Delayed-choice entanglement swapping with vacuum–one-photon quantum states

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We report the experimental realization of a recently discovered quantum-information protocol by Peres implying an apparent nonlocal quantum mechanical retrodiction effect. The demonstration is carried out by a quantum optical method by which each singlet entangled state is physically implemented by a two-dimensional subspace of Fock states of a mode of the electromagnetic field, specifically the space spanned by the vacuum and the one-photon state, along lines suggested recently by E. Knill *et al.* [Nature (London) **409**, 46 (2001)] and by M. Duan *et al.* [*ibid.* **414**, 413 (2001)].

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State entanglement, the most distinctive, fundamental feature of modern physics, is at the heart of the essential nonlocality of the quantum world, i.e., of the irremovable property of nature first discovered in 1935 by Einstein, Podolsky, and Rosen (EPR) and later formally analyzed by Bell [1] and recently by Hardy [2]. In the context of the modern fields of quantum information and computation, entanglement lies at the core of several important protocols and methods such as, for instance, quantum state teleportation, a fundamental process that has been implemented by different experimental approaches [3,4]. Very recently quantum teleportation with an unprecedentedly large "fidelity" has been experimentally demonstrated by adoption of the concept of "entanglement of one photon with the vacuum" by which each quantum superposition state, i.e., qubit, is physically implemented by a two-dimensional subspace of Fock states of a mode of the electromagnetic field, specifically the state spanned by the QED "vacuum" and the one-photon state [5]. This method requires a reformulation of the Hilbert space framework supporting the evolution of quantum information, and then the conception of appropriate devices and methods to implement the transformation algebra of states and operators. As a further clarification of the method in the perspective of future more complex applications, we investigate in the present Brief Report the procedure called entanglement swapping, in which the teleported state itself is entangled, i.e., where the teleported system is not even in its own state [6].

Let us first outline the swapping process in the above perspective. It is well known that the establishment of entanglement between two (or more) distant quantum systems does not necessarily require, as generally believed after the original EPR approach, a direct original interaction between these systems, but it can be realized by merely projecting by an appropriate joint measurement the independent entangled states pertaining to the separated systems, even in the absence of any previous mutual interaction. According to this scenario two separate observers Alice (A) and Bob (B) independently prepare two sets of entangled singlets. They perform on one component of each singlet an appropriate test of EPR nonlocality, e.g., a standard Bell-inequality test [1], a Hardy's no-inequality ladder test [2,3], or a continuous variables homodyne detection test [7]. The other two components of the singlets are sent to a third party Eve (E), who performs a joint test of her choice on the components received, one from A and one from B. By doing that Eve projects (i.e., "swaps") the states of the two originally nonentangled distant components in the hands of A and B onto an entangled state. Recently, it has been argued by Peres that the swapping process could be completed by Eve not necessarily at the time at which the two distant systems were tested by A and B but at any retarded delayed choice time [8]. Indeed, according to Eve's choice at a later time a fourth verification party Victor (V) can sort the samples already tested by A and B into subsets and can verify that each subset behaves as if it consisted of entangled pairs of distant systems that have never communicated in the past even indirectly via other systems. This may appear a paradoxical result, as we shall see.

