimal quantum cloning machine (UOQCM), and on the universal-NOT gate has been very thorough. In spite of all the success in the theoretical analysis of the bounds on optimal manipulations with individual qubits, it is extremely difficult to realize experimentally universal quantum machines. In the domain of quantum optics, this is possible by associating a cloning machine with a photon amplification process, e.g., involving inverted atoms in a laser amplifier or a nonlinear (NL) medium in a *quantum-injected optical parametric amplifier* (QIOPA) [13]. This can be done in virtue of the existing isomorphism between any logic state of a qubit and the polarization state of the photon. In the case of the mode nondegenerate QIOPA [13] it is generally supposed that N photons, prepared identically in an arbitrary quantum state (qubit) $|\Psi\rangle$ of *polarization* ($\boldsymbol{\pi}$) are injected into the amplifier on the input mode k_1. The amplifier then generates on the same output "cloning" (C) mode $M>N$ copies, or clones of the input qubit $|\Psi\rangle$. Correspondingly, the OPA amplifier generates on the output "anticloning" (AC) mode, $M-N$ states $|\Psi^{\perp}\rangle$, thus realizing a quantum NOT gate.

The work is organized as follows. In Sec. II the general theory of quantum-injected amplification is reviewed with emphasis on the dynamical conditions apt to ensure, in the present context, the *universality* property of the device, i.e., implying equal quantum efficiencies for *any* arbitrary input $\boldsymbol{\pi}$ state. Furthermore, the multiparticle superposition state [or *Schrödinger cat* (S cat) state] of the amplified field on modes C and AC will be investigated. In Sec. III the achievement of the universality property will be tested experimentally by injection of a "classical," Glauber *coherent* field. Details of the overall experiment will be given in this section. In Sec. IV the theory of the *optimal* cloning process and NOT gate will be outlined in a unitary and consistent fashion for the $N=1,M=2$ case and applied to the QIOPA scheme operating in a $\boldsymbol{\pi}$-*entangled* configuration [13]. In Sec. V the experimental demonstration for both optimal processes are reported, the values of the corresponding "fidelity" are evaluated, and are found in good agreement with the theoretical ones [4,13]. For the sake of completeness, in Sec. V the *universality* of the QIOPA apparatus will be tested again under the *quantum injection* of a single qubit. In conclusion, Sec. VI will be devoted to a theoretical discussion over the inner connections existing between the two basic quantum-information processes, here investigated contextually by the *same* overall dynamical process.

II. QUANTUM-INJECTED OPTICAL PARAMETRIC AMPLIFIER

In the present work the quantum-information carriers (*qubits)* are assumed to represent states of polarization ($\boldsymbol{\pi}$), and QE is expressed by the OPA parametric "gain" g. Let us investigate theoretically the dynamics of the QIOPA apparatus, making reference to Fig. 2 and to Ref. [13]. The active element of the device is a type-II NL crystal slab operating in

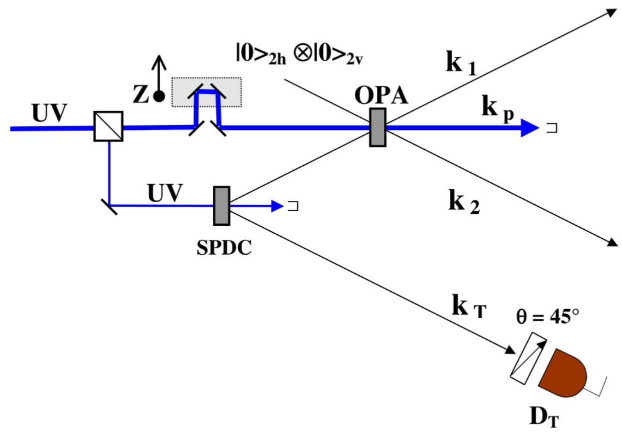

FIG. 2. (Color online) Schematic diagram of the *quantum-injected* optical parametric amplifier (QIOPA) in *entangled configuration*. The injection is provided by an external spontaneous parametric down conversion source (SPDC) of polarization ($\boldsymbol{\pi}$) entangled photon states.

noncollinear configuration. In these conditions the overall amplification taking place over the coupled electromagnetic (e.m.) modes k_j ($j=1,2$) is contributed by two equal and *independent* parametric amplifiers: OPA A and OPA A' inducing uncorrelated unitary transformations, respectively, on two couples of (time) t-dependent field operators $\hat{a}_1(t) \equiv \hat{a}_{1h}(t), \hat{a}_2(t) \equiv \hat{a}_{2v}(t)$ and $\hat{a}_1'(t) \equiv \hat{a}_{1v}(t), \hat{a}_2'(t) \equiv \hat{a}_{2h}(t)$ acting on the output modes $k_j(j=1,2)$ along the horizontal (H) and vertical (V) directions in the $\boldsymbol{\pi}$ plane. The interaction Hamiltonian may be expressed in the general form:

$$\hat{H}_I = i\hbar\chi[\hat{A} - e^{i\Phi}\hat{A}'] + \text{H.c.}, \tag{1}$$

where $\hat{A} \equiv \hat{a}_1(t)\hat{a}_2(t)$, $\hat{A}' \equiv \hat{a}_1'(t)\hat{a}_2'(t)$, and $g \equiv \chi t$ is a real number expressing the amplification gain proportional to the NL coupling term χ. The dynamics of OPA A and OPA A' is expressed correspondingly by the mutually commuting unitary squeeze operators: $\hat{U}_A(t) = \exp[-g(\hat{A}^{\dagger} - \hat{A})]$ and $\hat{U}_{A'}(t) = \exp[g(e^{-i\Phi}\hat{A}'^{\dagger} - e^{i\Phi}\hat{A}')]$ implying the following Bogoliubov transformations [13]:

$$\begin{bmatrix} \hat{a}_1(t) \\ \hat{a}_2(t)^{\dagger} \end{bmatrix} = \begin{bmatrix} C & S \\ S & C \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2^{\dagger} \end{bmatrix}; \begin{bmatrix} \hat{a}_1'(t) \\ \hat{a}_2'(t)^{\dagger} \end{bmatrix} = \begin{bmatrix} C & \tilde{S} \\ \tilde{S}^* & C \end{bmatrix} \begin{bmatrix} \hat{a}_1' \\ \hat{a}_2'^{\dagger} \end{bmatrix}, \tag{2}$$

where $C \equiv \cosh(g)$, $S \equiv \sinh(g)$, $\tilde{S} \equiv \epsilon S$, $\Gamma \equiv S/C$, $\tilde{\Gamma} = \epsilon\Gamma$, $\epsilon \equiv -e^{-i\Phi}$, and Φ is an externally adjustable *intrinsic phase* existing between the A and A' OPA devices. These transformations imply the time invariance of the interaction Hamiltonian and of the field commutors, i.e., $\hat{H}_I(t) = \hat{H}_I(0)$, $[\hat{a}_i(t), \hat{a}_j^{\dagger}(t)] = [\hat{a}_i, \hat{a}_j^{\dagger}] = \delta_{ij}$, $[\hat{a}_i'(t), \hat{a}_j'^{\dagger}(t)] = [\hat{a}_i', \hat{a}_j'^{\dagger}] = \delta_{ij}$, $[\hat{a}_i'(t), \hat{a}_j^{\dagger}(t)] = 0$ being $i,j = 1,2$ and $\hat{a}_{i,j} \equiv \hat{a}_{i,j}(0)$, $\hat{a}_{i,j}' \equiv \hat{a}_{i,j}'(0)$ the input fields at the initial time $t=0$, i.e., before the OPA interaction. The evolution in the Schrödinger picture of the state acted upon by the OPA system is determined by the overall operator $\hat{U}=\hat{U}_A\otimes\hat{U}_{A'}$ expressed in terms of the operators evaluated at $t=0$,

