Experimental test of the violation of local realism in quantum mechanics without Bell inequalities

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We report the results of an experiment in which postselected polarization-entangled photons were used to test the recent nonlocality proof by L. Hardy [Phys. Rev. Lett. **71**, 1665 (1993)]. The particular source of entangled photons used in the experiment has the advantage that it is very easy to prepare a pair of photons in a state with any degree of entanglement. Hardy's theorem, which only works for nonmaximally entangled states, was tested for a range of states. [S1050-2947(97)04907-X]

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Since the famous argument raised by Einstein, Podolsky, and Rosen (EPR) in 1935 concerning the ontological foundations of quantum reality, there have been many theoretical investigations ultimately leading to a number of experimental schemes to test local realism [1]. The Bell inequalities in 1965, the related work by Aspect and co-workers, and the recent, insightful arguments by Greenberger, Horne, and Zeilinger (GHZ) and by Mermin based on logical contradictions to be tested by single-outcome experiments without inequalities, were the most successful results of such an effort [2–6]. An interesting recent contribution by Hardy reaches, in our opinion, the highest attainable degree of simplicity and physical insight: *any* system consisting of only two spin-$\frac{1}{2}$ particles prepared in a nonmaximally entangled state admits a nonlocality proof not involving inequalities [7,8]. We believe that this result is so important and conclusive that it requires an adequate demonstration. Torgerson *et al.* have recently implemented Hardy's idea. In their experiment a ''postselected,'' nonmaximally entangled state was produced by having photons with orthogonal polarizations impinge on a normal (i.e., nonpolarizing) but unsymmetrical beam splitter. This produces a ''postselected'' entangled state with a *fixed* degree of entanglement [9]. The present work adopts a far more versatile scheme based on a method of ''postselected'' polarization entanglement and on a rather sophisticated detection apparatus. The experimental layout is shown in Fig. 1. Two photons with identical wavelengths and the same polarizations are generated by parametric downconversion (PDC). Their polarizations are rotated by angles ψ_1 (photon 1 of the pair) and ψ_2 (photon 2). Then they impinge on the input ports of a polarizing beam splitter (PBS). On emerging, the photons of any pair giving rise to a coincidence in the detection apparatus are in an entangled state. As will be seen below, it is possible to tune the degree of entanglement of this state by adjusting ψ_1 and ψ_2. Finally, the polarization (called π) of each photon is measured on each of the two modes i in the following way: first π is rotated by an angle θ_i, then the photon passes through a polarizing beam splitter with fixed orientation, PBS_i, followed by two detectors, D_i and $\bar{D}_i$ each coupled to one of the output ports of PBS_i.

We will now describe the experiment in more detail. The two photons A and B of a parametrically generated pair had identical horizontal linear polarization, $\pi_=$, and wavelength $\lambda = 727.6$ nm. The PDC process took place in a 7-mm-thick BBO (β-barium-borate) nonlinear (NL) crystal cut for type-I phase matching and excited by a 100-mW UV cw laser beam at $\lambda_p = 363.8$ nm. The two particles emitted over the i modes ($i = 1,2$) were selected by two $\Phi = 1$ mm aperture pinholes and sent through two independent Fresnel-rhomb π-rotator devices [$R(\psi_1)$, $R(\psi_2)$: Spectra-Physics Mod.310A] that rotated the respective polarization π by the angles ψ_1 and ψ_2 with respect to the horizontal laboratory axis. Then the beams were injected on the two input modes, i.e., orthogonally to two plane sides, of a cubic polarizing beam splitter [PBS: Spindler-Hoyer Mod.335523] where the state entanglement was realized for any photon pair that is ''postselected'' by the measurement apparatus, i.e., that produces a coincidence event registered by two detectors, each coupled respectively to one i mode ($i = 1$ or 2) at the output of PBS. The two beams emerging from the PBS on these output i modes finally excite two couples of detectors after being transmitted through two identical lossless π analyzers $A(\theta_1, \bar{\theta}_1)$ and $A(\theta_2, \bar{\theta}_2)$, $\bar{\theta}_i \equiv [\theta_i + 90°]$ that can be set independently at the angles θ_1 and θ_2 with respect to the horizontal laboratory axis. Each $A(\theta_i, \bar{\theta}_i)$ consisted of a Fresnel-rhomb π rotator $R(\theta_i)$ followed by a *polarizing* beam splitter PBS_i identical to the one adopted for the state entanglement. The detectors D_i and $\bar{D}_i$ were activated by the projections of the incoming field to be measured (i.e., belonging to the PBS output i mode) along the corresponding π angles θ_i and $\bar{\theta}_i$. Each detector may be conveniently identified by the field's π angle to which it reacts by setting henceforth $D_i \rightarrow D_i(\theta_i)$, $\bar{D}_i \rightarrow \bar{D}_i(\bar{\theta}_i)$. Likewise, each (analyzer+2 detectors) set, call it a D set, excited by the field associated with either one of the two PBS output i modes is identified correspondingly by $[D(\theta_i)]_i \equiv [D_i(\theta_i) + \bar{D}_i(\bar{\theta}_i)]$. The four cooled Si-avalanche detectors (EGG-SPCM-200-PQ) were selected to have nearly equal photon detection efficiencies (QE) $\zeta_1 \approx \zeta_1' \approx \zeta_2 \approx \zeta_2' \approx 55\%$ (at $\lambda = 650$ nm) and thermal noise rates $\nu_1 \approx \nu_1' \approx \nu_2 \approx \nu_2' \approx 50$ Hz. The four simultaneous output detector signals expressed corresponding