In the present work we report the experimental demonstration of the Peres delayed-choice process by applying the concept of entanglement of one photon with the vacuum [5]. The concept of nonlocality of a single photon, first introduced by Einstein in 1927 [9], has been thoroughly analyzed in the last decade by Tan *et al.* [10], by Hardy [11], and others in connection with the superposition state emerging from a beam splitter (BS) excited by a single photon at one of its input ports. In our view this state should indeed be interpreted as an *entangled state* by considering that in the domain of optics the *modes* of the electromagnetic (e.m.) field rather than the photons must be taken as the systems or components to be entangled. Thus, any single-particle superposition state expressed in the form $\Sigma_A = (2)^{-1/2}(|1\rangle_A|0\rangle_{A'} - |0\rangle_A|1\rangle_{A'})$ must be interpreted as a singlet entangling the mode pair $(k_A, k_{A'})$ which is excited by the Fock states $|1\rangle$ and $|0\rangle$, this last one expressing the QED vacuum state. If the same states $|0\rangle, |1\rangle$ are interpreted respectively as the logic zero, and 1 information states, the singlet Σ_A is viewed as an *ebit*, i.e., an entangled bit of quantum information [12]. Of course, in order to make use of the entanglement present in this picture we need to use the second quantization procedure of creation and annihilation of particles and/or use states that are superpositions of states with different numbers of particles. Another puzzling aspect of this second quantized picture is the need to define and measure the relative phase between states with different number of photons, such as the relative phase between the vacuum and one-photon states appearing in Eq. (2), below. To be able to control these relative phases we need, in gen-

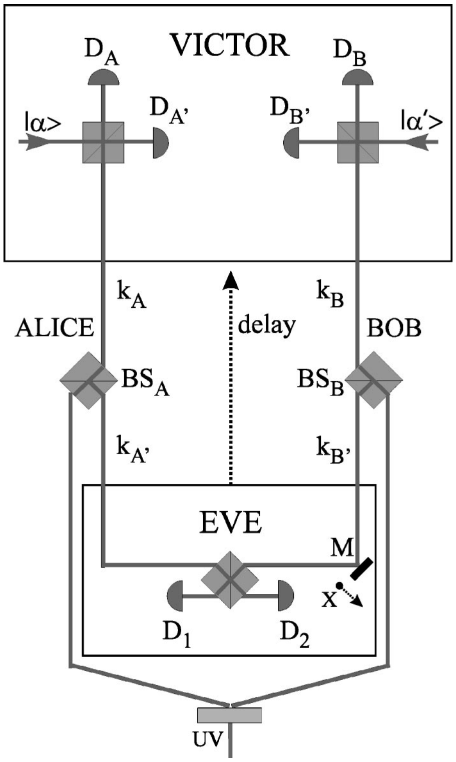

FIG. 1. Layout of an experimental demonstration of the delayed-choice entanglement-swapping process. In the actual experiment the two-homodyne apparatus was replaced by the two-detector set shown in Fig. 2, inset below.

eral, and in analogy with classical computers, to supply all gates and all sender-receiving stations of any quantum information network with a common synchronizing *clock* signal, e.g., provided by an ancillary photon or by an ancillary multiphoton, Fourier transformed coherent state [13]. Optionally, in simple cases, as in the present work, an *ad hoc* clock generator is not needed as the mutual phase information can be retrieved by a linear two-mode superposition in a beam splitter.

An example concerning the present experiment is illustrated by Fig. 1 which shows how the nonlocality implied by the quantum state of the overall system can be tested by the distant parties A and B via two coherent states $|\alpha\rangle \equiv ||\alpha|\exp i\theta\rangle$ and $|\alpha'\rangle \equiv ||\alpha|\exp i\theta'\rangle$ that can operate at the same time as clock states and as local oscillators (LOs) of the corresponding homodyne detectors performing the same test. The feasibility of a similar single-photon homodyne technique has been demonstrated recently [14].

Figure 1 shows the basic layout of the delayed-choice entanglement swapping experiment. Pairs of photons were generated by spontaneous parametric down-conversion excited by a single mode uv cw argon laser in a type I $LiIO_3$ crystal with the same wavelengths $\lambda = 727.6$ nm and with the same linear polarization (π). Each pair of photons, each of which is associated with an ultrashort optical pulse characterized by a coherence time $\tau_c = 0.1$ ps, was injected into two equal 50:50 beam splitters BS_A and BS_B characterized by equal *real* transmittivity and reflectivity parameters $t = r = 2^{-1/2}$. Precisely, each BS consisted of a 45° π-rotator fol-