$$\hat{U}_A=\exp[g(\hat{\sigma}_+ + \hat{\sigma}_-)],\quad \hat{U}_{A'}=\exp[g(\hat{\sigma}'_+ + \hat{\sigma}'_-)],\qquad(3)$$

by adopting the definitions $\hat{\sigma}_+=-\hat{A}^\dagger$, $\hat{\sigma}_-=\hat{A}$, $\hat{\sigma}_z=(1+\hat{a}_1^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2)\equiv(1+\hat{n}_1+\hat{n}_2)$, $\hat{\sigma}'_+=-\epsilon\hat{A}'^\dagger$, $\hat{\sigma}'_-=\epsilon^*\hat{A}'$, and $\hat{\sigma}_z=(1+\hat{n}'_1+\hat{n}'_2)$. In virtue of Eq. (2) the following commutation properties for the sets of the $\hat{\sigma}$ and $\hat{\sigma}'$ pseudo-spin operators hold: $[\hat{\sigma}_+,\hat{\sigma}_-]=\hat{\sigma}_z$, $[\hat{\sigma}_z,\hat{\sigma}_\pm]=\pm2\hat{\sigma}_\pm$ and $[\hat{\sigma}'_+,\hat{\sigma}'_-]=\sigma'_z$, $[\hat{\sigma}'_z,\hat{\sigma}'_\pm]=\pm2\hat{\sigma}'_\pm$, $[\hat{\sigma}_\pm,\hat{\sigma}'_\mp]=[\hat{\sigma}_\pm,\hat{\sigma}'_z]=0$. By further adopting the definitions: $\hat{\sigma}_x\equiv2^{-1/2}(\hat{\sigma}_+ + i\hat{\sigma}_-)$, $\hat{\sigma}_y\equiv2^{-1/2}(\hat{\sigma}_+ - i\hat{\sigma}_-)$, the following relevant commutors: $[\hat{\sigma}_x,\hat{\sigma}_y]=-i\hat{\sigma}_z$, $[\hat{\sigma}'_x,\hat{\sigma}'_y]=-i\hat{\sigma}'_z$ are recognized as those belonging to the symmetry group SU(1,1) [14]. The output field may be expressed, in virtue of an appropriate “operator disentangling theorem” in the following form [13,15]:

$$|\Psi\rangle=\{\exp\Gamma(\hat{\sigma}_+ + \hat{\sigma}'_+)\exp[-\ln C(\hat{\sigma}_z+\hat{\sigma}'_z)]\times\exp\Gamma(\hat{\sigma}_- + \hat{\sigma}'_-)\}|\Psi\rangle_{IN}.\qquad(4)$$

Take as input state into the QIOPA system the general *qubit*: $|\Psi\rangle_{IN}\equiv(\tilde{\alpha}|\Psi\rangle^\alpha_{IN}+\tilde{\beta}|\Psi\rangle^\beta_{IN})$, $|\tilde{\alpha}|^2+|\tilde{\beta}|^2=1$, defined in the (2×2)-dimensional Hilbert space of polarizations $\boldsymbol{\pi}$ on the two interacting modes k_1 and k_2 with basis vectors $|\Psi\rangle^\alpha_{IN}=|1\rangle_{1h}|0\rangle_{1v}|0\rangle_{2h}|0\rangle_{2v}\equiv|1,0,0,0\rangle$, $|\Psi\rangle^\beta_{IN}=|0,1,0,0\rangle$. Here the general product state $|x\rangle_{1h}\otimes|y\rangle_{1v}\otimes|z\rangle_{2h}\otimes|t\rangle_{2v}$ has been, and shall be henceforth, expressed by the shorthand $|x,y,z,t\rangle$. In virtue of the general *information preserving* property of any NL transformation of parametric type, the output state is again expressed by a *massive qubit* $|\Psi\rangle\equiv(\tilde{\alpha}|\Psi\rangle^\alpha+\tilde{\beta}|\Psi\rangle^\beta)$ [16]. This (pure) state is indeed a Schrödinger-cat state implying the quantum superposition of the orthonormal multiparticle states [13]:

$$|\Psi\rangle^\alpha\equiv C^{-3}\sum_{i,j=0}^{\infty}(-\Gamma)^{i+j}\epsilon^j\sqrt{i+1}|i+1,j,j,i\rangle;$$

$$|\Psi\rangle^\beta\equiv C^{-3}\sum_{i,j=0}^{\infty}(-\Gamma)^{i+j}\epsilon^j\sqrt{j+1}|i,j+1,j,i\rangle.\qquad(5)$$

Consider the density operator $\rho\equiv|\Psi\rangle\langle\Psi|$ and its reductions over the $\boldsymbol{\pi}$-*vector* spaces relative to the spatial modes k_1 and k_2: $\rho_1=\mathrm{Tr}_2\rho$; $\rho_2=\mathrm{Tr}_1\rho$. These ones may be expanded as a weighted superpositions of $p\times p$ matrices of order $p=(n+2)$, the relative weight $\Gamma^2=\tanh^2 g$ of each two successive matrices being determined by the parametric gain. $\Gamma^2=0$ for $g=0$ and approaches asymptotically the unit value for large g. In turn, the $p\times p$ matrices may be expressed as sum of 2×2 matrices as shown by the following expressions:

$$\rho_1=C^{-6}\sum_{n=0}^{\infty}\Gamma^{2n}\times\sum_{i=0}^{n}\begin{bmatrix}|\tilde{\beta}|^2(n-i+1) & \tilde{\alpha}^*\tilde{\beta}\sqrt{(i+1)(n-i+1)}\\ \tilde{\alpha}\tilde{\beta}^*\sqrt{(i+1)(n-i+1)} & |\tilde{\alpha}|^2(i+1)\end{bmatrix},\qquad(6)$$

written in terms of the Fock basis: $\{|i\rangle_{1h}|n-i+1\rangle_{1v},|i+1\rangle_{1h}|n-i\rangle_{1v}\}$. Correspondingly,

$$\rho_2=C^{-6}\sum_{n=0}^{\infty}\Gamma^{2n}\times\sum_{i=0}^{n+1}\begin{bmatrix}|\tilde{\beta}|^2(n-i+1) & \epsilon^*\tilde{\alpha}^*\tilde{\beta}\sqrt{(n-i+1)i}\\ \epsilon\tilde{\alpha}\tilde{\beta}^*\sqrt{(n-i+1)i} & |\tilde{\alpha}|^2 i\end{bmatrix}\qquad(7)$$

in terms of the Fock basis: $\{|n-i\rangle_{2h}|i\rangle_{2v},|n-i+1\rangle_{2h}|i-1\rangle_{2v}\}$. Interestingly enough, the value n of the sum indices appearing in Eqs. (6) and (7) coincides with the *number* of photon pairs generated by the QIOPA amplification. Note that all the 2×2 matrices in Eqs. (6) and (7) and the $p\times p$ matrices resulting from their sums over the *i index* are non-diagonal, as implied by the quantum superposition property of any Schrödinger-cat state. Note also that the OPA *intrinsic phase* Φ only affects the AC, i.e., the mode k_2.

A very powerful tool to inspect at a deeper level the Schrödinger-cat condition is the Wigner function $W\{\alpha,\beta\}$ of the output field for the QIOPA apparatus shown in Fig. 2. There $\{\alpha,\beta\}\equiv(\alpha_j,\ \alpha_j^*,\ \beta_j,\ \beta_j^*)$ are the complex, eight-dimensional phase-space variables expressing the overall QIOPA output field. A thorough analysis leading to an exact, closed form evaluation of $W\{\alpha,\beta\}$ was indeed carried out in Refs. [13,17]. As expected, the nondefinite positivity of $W\{\alpha,\beta\}$ demonstrated by these calculations shows explicitly the overall quantum character of our multiparticle, injected amplification scheme.