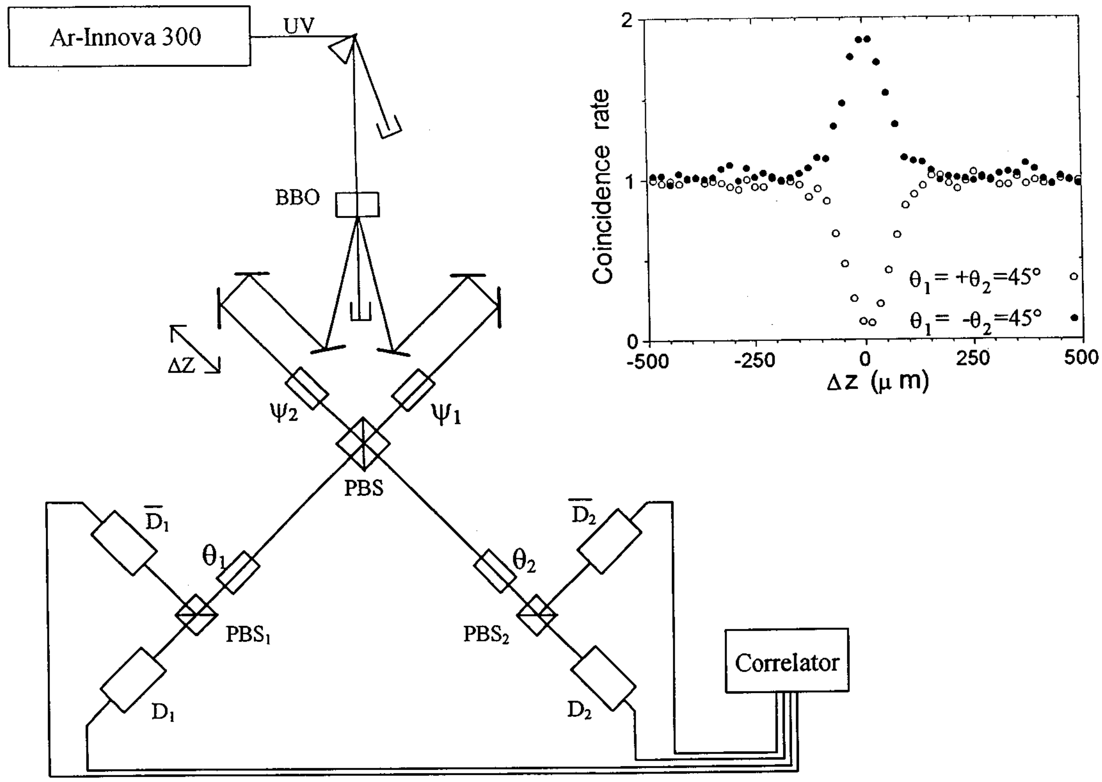

FIG. 1. Layout of the experimental apparatus.

samples of binary information related to positive photodetection (call it a "click" or bit "1") or negative detection ("no click" or bit "0") by the corresponding detector.