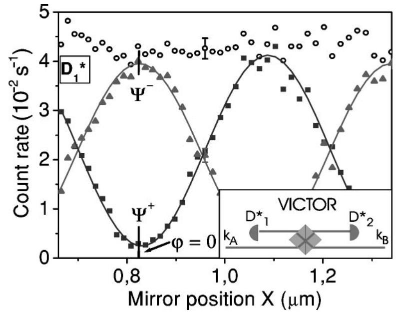

FIG. 2. Experimental results of the measurement of the count rate by detector D_1^* as a function of delayed settings of the phase φ determined by micrometric displacements X of the mirror M (open circles). The verification party Victor can sort the recorded pattern at a later time into two subsets showing two sinusoidal fringe patterns with opposite phases corresponding to the Bell states $\Psi^{\pm}$ for $\varphi = 0$. The visibility of the fringe patterns is $V = (91 \pm 2)\%$. The inset shows the two-detector apparatus that has been adopted to perform the EPR nonlocality test experimentally and for that purpose replaces, in a fully equivalent fashion, the double homodyne apparatus shown in Fig. 1.

lowed by a calcite crystal. As is well known [1,15], the product state character of each pair, $|\Phi\rangle = |1\rangle_A \otimes |1\rangle_B$, did not imply any interparticle EPR correlation, in agreement with the data reported in Fig. 2 (open circles). In other words, as far as the dynamics of the overall system is concerned, each photon pair could have been supplied equally well by any pair of distant sources. The state $|\Phi\rangle$ was transformed by the BS's into the product of two singlets defined over the pairs of output modes $(k_A, k_{A'})$ and $(k_B, k_{B'})$: $|\Phi\rangle = \Sigma_A \otimes \Sigma_B = \frac{1}{2}(|1\rangle_A |0\rangle_{A'} - |0\rangle_A |1\rangle_{A'}) \otimes (|1\rangle_B |0\rangle_{B'} - |0\rangle_B |1\rangle_{B'})$. The pure state $|\Phi\rangle$ may be expressed as a sum of products of Bell states defined in the two two-dimensional (2D) Hilbert subspaces spanned by the state eigenvectors to be measured, respectively, by the couple (Alice, Bob) and by Eve:

$$|\Phi\rangle = \Sigma_A \otimes \Sigma_B = \tfrac{1}{2}[\Phi^+ \otimes \Phi_E^+ - \Phi^- \otimes \Phi_E^- - \Psi^+ \otimes \Psi_E^+ + \Psi^- \otimes \Psi_E^-] \tag{1}$$

and the Bell states defined in the corresponding 2D Hilbert subspaces are [5]

$$\Phi^{\pm} = \frac{1}{\sqrt{2}}(|0\rangle_A |0\rangle_B \pm |1\rangle_A |1\rangle_B),$$

$$\Psi^{\pm} = \frac{1}{\sqrt{2}}(|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B), \tag{2}$$

$$\Phi_E^{\pm} = \frac{1}{\sqrt{2}}(|0\rangle_{A'} |0\rangle_{B'} \pm |1\rangle_{A'} |1\rangle_{B'}),$$

$$\Psi_E^{\pm} = \frac{1}{\sqrt{2}}(|0\rangle_{A'} |1\rangle_{B'} \pm |1\rangle_{A'} |0\rangle_{B'}).$$

Equation (1) shows how the original entanglement condition existing within the two separated systems ($k_A, k_{A'}$) and ($k_B, k_{B'}$) can be swapped to the "extreme" modes k_A and k_B by any joint Bell type measurement made by Eve on the intermediate modes ($k_{A'}, k_{B'}$). In the absence of such a measurement the overall state $|\Phi\rangle$ is a superposition while the one reaching the $(A+B)$ sector is a mixed state.