Universality. A necessary common property of the QIOPA system in the context of the present work is its *universality* (U), i.e., implying the *same* QE of the amplifying apparatus for any input, unknown qubit; that is, for a qubit spanning the entire Bloch sphere. In our experiment the qubits are assumed to represent states of polarization ($\boldsymbol{\pi}$), as said, and QE is expressed by the QIOPA parametric *gain g*. We shall find that universality implies an important symmetry property, namely, the invariance of the coupling Hamiltonian H_{int} under *simultaneous* general SU(2) transformations on the polarization $\boldsymbol{\pi}$ on the spatial modes k_j $(j=1,2)$ [13,18]. Assume that under a simultaneous general rotation R of $\boldsymbol{\pi}$ on both modes k_j, the field set $\{\hat{a}_j,\hat{a}'_j\}$ is changed into the set $\{\hat{a}_{Rj},\hat{a}'_{Rj}\}$ $(j=1,2)$. A general $R(\vartheta,\xi)$ transformation, expressed in terms of complex parameters for which $|\vartheta|^2+|\zeta|^2=1$, relates the two field sets as follows:

$$\begin{bmatrix}\hat{a}_{R1}\\ \hat{a}'_{R1}\end{bmatrix}=R^{\dagger}\begin{bmatrix}\hat{a}_1\\ \hat{a}'_1\end{bmatrix}R=\begin{bmatrix}\vartheta & \zeta\\ -\zeta^* & \vartheta^*\end{bmatrix}\begin{bmatrix}\hat{a}_1\\ \hat{a}'_1\end{bmatrix};$$

$$\begin{bmatrix}\hat{a}'_{R2}\\ \hat{a}_{R2}\end{bmatrix}=R^{\dagger}\begin{bmatrix}\hat{a}'_2\\ \hat{a}_2\end{bmatrix}R=\begin{bmatrix}\vartheta & \zeta\\ -\zeta^* & \vartheta^*\end{bmatrix}\begin{bmatrix}\hat{a}'_2\\ \hat{a}_2\end{bmatrix}. \quad (8)$$

It can be easily checked that Eq. (1) can be re-expressed in terms of the new field set into the invariant form: $\hat{H}_{RI}=i\hbar\chi[\hat{A}_R-\hat{A}'_R]+\text{H.c.}$, where $\hat{A}_R\equiv\hat{a}_{R1}\hat{a}_{R2}$, $\hat{A}'_R\equiv\hat{a}'_{R1}\hat{a}'_{R2}$, *only* by setting the OPA *intrinsic phase* $\Phi=0$, i.e., $\epsilon=-1$. Interestingly, note that the same dynamical condition $\Phi=0$ implies the well-known SU(2) invariance of the $\boldsymbol{\pi}$-entangled ‘‘singlet states’’ generated by spontaneous parametric down-conversion (SPDC), that is, by the OPA when it is not *quantum injected* (or, when it is only ‘‘injected’’ by the *vacuum field* on both input modes k_j).

Since, in general the input $N\geqslant 1$ qubits injected into the amplifier are quantum superpositions of $\boldsymbol{\pi}$ states, the dynamical condition $\Phi=0$ finally implies the *universality* of the overall cloning and universal-NOT transformations. For the sake of clearness in the future discussions, we find convenient to recast the invariant Hamiltonian with $\Phi=0$, in the following form:

$$\hat{H}_{int}=i\hbar\chi(\hat{a}_{\pi}\hat{b}_{\pi\perp}-\hat{a}_{\pi\perp}\hat{b}_{\pi})+\text{H.c.}, \quad (9)$$

where $\hat{a}$ and $\hat{b}$ are the overall field operators acting, respectively, on the output modes k_1 and k_2. For reasons that will become clear in the following sections these modes are referred to as the C and the AC modes, respectively. Furthermore, since $g=\chi t$ is independent of any *unknown* polarization state of the injected field, we have denoted the creation $\hat{a}^{\dagger}$, $\hat{b}^{\dagger}$ and annihilation $\hat{a}$, $\hat{b}$ operators of a single photon in modes k_1, k_2 with subscripts π or $\pi^{\perp}$ to indicate the invariance of the process with respect to the polarization states of the input particles. Of course, the SU(2) transformation for the fields $\hat{a}$ is again expressed as follows:

$$\begin{bmatrix}\hat{a}_{R\pi}\\ \hat{a}_{R\pi\perp}\end{bmatrix}=R^{\dagger}\begin{bmatrix}\hat{a}_{\pi}\\ \hat{a}_{\pi\perp}\end{bmatrix}R=\begin{bmatrix}\vartheta & \zeta\\ -\zeta^* & \vartheta^*\end{bmatrix}\begin{bmatrix}\hat{a}_{\pi}\\ \hat{a}_{\pi\perp}\end{bmatrix}, \quad (10)$$

and $|\vartheta|^2+|\zeta|^2=1$. The same R transformation is valid for the fields $\hat{b}$. The polarization conditions π and $\pi^{\perp}$ will be expressed, respectively, by the field state vectors $|\Psi\rangle$ and $|\Psi^{\perp}\rangle$ on the C and AC output modes of the apparatus. As a final remark, note that the analysis expressed by the present section implies the classical, undepleted, coherent field hypothesis for the ultraviolet (uv) pump field of the parametric process. A more complete Manley-Rowe theory including the uv ‘‘pump’’ depletion could be undertaken in the present context [19]. It is expected to preserve the quantum character of the interaction and lead to interesting results, e.g., a measurement of the number of pump photons lost in the interaction would imply the exact knowledge of the number of the generated pairs. However, the corresponding realization is physically unreasonable in the present case as the number of photons associated with any uv pump pulse is $\approx 10^{10}$.

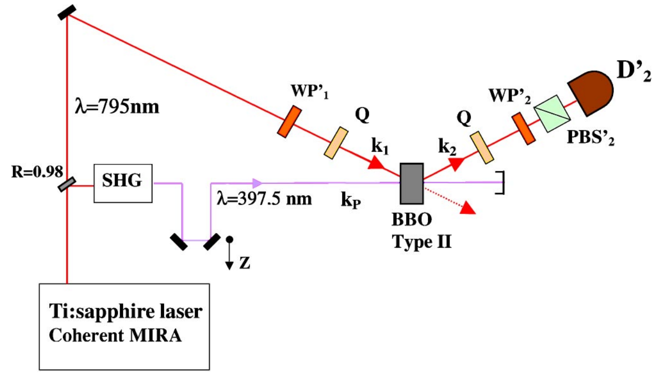

FIG. 3. (Color online) Injection apparatus of attenuated coherent optical pulses into a QIOPA to demonstrate the universality of the parametric amplification.

III. TEST OF THE ‘‘UNIVERSALITY’’ CONDITION

As already remarked, the *universality* condition implies for the OPA amplifier the SU(2) invariance of $\hat{H}_{int}$ when the spatial orientation of the OPA crystal makes it available for SPDC creation of two-photon entangled *singlet* states. Indeed the universality condition in amplifying physical devices is quite a peculiar property that can only be realized by a very small number of arrangements set in very special conditions [20]. Luckily enough, the QIOPA apparatus has been found to possess this property. In fact, the OPA application represents the first actual realization of the universality condition for $\boldsymbol{\pi}$ *qubits* [9,13,21]. In spite of the ‘‘microscopic’’ quantum theoretical approach adopted in the preceding section theory, we should note that in the present context the universality property is indeed a ‘‘macroscopic’’ classical feature of the OPA device. Thus it can be tested equally well either by injection of a quasiclassical, e.g., a *coherent* (Glauber) field, or by injection of a quantum state, e.g., a Fock state. Because of the relevance of the universality property, we shall undertake the experimental demonstration in both ways. The classical test will be described in the present section while analogous tests carried out by single-photon Fock states will be reported later in the paper. In order to do that, let us first venture into a detailed description of the excitation laser and of the QIOPA apparatus adopted throughout the present work, Fig. 3.