The four signals were simultaneously registered within a common gate time interval $\Delta T = 10$ ns, and then stored in four registers of computer memory in correspondence with each coincidence event registered between the two D sets, $[D(\theta_i)]_i$. This coincidence technique assured the measurement on the pair system only when both output modes were simultaneously excited and greatly reduced the effect due to the detector thermal noise [10]. The photon wave packets were made to overlap on the scattering zone of PBS by a careful alignment of the corresponding interfering beams through couples of pinholes with $\Phi = 1$ mm set at a mutual distance of 1.5 m. The time overlap was realized by a "trombone" time compensator driven by a computer-controlled motorized slide over a linear displacement ΔZ by steps of 0.1 μm. The beams were selected at the output of the BBO crystal through two equal IF filters with bandwidth $\Delta\lambda = 2$ nm that determined the photon coherence time: $\tau_c \approx 560$ fs. The exact value of ΔZ corresponding to wave-packet overlap was ascertained by counting correlations established between the two $[D(\theta_i)]_i$.

The inset of Fig. 1 shows the cancellation (or the enhancement) of counting correlations arising from the negative (or positive) two-photon interference set by the state-entangling PBS having selected correspondingly the π-rotated angles $\psi_1 = 45° = \psi_2$ and $\theta_1 = \theta_2 = 45°$ (or $\theta_1 = -\theta_2 = 45°$). This result may be considered a useful by-product of the π-entanglement process representing the conceptual basis of the present experiment [11]. Note that since all angles, θ_i and ψ_i, can be set independently with no constraints and the entanglement coefficients determined by our scheme are real trigonometric functions of ψ_i, as will be seen below, our setup is very flexible and allows a complete analysis of Hardy's argument. This allows assessment of the actual sensitivity of the optimum entanglement (i.e., which gives the maximum effect with regard to the size of the parameter in terms of which the "logical contradiction" found by Hardy is expressed).

The theoretical analysis of the experiment may be formulated by following the evolution of the state vector of the pair system in Schrödinger representation. The polarization dynamics of particle A belonging to mode 1 before interaction with $[R(\psi_1) + \mathrm{PBS}]$ is described in a two-dimensional Hilbert space ${}^A H_2$ spanned by the two orthonormal basis vectors, or column matrices: $|1_A\rangle_= \equiv \binom{1}{0}$, $|1_A\rangle_\perp \equiv \binom{0}{1}$ representing horizontal and vertical linear π's, respectively [12,13]. Another ${}^B H_2$ space and basis vectors $|1_B\rangle_= \equiv \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]$, $|1_B\rangle_\perp \equiv \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]$ are also defined for particle B, belonging to mode 2 before interaction. The pair state vector after PDC generation and before interaction is then described in the four-dimensional space ${}^A H_2 \otimes {}^B H_2$ by the nonentangled, product state $|\Psi\rangle_{\mathrm{in}} = \binom{1}{0} \otimes \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]$. The change induced on that state by interaction with the set $[2R(\psi_i) + \mathrm{PBS}]$ is conveniently described by the corresponding unitary transformation $\mathbf{U} = \mathbf{U}_{\mathrm{PBS}} \mathbf{U}_R$, $\mathbf{U}^{-1} = \mathbf{U}^\dagger$ acting on the input-state column vector: $\mathbf{w}' \equiv \{(1 \quad 0)[1 \quad 0]\}^\dagger \equiv \{1 \quad 0 \quad 1 \quad 0\}^\dagger$, i.e., by the evaluation of the product $\mathbf{w} = \mathbf{U}\mathbf{w}'$. The two unitary matrices accounting for the π rotation and for the PBS peration are given respectively in the form

$$\mathbf{U}_R \equiv \begin{bmatrix} \cos\psi_1 & -\sin\psi_1 & 0 & 0 \\ \sin\psi_1 & \cos\psi_1 & 0 & 0 \\ 0 & 0 & \cos\psi_2 & -\sin\psi_2 \\ 0 & 0 & \sin\psi_2 & \cos\psi_2 \end{bmatrix},$$

$$\mathbf{U}_{\mathrm{PBS}} \equiv \begin{bmatrix} \xi & 0 & i\varepsilon & 0 \\ 0 & \varepsilon' & 0 & -i\xi' \\ i\varepsilon & 0 & \xi & 0 \\ 0 & -i\xi' & 0 & \varepsilon' \end{bmatrix}.$$