Suppose that one of the two detectors D_j of the Eve sector clicks, i.e., measures the state Ψ_E^-, say. A sudden state reduction occurs that projects the overall system onto the corresponding entangled Bell state: $|\Phi\rangle \Rightarrow \Psi^-$. Eve's apparatus consisting of a 50:50 beam splitter and of a φ phase shifter is apt to perform this task with a 50% efficiency. Indeed, it can be easily found by applying the standard BS theory [5] that the realization of the one-photon Bell state Ψ_E^- (or Ψ_E^+) over the input modes $k_{A'}, k_{B'}$ determines a click by D_1 (or D_2). It is also well known that the states $\Phi_E^{\pm}$ corresponding to a two-photon excitation of Eve's sector cannot be discriminated by any linear device [16]. Note, however, that the present experiment is *noise free* since a two-photon excitation of Eve's sector implies no detections by the $(A+B)$ sector, an event easily discarded by the electronic apparatus. An additional degree of freedom under Eve's control, indeed an optional "delayed choice," was provided by the micrometric displacement ΔX of the mirror *M*, activated by a piezoelectric transducer. This one induced a corresponding phase shift $\Delta\varphi = (2)^{3/2}\pi\lambda^{-1}\Delta X$ between the modes ($k_{A'}, k_{B'}$). Optionally, the same task can be accomplished by a fast electro-optic (EO) phase modulator, as we shall see. The four detectors adopted in the experiment were equal Si avalanche EG&G SPCM200 modules with quantum efficiencies 0.45.

Suppose that a complete EPR nonlocality test is performed by Alice and Bob by means of the two optical homodyne devices shown in Fig. 1, according to the scheme by Tan *et al.* [10]. Assume that the eigenvalues of the clock LO coherent states are $\alpha = |\alpha| \exp\theta$, $\alpha' = |\alpha| \exp\theta'$. By a simple extension of a previous analysis [10] it can be shown that, if Eve's detector D_1 clicks, i.e., Ψ^- is realized, the probability of a coincidence involving the detectors D'_A and D_B is

$$\langle\Psi^-|I_{A'}I_B|\Psi^-\rangle = \tfrac{1}{4}|\alpha|^2\{|\alpha|^2 + [1+\cos(\theta'-\theta+\varphi)]\}. \tag{3}$$

Rather than performing the difficult double homodyne experiment, in our case Alice and Bob carried out an equally significant EPR nonlocality test by mixing the modes (k_A, k_B) by a 50:50 BS coupled to the detector pair D_1^*, D_2^* (Fig. 2, inset). In analogy with Eve's apparatus, this device may be thought to perform a test on the Bell states $\Psi^{\pm}$ spanning the Hilbert subspace pertaining to the 2D manifold (k_A, k_B). At the same time it also provides the necessary synchronizing clock effect, as we said. Consider, for instance, the photodetection by D_1^*. Note first that the coincidence probability of simultaneous clicks by D_1^* and D_1 is found by standard theory to be expressed by [5]

$$\langle\Psi^-|I_D I_{D*}|\Psi^-\rangle = \tfrac{1}{2}[1+\cos\varphi],$$

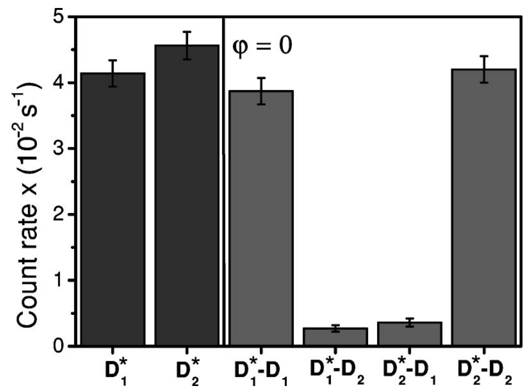

FIG. 3. Histograms showing the measured detection count rates by D_1^* and D_2^* and the accuracy affecting the experimental determination of the Bell states $\Psi^{\pm}$ obtained by the delayed coincidence rates involving all detector pairs D_i^*-D_j $(i,j=1,2)$, for $\varphi = 0$.