Apparatus. The main source of all experiments reported by this work was a Ti:sapphire mode-locked pulsed laser (Coherent MIRA), providing by second-harmonic generation (SHG), the pump field for the quantum-injected optical parametric amplifier QIOPA associated with the spatial mode having wave vector k_p and wavelength $\lambda_p=397.5$ nm, i.e., in the uv range of the spectrum. The average uv power was 0.25 W, the pulse repetition rate was 76 MHz, and the time duration of each uv pulse was $\tau=140$ fs. The OPA active element [consisting of a 1.5-mm-thick NL crystal of

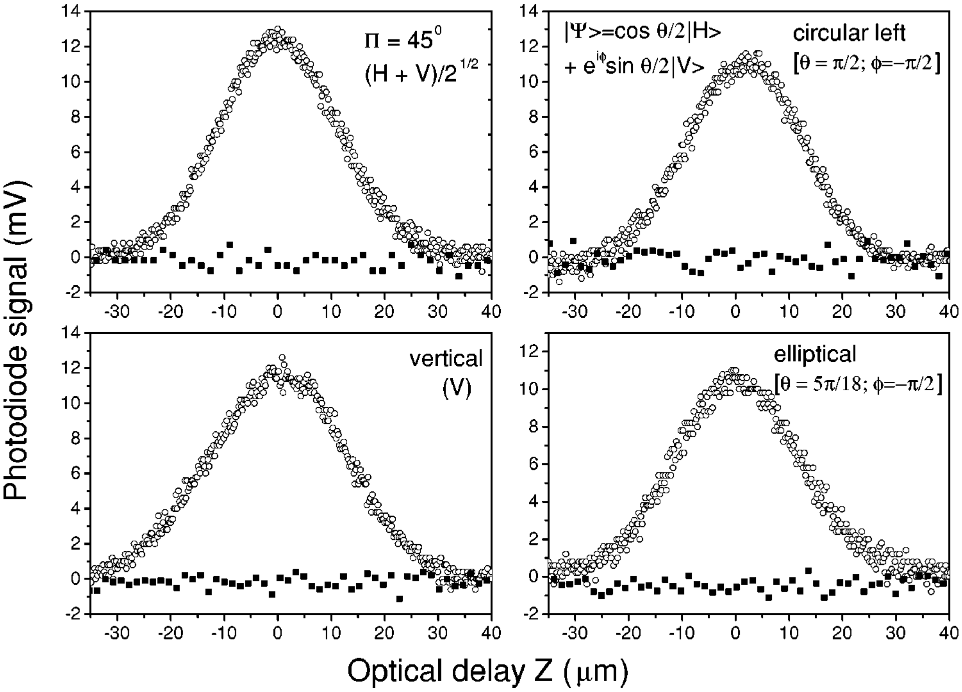

FIG. 4. Experimental verification of the universality of the OPA system by injection of **π** *qubits* represented by several significant points on the Bloch sphere. The injected state is encoded in a very attenuated coherent laser beam.

β-barium borate (BBO) cut for type-II phase matching] was able to generate, by SPDC, ***π**-entangled* pairs of photons. Precisely, the OPA *intrinsic phase* was set as to generate by SPDC, "singlet" entangled states on the output optical modes: $\Phi=0$. The photons of each pair were emitted with equal wavelengths $\lambda=795$ nm over two spatial modes k_1 and k_2 making an internal angle$=8°$. In all experiments the time t optical walk-off effects due to the birefringence of the NL crystal were compensated by inserting in the modes k_1 and k_2 fixed X-cut, 4.8-mm-thick quartz plates. All adopted photodetectors D, but D_2' (Fig. 3), were equal SPCM-AQR14 Si-avalanche nonlinear single-photon units with nearly equal QE's$\cong 0.55$. One interference filter with bandwidth $\Delta\lambda=6$ nm was placed in front of each detector D. Only the detector D_2' was a linear Si photodiode SGD100. Polarizing beam splitters (PBS) in Figs. 3 and 5 were adopted as measurement devices providing the polarization analysis.

Universality test by classical field injection. The universality test was carried out by injection of the strongly attenuated laser beam, with wavelength $\lambda=795$ nm contributed via a beam splitter by the main mode-locked source and directed along the OPA injection mode k_1, Fig. 3. The parametric amplification with gain $g=0.11$ was detected at the OPA output mode k_2 by the linear Si photodiode SGD100 (D_2'), filtered by an interference filter with bandwidth $\Delta\lambda=3$ nm. The temporal overlap in the NL crystal of the pump and of the "injection" pulses was assured by micrometric displacements **Z** of a two-mirror "optical trombone." The pulse shapes reported as function of **Z** in Fig. 4, as well as in Figs. 6 and 7, are indeed the *signature for actual amplification*, i.e., arising from the effective time and space overlap in the NL crystal of the uv pump pulse and of the optical pulses with $\lambda=795$ nm injected into the OPA. Different ***π** states* of the injected field, formally expressed also as insets of Figs. 4, 6, and 7, were prepared by a single wave plate WP_1', corresponding to a suitable optical retardation, equal to $\lambda/2$ and $\lambda/4$ between the two orthogonal basis states, i.e., H and V. The OPA amplified output states were detected by an apparatus inserted on mode k_2 and consisting of the set (WP_2' + **π** *analyzer*), the last device being provided by the polarizing beam splitter PBS_2'.

The universality condition is demonstrated by the plots of Fig. 4 showing the amplification pulses detected by D_2' on the OPA output (AC) mode k_2. Each plot corresponds to a definite state of polarization (**π**) of the field injected on mode k_1: $[\cos(\theta/2)|H\rangle+\exp(i\varphi)\sin(\theta/2)|V\rangle]$. Precisely, the polarization was set to be either linear (that is, $\theta=\pi/2,\pi;\varphi=0$) or circular (that is, $\theta=\pi/2;\varphi=-\pi/2$), or very generally elliptical: $\theta=5\pi/18,\varphi=-\pi/2$. We may check in Fig. 4 that, in spite of the quite different injected states, the amplification curves are almost identical. Each coherent pulse injected on the mode k_1 was amplified into an average photon number $M'=5\times 10^3$ on the output mode k_2.

IV. UNIVERSAL OPTIMAL QUANTUM CLONING MACHINE AND UNIVERSAL-NOT GATE

The first machine which has been investigated theoretically was the *optimal universal quantum cloner* of qubits. In the simple case investigated in the present work, namely, of cloning $N=1$ input into $M=2$ output *qubits*, the action of the cloner can be described by a simple covariant transformation [22]

$$|\Psi\rangle|\downarrow\rangle_C|\downarrow\rangle_{AC}\Rightarrow\sqrt{2/3}|\Psi\rangle|\Psi\rangle|\Psi^{\perp}\rangle_{AC} - \sqrt{1/3}|\{\Psi,\Psi^{\perp}\}\rangle|\Psi\rangle_{AC}, \qquad (11)$$