In the last matrix the (real) stray reflectivity, $\varepsilon=(1-\zeta^2)^{1/2}\approx 0$, and transmittivity parameters, $\varepsilon'=(1-\zeta'^2)^{1/2}\approx 0$, for π horizontal and π vertical respectively, are introduced to account for the nonideal behavior of the PBS operation based on the beam-polarizing effect of any semiconductor multilayer coatings, as in our case. While this consideration is important for the correct analysis of the experimental results, let us assume here the ideal case for simplicity: $\varepsilon=\varepsilon'=0$, $\zeta=\zeta'=1$ [14]. It is verified that, owing to the $\mathbf{U}$ transformation, the state on the input mode 1 $\binom{1}{0}$, evolves, at the output of PBS, into the state $\{\cos\psi_1\binom{1}{0}-i\sin\psi_1[{}^0_1]\}$ and the state on the input mode-2, $[{}^1_0]$, into $\{-i\sin\psi_2\binom{0}{1}+\cos\psi_2[{}^1_0]\}$. The states appearing first within the two sets of curly brackets are realized at the output of PBS on mode 1 while the last states are realized on the output mode 2. Then the original product state generated by PDC, $|\Psi\rangle_{\mathrm{in}}$, is transformed at the output of PBS into a sum of an "entangled state," where the two photons of each PDC-generated couple appear on different modes at the output of PBS, and of two product states where both photons of each couple emerge from PBS on the *same* output mode. Since the latter two-photon states are not recognized as such by the adopted nonlinear avalanche detectors, i.e., unable of discriminating couples against single excitation photons, our coincidence method determines a "postselection" of a subensemble of photon pairs in an entangled state [15]. Accordingly, we restrict ourselves to considering here this subensemble and we may conveniently cast the corresponding effective state vector in the normalized form [15]. $|\Psi\rangle=\{\alpha|1_A\rangle_=\otimes|1_B\rangle_= +\beta|1_B\rangle_\perp\otimes|1_A\rangle_\perp\}$, where states appearing at the left of the direct product sign refer to mode 1 and the (real) entanglement coefficients are $\alpha=\eta\cos\psi_1\cos\psi_2$; $\beta=-\eta\sin\psi_1\sin\psi_2$. $\eta=\sqrt{2}[\cos^2(\psi_1+\psi_2)+\cos^2(\psi_1-\psi_2)]^{-1/2}$ is a normalization parameter. The normalization of the state function of the detected pairs is found convenient, in spite of the state postselection, in order to provide a quantum theoretical evaluation of the detection probabilities; i.e., these probabilities are relative to the ensemble of pairs postselected in the experiment. The joint probability for detection by $D_1(\theta_1)$, $D_2(\theta_2)$ is given by $P_{12}(\theta_1,\theta_2)=\zeta_1\zeta_2[\alpha\cos\theta_1\cos\theta_2+\beta\sin\theta_1\sin\theta_2]^2$ and is shown in Fig. 2 for the *optimum* value of the entanglement parameter $\gamma(\psi_1,\psi_2)\equiv(\alpha/\beta)=-0.4643$, as we shall see. Here $P_{12}(\theta_1,\theta_2)+P_{12}(\theta_1,\theta_2)=P_{12}[\theta_1,(\theta_2+\theta_2)]$ expresses the probability of one positive detection event, i.e., a "click", registered by $D_1(\theta_1)$ in coincidence with a "click" registered by either one detector of the set $[D(\theta_2)]_2$. Owing to the lossless character of the π analyzer $A(\theta_2,\theta_2)$, $P_{12}[\theta_1,(\theta_2+\theta_2)]$ is determined by registering *simultaneous*