proportional to the expression (3) obtained for the homodyne devices by setting $\theta = \theta'$ and $|\alpha|^2 \ll 1$. Similar results are found for the other three coincidence combinations involving D_i^* and D_j $(i,j=1,2)$. Let Alice and Bob carry out an experiment aimed at the measurement of the rate of detection by D_1^* (Fig. 2). Since the two systems to be tested (k_A, k_B) lack any original nonlocal character, it is natural to expect a total insensitivity to any change of local parameters acting on remote parts of the apparatus, such as, for instance, the phase shift $\Delta\varphi$. This is indeed shown by the experimental data (open circles) given in Fig. 2. However, had the verification party, Victor, kept the record of the individual outcomes of both pairs (D_1, D_2) and (D_1^*, D_2^*), at a later time he could sort into two subsets the already tested samples detected by Alice and Bob. Figures 2 and 3 show that indeed each subset behaves as if it consisted of entangled pairs of distant systems. Note that these have never communicated in the past even indirectly via other systems. Furthermore, as pointed out by Peres, after Alice and Bob have recorded the results of all their measurements, Eve still has the freedom of deciding which experiment she will perform [8]. This one may consist of a standard Bell measurement, or a joint measurement with a $\Delta\varphi$ shift, or a positive operator valued measure measurement [15] or one of the exotic, interesting single-photon nonlocality tests suggested by Hardy [11]. Indeed, in the present experiment, owing to a spatial displacement of the corresponding detector sets, Eve's action could take place with a time delay $\Delta\tau \approx 3$ ns $\gg \tau_c$ with respect to the time of the state reduction event determined by the test performed by Alice and Bob. In other words, since in our case $\Delta\tau$ was about 3×10^3 larger than τ_c, the photon *coherence time*, the swapping process was completed by Eve's apparatus long after the complete annihilation of the particle measured by $(A+B)$. In order to offer an even more convincing demonstration, a sophisticated $\Delta\tau = 20$ ns delay apparatus has been realized allowing delayed fast $\Delta\varphi$ changes by a randomly driven EO phase modulator (Inrad 621-040 with $\Delta\varphi = \pi \equiv \Delta\varphi_{\lambda/2}$ driven by 400 V rectangular pulses) triggered by the $(A+B)$ detection apparatus, i.e., long *after* the completion of the $A+B$ test. Note in Fig. 2 that shifts $\pm\Delta\varphi_{\lambda/2}$ correspond to the detection interchanges $\Psi^- \rightleftarrows \Psi^+$. This makes the original Peres argument, conceived for standard Bell-inequality tests of $(2\otimes 2)$D Hilbert photon π states, fully consistent with the present experiment [1,8].

How then could Eve's delayed choice determine data already irrevocably recorded? According to Peres, it is meaningless to assert that two quantum systems are entangled without specifying their state, or to assert that a system is in a pure state without specifying that state or to attribute an objective meaning to the quantum state of a single system. If these prescriptions are forgotten one may encounter paradoxes such as the one seen here: a past event may sometimes appear to be determined by future actions [8]. For better clarification it is perhaps worth recalling here that ''A phenomenon is not a phenomenon until is a measured phenomenon'' (J. A. Wheeler), asserting the inanity of any intellectual speculation involving mental modeling of the inner evolution of a quantum superposition process. Furthermore, as pointed out by Popescu and Jozsa [12], no apparent retrodictive process, e.g., associated with quantum evolution in presence of the EPR nonlocality in a teleportation process, can finally lead to paradoxes or contradictions of causality because of the inherent inaccessibility of the quantum information.

In conclusion, we have illustrated experimentally an enlightening aspect of quantum EPR nonlocality. For instance, the application of our methods of high fidelity quantum teleportation and entanglement swapping to modern quantum repeaters will certainly improve in the near future the technology of quantum communication at large distances [17–19].

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