where the first (unknown) state vector in the left-hand side of Eq. (11) corresponds to the system to be cloned, the second state vector describes the system on which the information is going to be copied ("blank" qubit), represented by the C channel, the mode k_1, while the third state vector represents the cloner. The state $|\Psi^{\perp}\rangle$ is the *antipode* of $|\Psi\rangle$ on the Bloch sphere. In the present work the cloner is a qubit, associated with the AC channel. The blank qubit and the cloner are initially in the known state $|\downarrow\rangle$. At the output of the cloner we find two qubits (the original and the copy) in the state $\rho = 2/3|\Psi\rangle|\Psi\rangle\langle\Psi|\langle\Psi| + 1/3|\{\Psi,\Psi^{\perp}\}\rangle\langle\{\Psi,\Psi^{\perp}\}|$, where $|\{\Psi,\Psi^{\perp}\}\rangle$ is the completely symmetric state of two qubits. The density operator ρ describes the best possible approximation of the perfect cloning process, i.e., the state $|\Psi\rangle|\Psi\rangle$. The fidelity of this transformation does not depend on the state of the input and reads $F = \mathrm{Tr}(\rho_1 \hat{n}_{1\pi})/\mathrm{Tr}(\rho_1 \hat{n}_1) = 5/6 \approx 0.833$. The cloner itself after the cloning transformation is in the state $\rho_{AC} = 1/3|\Psi^{\perp}\rangle\langle\Psi^{\perp}| + 1/3 \times \mathbf{I}$, where $\mathbf{I}$ is the unity operator. This last density operator is the best possible approximation of the action of the spin-flip (universal-NOT) operation permitted by the quantum mechanics. As we shall see in the following sections, for $N=1$ input states the fidelity of the universal-NOT transformation is $F^* = \mathrm{Tr}(\rho_2 \hat{n}_{2\perp})/\mathrm{Tr}(\rho_2 \hat{n}_2) = 2/3 \approx 0.666$ and is equal to the fidelity of estimation [4]. The universal quantum machine has been subsequently generalized for the case of multiqubit inputs, when the machine takes as an input $N>1$ identically prepared qubits and generates either $M>N$ clones or otherwise transformed qubits.

We may establish a close connection of the above results for cloning with the adopted *universal* QIOPA system, by considering, for instance, the simple case of a linearly polarized *single photon* $N=1$ injected into the OPA on the input mode k_1 with H polarization, $\boldsymbol{\pi}=\mathbf{h}$. In virtue of the *universality* this very particular injection condition indeed expresses the general behavior of the amplifier. By the analysis reported in Sec. II, the amplification leads to the output state $|\Psi\rangle = \hat{U}|\Psi\rangle_{IN} = \hat{U}|1\rangle_{1h}|0\rangle_{1v}|0>_{2h}|0\rangle_{2v} \equiv \hat{U}|1,0,0,0\rangle$. This last expression implies one photon on the input k_{1h} *mode* with $\boldsymbol{\pi}=\mathbf{h}$ and zero photons on the three other input modes k_{1v}, k_{2h}, k_{2v}. Let us adopt the isomorphism $|\Psi\rangle|\downarrow\rangle = |1\rangle_{1h}|0\rangle_{1v}$, $|\Psi^{\perp}\rangle|\downarrow\rangle = |0\rangle_{1h}|1\rangle_{1v}$ and $|\{\Psi,\Psi^{\perp}\}\rangle = |1\rangle_{1h}|1\rangle_{1v}$; we shall find that the output state of the OPA amplifier, $|\Psi\rangle_{out}$, expressed by Eq. (5) is identical, in the first order of the small parameter Γ, to the output of the cloner transformation expressed by Eq. (11). Indeed, the input mode $|1,0,0,0\rangle$ evolves into the first order state:

$$\hat{U}|1,0,0,0\rangle \approx |1,0,0,0\rangle - \Gamma(\sqrt{2}|2,0\rangle_{k_1} \otimes |0,1\rangle_{k_2} - |1,1\rangle_{k_1} \otimes |1,0\rangle_{k_2}), \tag{12}$$

where for clarity we set $|x,y\rangle_{k_j} \equiv |x\rangle_{jh} \otimes |y\rangle_{jv}$ and $j=1,2$. The approximation for the state vector describing the two k_j modes at times $t>0$ is sufficient in all cases in which the coupling value is small, as in our experiment: $g \sim 0.11$. The two addenda at the right-hand side of Eq. (12) represent, respectively, the process where the input photon in the mode k_1 did not interact in the nonlinear medium, followed by the first-order amplification process in the OPA. Here the state $|2,0\rangle_{k_1}$ describing two photons of the *cloning* mode k_1 in the polarization state $\boldsymbol{\pi}$ corresponds to the state $|\Psi\Psi\rangle$. This state-vector describes the $1 \rightarrow 2$ *cloning* of the original injected photon. The vector $|0,1\rangle_{k_2}$ describes the state of the *anticloning* mode k_2 with a single photon with the polarization $\boldsymbol{\pi}^{\perp}$ corresponding to the state $|\Psi^{\perp}\rangle_{AC}$. This state vector represents the *flipped* version of the input. To see that the stimulated emission is indeed responsible for creation of the flipped qubit, let us compare the result above with the output of the OPA when the vacuum is injected into the nonlinear crystal, i.e., the SPDC process. In this case, we obtain to the same order of approximation:

$$\hat{U}|0,0,0,0\rangle \approx |0,0,0,0\rangle_{k_1} - \Gamma(|1,0\rangle_{k_1} \otimes |0,1\rangle_{k_2} - |0,1\rangle_{k_1} \otimes |1,0\rangle_{k_2}). \tag{13}$$

We see that the flipped qubit described by the state vector $|0,1\rangle_{k_2}$ in the right-hand sides of Eqs. (12) and (13) does appear with different amplitudes corresponding to the ratio of the probabilities to be equal to $R=2:1$. Our experiment, which deals with the injection of $N=1$ photon, indeed consists of the measurement of R, as we shall see. According to the analysis reported in Sec. II, the cloning and the universal-NOT operations are not altered by the multiplication of $\hat{H}_I$ by any overall phase factor.

As anticipated in Sec. I, it is well known that a method for flipping qubits alternative to the present universal-NOT gate consists of manufacturing the orthogonal qubits on the basis of the result of the measurement on the input qubits. The fidelity F^* of the two alternative methods is indeed the same [3,4]. However, in the universal-NOT gate the information encoded in the input qubit is not lost in the irreversible state reduction implied by the measurement. It is just redistributed into several qubits at the output. Since this information redistribution is governed by a unitary transformation, the process is in principle reversible, which is definitely not true in the case of measurement-based flipping operation. We also want to stress that, to the best of our knowledge, the recent universal-NOT experiment reported by us in Ref. [9] and the present one are the first systematic attempts to realize a unitary approximation to an *antiunitary* operation on a subspace. Note that the possibility of realizing any antiunitary operation enables one to realize *all of them* by multiplication with unitaries. Obviously, the fidelity of the gate is strictly determined by the structure, i.e., rules, of quantum mechanics, as shown. This stresses the fundamental importance to understand the action of the universal-NOT gate in view of a clarification of the structure of quantum mechanics in connection with the CP-map topology, as we shall see [1].

A more general analysis can be undertaken by extending the isomorphism discussed above to a larger number of input and output particles, N and M. In this case too it is found that the QIOPA amplification process $\hat{U}_{AA'}$ in each order of the decomposition into the parameter Γ corresponds to the $N \rightarrow M$ cloning process. Precisely, in this case $M \geqslant N$ output particles are detected over the output C mode k_1. Correspondingly, $M-N$ particles are detected over the output AC mode k_2. Indeed, the *optimum cloning* output state determined by the theory of the *quantum-injected* OPA is found,

$$|\Psi_N\rangle_M = \sum_{m=0}^{M-N} (-1)^m P_N^M(m)|M-m,m,m,M-N-m\rangle, \tag{14}$$

where $P_N^M(m)=[\binom{M-m}{N}/\binom{M+1}{N+1}]^{1/2}$. By a simple rearrangement of the Fock state degeneracies, Eq. (14) is found to agree exactly with the general expression of the output state of the QIOPA given by Eq. (5) [13,18]. In this general case *optimal cloning fidelity* is found [23,24], $F=(NM+M+N)/(MN+2M)$. For $N=1$, $M=2$ is $F=5/6\approx 0.833$. It appears clear from the above analysis that the effect of the input *vacuum field*, which is *necessarily* injected in any *universal* optical amplifier, is indeed responsible to reduce the fidelity of the quantum machines at hand. More generally, the vacuum field is in exact correspondence with, and must be interpreted as, the amount of quantum fluctuations which determines the upper bounds to the *fidelity* determined by the very CP-map structure of quantum mechanics.