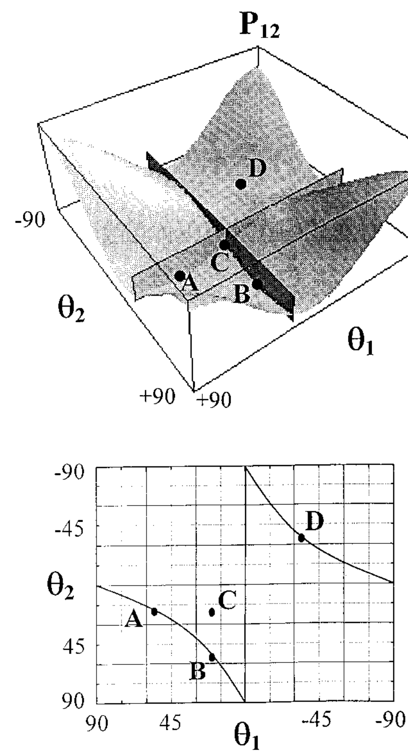

FIG. 2. Tridimensional plot of the joint detection probability. $z=P_{12}(\theta_1,\theta_2)$ for "optimum" entanglement ($\gamma=-0.4642$) as a function of angular settings of the (lossless) photon-polarization analyzers placed in front of the detectors. This γ value is obtained by setting the rotators at $\psi_1=\psi_2=55.5°$. A projection of that probability surface on the plane $z=0$ shows the loci of the points for which: $P_{12}(\theta_1,\theta_2)=0$.

coincidences involving detectors $D_1(\theta_1)$ and $D_2(\theta_2)$ or $D_1(\theta_1)$ and $D_2(\theta_2)$. This measurement is allowed by the high flexibility of our system. In the context of our π-entanglement scheme, the Hardy argument is expressed by the following statements:

(a) For a certain setting of the experimental parameters that determine the state entanglement of the pair of particles (e.g., for a certain value of α and β), let us find a setting of both π analyzers $A(\theta_i,\theta_i)$ such that $P_{12}(\theta_1',\theta_2)=0$. Then the corresponding *relative conditional probability* of detecting the pair on the i modes is equal to unity: $\mathbf{P}_{12}(\theta_1'|\theta_2)\equiv\{P_{12}(\theta_1',\theta_2)/P_{12}[(\theta_1'+\theta_1'),\theta_2]\}=1$. This last expression means that if a photon is detected by the detector $D_2(\theta_2)$ then, assuming that the other photon of the pair is actually detected, it will certainly be detected by detector $D_1(\theta_1)$.

(b) For the same values of α and β, i.e., without disturbing the state preparation, find another setting of both $A(\theta_i,\theta_i)$ such that $P_{12}(\theta_1,\theta_2')=0$. Then $\mathbf{P}_{12}(\theta_1|\theta_2')\equiv\{P_{12}(\theta_1,\theta_2')/P_{12}[\theta_1,(\theta_2'+\theta_2')]\}=1$. The measure of the relative conditional probabilities in our work is straightforward, as we shall see, and implies a large reduction of the effect of the detector quantum efficiencies from the measurement's results. Then, according to EPR realism, since the conditional probabilities of finding coincidences for a particle pair filtered through π analyzers with settings (θ_1',θ_2) and (θ_1,θ_2') are equal to unity, the field's polarization may be considered an intrinsic ''element of physical reality'' characterizing the particles [1].

(c) Assume that the two previous experiments are connected by the prescription that for the *same* values of α and β adopted for (a) and (b) we have $P_{12}(\theta_1,\theta_2)>0$.

(d) According to local realism, the one-to-one correlations (θ_1',θ_2) and (θ_1,θ_2') in conjunction with (c) imply by way of classical logic that $P_{12}(\theta_1',\theta_2')>0$ for the same values α and β. However, we shall see that we can find a couple of angles that virtually realize the ''paradoxical'' value: $P_{12}(\theta_1',\theta_2')=0$ in the ideal case (i.e., in the absence of accidental coincidences due to detector noise). It is also possible to show that this is always possible for a pair of spin-$\frac{1}{2}$ particles prepared in a nonmaximally entangled state. The solution of the contradiction according to quantum mechanics is, of course, the following: the *are no* ''elements of physical reality''in the EPR sense. The particles do not possess any intrinsic polarization attribute in the absence of measurement. Then, the results of experiments (a), (b), (c), and (d) are not mutually connected by any requirement of ontological or objective logical consistency.

Before proceeding further it is important to consider what happens in the ideal case, i.e., with a perfect ''visibility'' and in the absence of accidental coincidences, when the relative conditional probabilities are measured in our experiment. Let us consider, for instance, the measurement (a). Set the two π analyzers, respectively, at angles θ_1',θ_2, and also set the gated counting correlator to register the D output data only when one coincidence is detected between $D_2(\theta_2)$ and the full, two-detector set $[D(\theta_1')]_1$. Look now at the four printed correlated strings of binary data generated by the four detectors in the course of an experimental run in which, say, $N=10^3$ coincidences are detected. No matter how long the time period $\Delta\tau$ taken to perform the experiment has been, at the end we find that the string of data relative to $D_1(\theta_1')$ consists of N bits ''1'' and the one relative to $\overline{D}_1(\theta_1')$ of N bits ''0'' [17]. In other words, every time $D_2(\theta_2)$ registers a ''click'' on mode 2, always [''with certainty, i.e., with probability equal to unity,'' in the words by Einstein *et al.* [1]] a ''click'' is also registered by $D_1(\theta_1')$ on mode 1 and never by $\overline{D}_1(\theta_1')$. This procedure virtually realizes, in the ideal case, the operational determination of any ''element of physical reality'' according to the definition by EPR [10,15,16]. The method has been adopted to determine the angles $\theta_1',\theta_2,\theta_1,\theta_2'$ for which $\mathbf{P}_{12}(\theta_1'|\theta_2)=\mathbf{P}_{12}(\theta_1|\theta_2')=1$ once the photon wave packet overlapping in the PBS has been assured, $\Delta Z=0$. The above arguments are now applied to the experiment.

Conditions (a), (b), and (d) lead to imposing, respectively, the following relations on the detection angles: $\gamma(\tan\theta_2/\tan\theta_1')=(\tan\theta_1/\tan\theta_2')=-(\tan\theta_1'\times\tan\theta_2')$, leading to $(\tan\theta_1\times\tan\theta_2)=-\gamma^3$. By these conditional relations, the joint detection probability for a pair π analyzed along the angles θ_1,θ_2 may be expressed, as a function of θ_1 and γ, by $f(x,y)=[x/(1+x)][(1-x)^2/(1+x^3y)][y/(1+y)]$, where: $x\equiv\gamma^2$, $y\equiv\cot\text{an}^2\theta_1$ (Fig. 2). A close investigation of the surface $z=f(x,y)$ with evaluation of the Hessian determinant, i.e., the discriminant of the quadratic form expressing the differential d^2f: $H(\overline{P})\equiv\{f_{xx}(\overline{P})f_{yy}(\overline{P})-[f_{xy}(\overline{P})]^2\}$

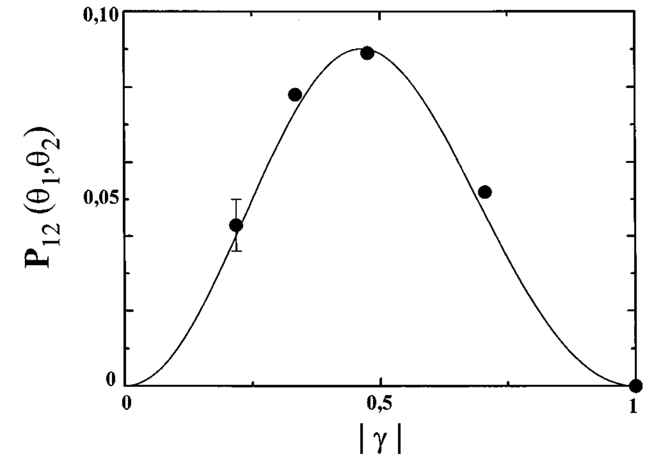

FIG. 3. Probability $P_{12}(\theta_1,\theta_2)=P_{12}[\theta_1(|\gamma|),|\gamma|)]\equiv P_{12}(|\gamma|)$ expressing Hardy's contradiction as function of the absolute value of the state-entanglement parameter $|\gamma(\psi_1,\psi_2)|$. The experimental points correspond to the result of different experiments each corresponding to different realizations of the input π rotation angles, ψ_1, ψ_2.

shows that the point $\overline{P(x,y)}$ where the derivatives $f_x(x,y)=f_y(x,y)=0$, is an elliptic point corresponding to a relative maximum determined by the interception of the lines [18]. $(a-by)=0=(1-x^3y^2)$, with $a\equiv(1-3x-2x^2)$, $b\equiv(2$