At last note that, luckily enough, the QIOPA apparatus is found to be an ideal system to demonstrate, in a peculiar unifying manner, the relevant features of the most interesting machines investigated thus far, the UOQCM and the universal-NOT gate. The present work also establishes an interesting connection between the technical *engineering* of parametric amplifiers and the abstract quantum measurement theory. Interestingly, an alternative scheme should be offered by the ''Faraday mirror'' that supposedly performs the action of returning the orthogonal polarization to any given input $\boldsymbol{\pi}$-encoded qubit [10].

However, very important, we should consider that associating a *cloning* process with a *particle amplification* process is conceptually *incorrect* [18]. The point is that in QED N photons belonging to the *same* e.m. mode cannot be properly associated with N spin-$\frac{1}{2}$ particles because they actually are *indistinguishable* objects. This means that, within the present model, a cloned state say $|\Psi\rangle|\Psi\rangle|\Psi\rangle|\Psi\rangle$, is indeed represented by the *symmetrized* Fock state $|N\rangle$, i.e., bearing in this case the *Bose degeneration*: $N=4$. In quantum information this consists of a substantial limitation as the information content of a Fock state $|N\rangle$ is $N+1$ bits, i.e., generally a far smaller number that the value 2^N corresponding to the case of N spin-$\frac{1}{2}$ particles that can be *addressed* separately without ambiguity. At present we do not know of any *amplification* scheme realizing a *real* quantum cloning process by adopting photons, atoms, or other information carrying particles.

V. EXPERIMENTAL UOQCM AND UNIVERSAL-NOT GATE

The main laser apparatus and the basic structure of the NL parametric amplifier were already described in Sec. III. Figure 5 shows the QIOPA apparatus, arranged in a *self-injected* configuration and adopted to realize *simultaneously* the UOQCM and the universal optimal NOT gate. The uv pump beam, backreflected by a spherical mirror M_p with 100% reflectivity and micrometrically adjustable position **Z**, ex-

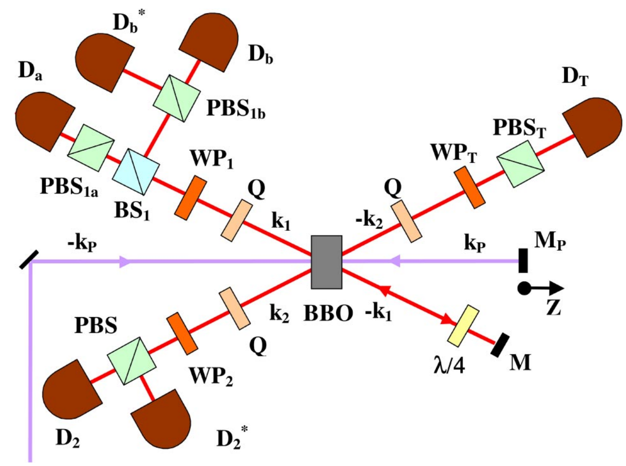

FIG. 5. (Color online) Schematic diagram of the *universal optimal cloning machine* (UOQCM) realized on the cloning (C) channel (mode k_1) of a *self-injected* OPA and of the universal-NOT (universal-NOT) gate realized on the anticloning (AC) channel k_2. Micrometric adjustments of the coordinate **Z** of the uv mirror M_p ensured the time superposition in the active NL crystal of the uv 140-fsec pump pulses and of the single-photon pulse injected via backreflection by the fixed mirror *M*.

cited the NL OPA crystal amplifier in both directions $-k_p$ and k_p, i.e., correspondingly oriented towards the right (R) and the left (L) sides of the figure. A SPDC process excited by the $-k_p$ pump mode created single pairs of photons with equal wave lengths $\lambda=795$ nm in entangled singlet states of linear polarization, $\boldsymbol{\pi}$. One photon of each pair, emitted over $-k_1$, was reflected by a spherical mirror M into the NL crystal where it provided the $N=1$ *quantum injection* into the OPA excited by the uv-pump beam associated with the backreflected mode k_p. Because of the low pump intensity, the probability of the unwanted $N=2$ photon injection has been evaluated to be 10^{-2} smaller than the one for $N=1$. The twin SPDC generated photon emitted over $-k_2$ was selected by the devices (wave plate and polarizing beam splitter: $\mathrm{WP}_T+\mathrm{PBS}_T$), detected by D_T and provided the ''trigger'' of the overall conditional experiment. Because of the EPR nonlocality implied by the SPDC emitted singlet state, the selection on mode $-k_2$ provided the realization on k_1 of the polarization states $|\Psi\rangle_{IN}$ of the injected photon. By adopting $\lambda/2$ or $\lambda/4$ WPs with different orientations of the optical axis, the following states were injected: $|H\rangle$, $2^{-1/2}(|H\rangle+|V\rangle)$, and $2^{-1/2}(|H\rangle+i|V\rangle)$. The three *fixed* quartz plates Q inserted on the modes k_1, k_2, and $-k_2$ provided the compensation for the unwanted walk-off effects due to the birefringence of the NL crystal. An additional walk-off compensation into the BBO crystal was provided by the $\lambda/4$ WP exchanging on the mode $-k_1$ the $|H\rangle$ and $|V\rangle$ $\boldsymbol{\pi}$ components of the injected photon.

As we shall see, the goal of the present *cloning* experiment was to measure, under injection of the state $|\Psi\rangle$, the OPA amplification R on the output C mode carrying the *same* $\boldsymbol{\pi}$ condition corresponding of the input state. Contextually, no amplification should affect the output state corresponding to the polarization $\boldsymbol{\pi}^{\perp}$ orthogonal to $\boldsymbol{\pi}$. In order to perform

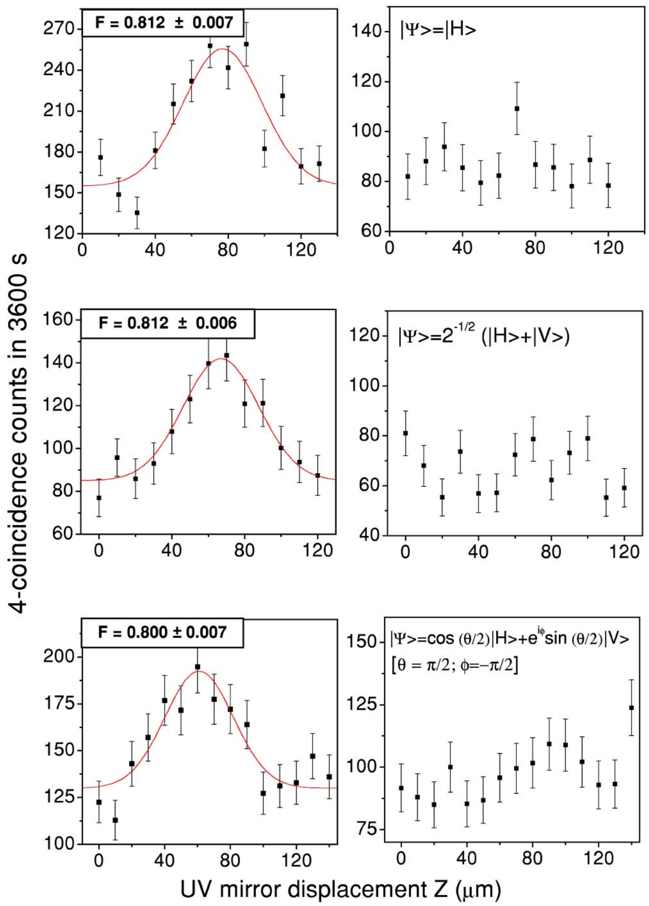

FIG. 6. (Color online) Demonstration of the UOQCM by single photon qubit self-injection and four-detector coincidences. The plots on the lhs and on the rhs of the figure mutually correspond. The rhs plots correspond to "noise". The values of the fidelity of the process F evaluated by each test are expressed in the upper side of the lhs plots while the corresponding test qubits are expressed in rhs plots.

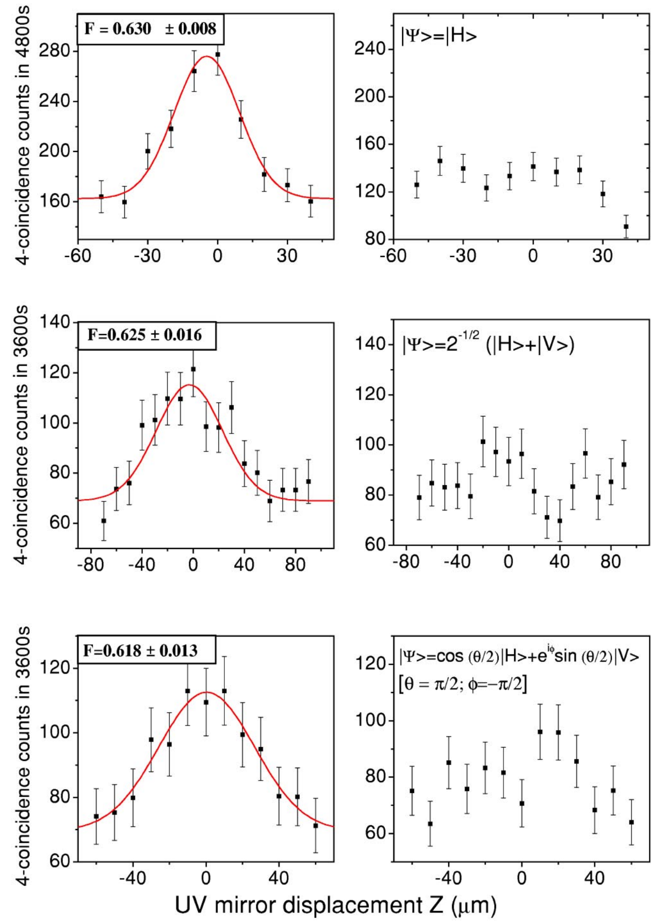

FIG. 7. (Color online) Demonstration of the universal optimal NOT gate by single photon qubit self-injection and four-detector coincidences. The plots on the lhs and on the rhs of the figure mutually correspond. The rhs plots correspond to "noise". The values of the fidelity of the process, F^*, and the expression of the corresponding test qubits are shown as in Fig. 6.

this task, the PBS_2 was removed on the mode k_2 and the photons on the same mode detected by a single detector D_2. The two-cloning photons associated with the C mode, k_1 were separated by means of a 50:50 conventional BS and their polarization states were analyzed by the combinations of WP_1 and of PBS_{1a} and PBS_{1b}. For each injected state of polarizations $|\Psi\rangle$, WP_1 was set in order to detect $|\Psi\rangle$ by detectors D_a and D_b and to detect the state orthogonal to $|\Psi\rangle$, viz., $|\Psi^{\perp}\rangle$ by D_b^*. Hence any coincidence event detected by D_a and D_b corresponded to the realization of the state $|\Psi\Psi\rangle_1$ over the C mode, while a coincidence detected by D_a and D_b^* corresponded to the state $|\Psi\Psi^{\perp}\rangle_1$.

The measurement of R was carried out by four-coincidence measurements involving simultaneously the detectors of the set $[D_2,D_T,D_a,D_b]$ and the one of the set $[D_2,D_T,D_a,D_b^*]$. The adopted four-coincidence electronic apparatus had a time resolution $\tau=3$ nsec.

The experimental data reported in the left side of Fig. 6 correspond to the four-coincidence measurement by $[D_2,D_T,D_a,D_b]$, that is, to the emission over the C mode of the state $|\Psi\Psi\rangle_1$ under injection of the input state $|\Psi\rangle_{IN}$. As for all experiments reported in Figs. 4, 6, 7, the resonance peaks found by this last measurement identified the position Z of the uv retroreflecting mirror for which the spatial and temporal superposition of the uv-pump pulses and of the injected single-photon pulses was realized. In other words the peaks identified the actual realization of the *quantum-injected* amplification of the OPA apparatus. The right-hand side (rhs) of Fig. 6 reports the corresponding four-coincidence data obtained by the set $[D_2,D_T,D_a,D_b^*]$, i.e., implying the realization on the C mode of the state $|\Psi\Psi^{\perp}\rangle_1$. As expected from theory, no amplification peaks are present in this case, here referred to as "noise." The value of the signal-to-noise ratio R determined by the data shown in the left and right sides of Fig. 6 in correspondence with each injection condition $|\Psi\rangle_{IN}$ was adopted to determine the corresponding value of the *cloning fidelity* $F=\mathrm{Tr}(\rho_1\hat{n}_{1\pi})/Tr(\rho_1\hat{n}_1)=(2R+1)/(2R+2)$ implied by Eqs. (12) and (13) with the definitions $\hat{n}_1=\hat{n}_{1\pi}+\hat{n}_{1\perp}$, $\hat{n}_{1\pi}\equiv\hat{a}_{\pi}^{\dagger}\hat{a}_{\pi}$, $\hat{n}_{1\perp}\equiv\hat{a}_{\pi\perp}^{\dagger}\hat{a}_{\pi\perp}$. Precisely, the values of the signal-to-noise ratio R were determined as the ratio of the peak values of the plots on the left-hand side (lhs) of Fig. 6 and of the values of the corresponding plots on the rhs measured at the same value of **Z**. The results are $F_H = 0.812 \pm 0.007$; $F_{H+V} = 0.812 \pm 0.006$; $F_{left} = 0.800 \pm 0.007$, to be compared with the theoretical "optimal" value $F_{th} = 5/6 \approx 0.833$ corresponding to the limit value of the amplification ratio $R = 2:1$ [21,25].

The universal-NOT gate operation has been demonstrated by restoring the PBS_2 on the AC mode, k_2 coupled to the detectors D_2 and D_2^*, via the WP_2, as shown in Fig. 5. The $\boldsymbol{\pi}$ analyzer consisting of ($PBS_2 + WP_2$) is set as to reproduce the *same* filtering action of the analyzer ($PBS_T + WP_T$) for the trigger signal. On the C channel, k_1, the devices PBS_{1a} and PBS_{1b} were removed and the field was simply coupled by the *normal* beam splitter BS_1 to the detectors D_a and D_b. A coincidence event involving these ones was the *signature* for a *cloning* event. The values of the signal-to-noise ratio R^* evaluated on the basis of the data of the four-coincidence experiments involving the sets $[D_2, D_T, D_a, D_b]$ and $[D_2^*, D_T, D_a, D_b]$ and reported in Fig. 7, were adopted to determine the corresponding values of the universal-NOT *fidelity* $F^* = \mathrm{Tr}(\rho_2 \hat{n}_{2\perp})/\mathrm{Tr}(\rho_2 \hat{n}_2) = R^*/(R^*+1)$ implied by Eqs. (12), (13), and $\hat{n}_2 = \hat{n}_{2\pi} + \hat{n}_{2\perp}$, $\hat{n}_{2\pi} \equiv \hat{b}_\pi^\dagger \hat{b}_\pi$, $\hat{n}_{2\perp} \equiv \hat{b}_{\pi\perp}^\dagger \hat{b}_{\pi\perp}$. The results are $F_H^* = 0.630 \pm 0.008$; $F_{H+V}^* = 0.625 \pm 0.016$; $F_{left}^* = 0.618 \pm 0.013$ to be compared with the theoretical "optimal" value $F^* = 2/3 \approx 0.666$ again corresponding to the limit value $R = 2:1$. The four-coincidence method was adopted in all experiments described in this section because it allowed a better spatial mode filtering of the system, leading to a larger experimental value of R. A three-coincidence cloning experiment involving the sets $[D_2, D_T, D_b]$ and $[D_2, D_T, D_b^*]$ was also carried out successfully but attaining a small amplification ratio $R = 1.18:1$. An analogous result was obtained by a three-coincidence universal-NOT gate experiment. Note also that the height of the coincidence signal in Figs. 6 and 7 does not decrease towards zero for large values of **Z**, as expected. This effect is attributable entirely to the limited time resolution of the four-coincidence apparatus. The effect would disappear if the resolution could be pushed into the subpicosecond range, precisely of the order of the time duration of the interacting pump and injection pulses: $\tau' \simeq 140$ fsec. Such a resolution is hardly obtainable with the present technology.