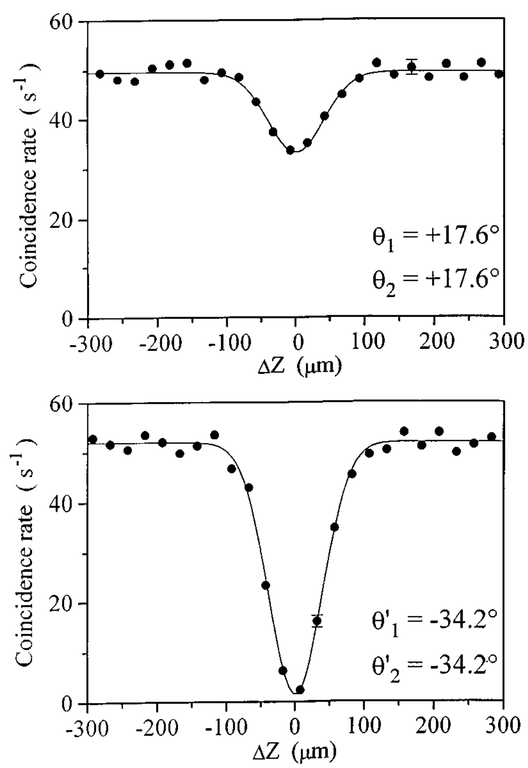

FIG. 4. Two-photon interference plots relative to the conditions expressing the logical contradiction: $P_{12}(\theta_1',\theta_2')=0$ and $P_{12}(\theta_1,\theta_2)>0$. The value of this maximum and the value of the minimum of the corresponding function expressing the coincidence counting rate vs ΔZ, i.e., the related two-photon interference visibility, are found consistent with the experimental value $P_{12}(\theta_1,\theta_2)=0.093\pm0.005$.

$+3x-x^2)x^3$. By solving a seventh degree polynomial equation we find, by the given substitutions, the ''optimum'' values $\gamma=-0.4643$, $\theta_1=\theta_2=17.55°$, that determine the maximum value of the joint probability: $P_{12}(\theta_1,\theta_2)=0.0902$. The behavior of this most relevant quantity (whose nonzero value precisely establishes the contradiction with EPR local realism) near its maximum as a function of the value of the entanglement parameter $|\gamma(\psi_1,\psi_2)|\equiv|\alpha(\psi_1,\psi_2)/\beta(\psi_1,\psi_2)|$ is shown in Fig. 3. Precisely there is shown $P_{12}[\theta_1(|\gamma|),|\gamma|]\equiv P_{12}(|\gamma|)$ where the functional relation $\theta_1=\theta_1(\gamma)$ is supplied by either one of the two conditions expressing the maximization procedure, e.g., $f_y(x,y)=0$. The experimental points on that curve have been obtained by different experiments each one corresponding to a different realization of the π rotation angles ψ_1, ψ_2. This shows the sensitivity of the maximum condition to variation of the properties of the system's state entanglement, here easily obtained by simple changes of the settings of one of, or both, input Fresnel-rhomb π rotators. Note in Fig. 3 that $P_{12}(|\gamma|)=0$ at the extreme values $|\gamma|=0$ and $|\gamma|=1$ implying the input condition of product state and Bell state, respectively. These are precisely the conditions for which Hardy's nonlocality argument cannot be applied, as stated [7,8].

The points appearing on the tridimensional plot in Fig. 2 of the joint detection probability drawn for ''optimum'' γ as a function of general angular settings of the π analyzers θ_1,θ_2 correspond to couples of angles involved in the given conditions expressing the Hardy argument. For instance, the point $\mathbf{A}(\overline{\theta_1'},\theta_2)$ corresponds to condition (a), by which $P_{12}(\mathbf{A})=0$. According to the above discussion and assuming again the ideal case, i.e., absence of noise and losses in the experiments, the EPR ''element of reality'' is then expressed by the conjugated point $\mathbf{A}'(\theta_1',\theta_2)$ with $\theta_1'=(\overline{\theta_1'}-90°)$ for which the conditional probability is $\mathbf{P}_{12}(\mathbf{A}')=1$. A similar consideration is made for the point $\mathbf{B}$ of Fig. 2 and for its conjugated $\mathbf{B}'$. The calculated values of the couples of angles involved in the contradiction are [$\theta_1'=-34{,}17°$, $\theta_2=+17.55°$: $\mathbf{P}_{12}(\mathbf{A}')\equiv\mathbf{P}_{12}(\theta_1'|\theta_2)=1$]; [$\theta_1=+17.55°$, $\theta_2'=-34{,}17°$: $\mathbf{P}_{12}(\mathbf{B}')\equiv\mathbf{P}_{12}(\theta_1|\theta_2')=1$]; [$\theta_1'=\theta_2'=-34{,}17°$: $P_{12}(\mathbf{D})\equiv P_{12}(\theta_1',\theta_2')=0$]: [$\theta_1=\theta_2=+17.55°$: $P_{12}(\mathbf{C})\equiv P_{12}(\theta_1,\theta_2)=0.0902$]. Figure 4 shows the experimental realization of the conditions accounting for the contradiction $P_{12}(\mathbf{D})\equiv P_{12}(\theta_1',\theta_2')=0.005\pm0.001$; $P_{12}(\mathbf{C})\equiv P_{12}(\theta_1,\theta_2)=0.093\pm0.005$. The experimental values for the other two points of diagram of Fig. 2 are $P_{12}(\mathbf{A})\equiv P_{12}(\overline{\theta_1'},\theta_2)=(0.006\pm0.001)=P_{12}(\mathbf{B})\equiv P_{12}(\theta_1,\overline{\theta_2'})$. Then the ''contradictory'' value of the probability $P_{12}(\mathbf{C})$ is found to be 14 standard deviations larger than the one evaluated by local realistic theories [19].