Universality test by quantum injection. As a concluding remark, note that all experimental results reported in the left sides of Figs. 6 and 7 show an amplification efficiency, viz., a value of the signal-to-noise ratio R, which is almost identical for the adopted different $\boldsymbol{\pi}$ conditions corresponding to the input state $|\Psi\rangle_{IN}$. This very significant result represents the first demonstration of the "universality" of the QIOPA system carried out by *quantum injection* of a *single* qubit. It is alternative to the *classical field* universality test reported earlier in Sec. III and in Refs. [9,21].

VI. INTERPRETATION

A remarkable, somewhat intriguing aspect of the present work is that both processes of quantum cloning and of the universal-NOT gate are realized *contextually* by the *same* physical apparatus, by the same unitary transformation and correspondingly by the same quantum logic network. To the best of our knowledge it is not well understood yet why these *forbidden* processes can be so closely related. We may try to enlighten here at least one formal aspect of this correlation [26].

Remind first that the two processes are detected in our experiment over the two $\boldsymbol{\pi}$*-entangled* output C and AC channels, physically represented by the two corresponding wave vectors $\boldsymbol{k}_j$ $(j=1,2)$. In addition, the overall output vector state $|\Psi\rangle$ given by Eq. (5) is a *pure state* since the *unitary* evolution operator $\hat{U} = \hat{U}_A \otimes \hat{U}_{A'}$ [Eq. (3)] acts on a *pure* input state. As a consequence, the reduced density matrices ρ_1 and ρ_2 have the same eigenvalues [11] and the entanglement of the *bipartite* state $|\Psi\rangle$ can be conveniently measured by the *entropy of entanglement:*

$$E(\Psi) = S(\rho_1) = S(\rho_2), \tag{15}$$

where $S(\rho_j) = -\mathrm{Tr}\, \rho_j \log_2 \rho_j$ is the Von Neumann entropy of either the C or AC subsystem, $j(j=1,2)$ [26]. We may comment on this result by considering first the approximate *cloning* process performed by our *universal optimal machine* (UOQCM) acting on the C channel, k_1. What has been actually realized in the experiment was a procedure of "linearization" of the cloning map which is *nonlinear* and, as such, cannot be realized exactly by nature [1,5,6]. By this procedure a *mixed-state* condition of the output state ρ_1 was achieved corresponding to the *optimal* limit value of the entropy $S(\rho_1) > 0$. According to Eq. (15) the *same* degree of *mixedeness* affects *also* the output state realized on the AC channel, as implied by the same limit values of the signal-to-noise ratios, $R = R^*$, affecting the experimental plots shown in Figs. 6 and 7. Since on the AC channel an approximate CP map is realized which is generally distinct from any process realized on the C channel, e.g., here the *cloning* process, Eq. (15) appears to establish a significant *symmetry condition* in the context of axiomatic quantum theory [1,27]. In this same context it has been noted that the transformation investigated in the present work connecting the cloning and universal-NOT processes also realizes contextually the *optimal entangling* process and the *universal probabilistic quantum processor* [28].

Note in this connection that the adoption of the *classical* OPA apparatus within an investigation pushed at the extreme quantum measurement limits, enlightened the conceptual significance of the quantum noise, originating in this case from the QED vacuum field. We remind here that the failure of an old proposal for superluminal signaling by an *amplified* Einstein, Podolsky, Rosen scheme was attributed to the impossibility of realizing a single-photon *ideal* amplifier, i.e., one for which the signal-to-noise ratio would be $R > 2$ [6,20].

For the sake of further clarification, let us analyze the simple case of a *mode-degenerate* QIOPA amplifier in which *only* two e.m. modes $k_\parallel$ and $k_\perp$ with the same wavelength $\lambda = 2\lambda_p$ interact in the NL crystal. In Ref. [29] this case has been realized by a type-II BBO crystal cut for *collinear* emission over a *single* wave vector $\boldsymbol{k}$ and the modes $k_\parallel$ and

$k_\perp$ express two orthogonal states of polarization, e.g., of linear $\boldsymbol{\pi}$. This condition is still represented by the device shown in Fig. 2 in which the modes k_1 and k_2 are made to collapse into a single k. In general, the *quantum injection* is still provided there by a qubit encoded in a single photon, $|\Psi\rangle_{IN} \equiv (\tilde{\alpha}|1,0\rangle + \tilde{\beta}|0,1\rangle)$, where again $|\tilde{\alpha}|^2+|\tilde{\beta}|^2=1$ and $|x,y\rangle \equiv |x\rangle_\parallel \otimes |y\rangle_\perp$. For the present purpose the problem may be simplified, with no loss of generality by assuming $\tilde{\alpha}=1$, $\tilde{\beta}=0$. Applying the theory given in Sec. II, we get the output state to the first order of approximation,

$$|\Psi\rangle \approx |1,0\rangle - \sqrt{2}\Gamma|2,1\rangle, \tag{16}$$

to be compared with Eq. (12). Likewise, the no-injection condition leads to the output state $|\Psi\rangle_0 \approx |0,0\rangle - \Gamma|1,1\rangle$, to be compared with Eq. (13). The "no-interaction" first term is easily discarded by a coincidence measurement [29]. The second term consists of a *pure* state realizing both *exact* cloning and *qubit flipping* on the output modes. This means that any test aimed at the detection of the two processes *is not affected* by quantum noise. However, the device works *just* for *one particular* input qubit, i.e., for one particular choice of the parameters $\tilde{\alpha}$ and $\tilde{\beta}$. Any other choice would lead to a different amplification quantum efficiency. In short, the device is not an *universal machine*. Referring ourselves to the theoretical results of Sec. II, this happens because the interaction Hamiltonian of the present OPA system, $\hat{H}_I = i\hbar\chi\hat{A} + \text{H.c.}$, $\hat{A} \equiv \hat{a}_\parallel \hat{a}_\perp$ cannot be made formally invariant under displacements of the injected qubit over the entire Bloch sphere, Fig. 1. Indeed, at least four nonlinearly interacting e.m. modes are needed to attain such invariance of the optical amplifier/squeezing Hamiltonian. All these results are fully consistent with the general theory of the "optimal quantum machines" [4,22].

In summary, the present work shows once again, in a unifying manner, that the concepts of no signaling, i.e., causality, linearity, and complete positivity have deep interconnections within the inner structure of quantum mechanics [1,6,9]. We believe that the actual results, the suggestions, and the open problems contributed by the present work could be useful by setting measurement bounds and fundamental performance limitations in the domain of quantum information. Furthermore, they should somewhat contribute to a clarification of some structural aspects of axiomatic quantum theory.

ACKNOWLEDGMENTS

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[1] K. Kraus, *States, Effects and Operations* (Springer-Verlag, Berlin, 1983); see J. Preskill, Lecture Notes on Quantum Computation, http://www.theory.caltech.edu/people/ph229/#lecture. A CP-map $\Lambda(\rho)$ preserves *positivity* for: (a) *local* state in the Hilbert space *H*, (b) when *tensor-multiplied* by the identical map **I** acting on *any* Hilbert space *K*, the extended map $\Lambda(\rho)\otimes\mathbf{I}$ is positive for any state in the entangled space $H \otimes K$, for *any* extension of *K*. A *positive map* (*P map*) only satisfies property (a).

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