In summary, we have reported a demonstration of the essential violation of the EPR local realism in the microscopic world within the assumption that the measured events are faithful reproductions of the existing photon pairs and by ignoring the effects of noise and losses [15]. In the present work the adopted method allows by its large flexibility a fairly complete investigation of the effect of the size of any degree of state entanglement on the basic system's property implying a relevant logical contradiction. Far more generally, the present method appears to be of broad interest in quantum optics as it can be adopted for the investigation of any set nonmaximally entangled stated, e.g., as for any stochastic superposition implying the condition of mixed states [19]. We believe that the present work may represent a relevant contribution toward the demonstration of the validity of the standard quantum theory in the context of one of its most intriguing aspects, nonlocality.

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[13] The right-hand particle π convention is assumed through this work.

[14] The stray parameters of PBS have been measured by direct detection of the transmitted and reflected photon-*A* exciting PBS on input mode 1 with π horizontal or vertical, in coinci-

dence with detection of photon-B on mode 2 *right after* PDC. The role of the two modes was then interchanged for double checking. By the measured values $[\varepsilon = 0.007 \pm 0.002;\ \varepsilon' = 0.005 \pm 0.002]$ the deviation of the final results of the experiment respect to theory has been determined. The given expression of $\mathbf{U}_{\mathrm{PBS}}$ is valid when zero or one photon is present on each i mode.

[15] Because of this state postselection, implying the nondetection by the system of a relevant number of PDC generated photon pairs and thus a reduced overall quantum efficiency of the apparatus, the present experiment cannot be considered a loophole-free experiment: cf. P. H. Eberhard, Phys. Rev. A **47**, 748 (1993); L. De Caro and A. Garuccio, *ibid.* **50**, R2803 (1994). Note that the Hardy argument can be transformed into a Bell-type inequality: A. Garuccio, Phys. Rev. A **52**, 2535 (1995). Thus far all experiments on the violation of Bell's-type inequalities have implied the same postselection process encountered here. The only exception is the recent work adopting the method of ''direct entanglement'' allowed by type-II NL crystals: P. Kwiat, K. Matte, H. Weinfurter, A. Sergienko, A. Zeilinger, and Y. Shih, Phys. Rev. Lett. **75**, 4337 (1995); and D. Boschi, F. De Martini, and G. Di Giuseppe in *Quantum Interferometry*, edited by F. De Martini, G. Denardo, and Y. Shih (VCH Publishing group, Berlin, 1996).

[16] While $\mathbf{P}_{12}(\theta_1', \theta_2)$ is independent of any (Q.E.), $\Delta\tau$ is inversely proportional to the product of all (Q.E.)'s involved in the experiment; i.e., related to laser, NL crystal, optics, detectors, etc.

[17] Note that by the adoption, made in Ref. [9], of *only* two $[(\text{lossy } \pi\text{-analyzer}) + (\text{single detector})]_i$ systems coupled to the corresponding output i modes, the trivially tautological registration of coincident ''clicks'' by the only two detectors active in the experiment, call them $D_1(\theta_1')$ and $D_2(\theta_2)$, would not exclude the presence of all the significant events involving a missed coincidence between a ''click'' registered on one mode and a twin photon *rejected* (i.e., lost, absorbed) on the other. Then the latter technique could not allow the unambiguous operational determination of any ''element of physical reality.''

[18] E. Goursat, *A Course in Mathematical Analysis* (Dover, New York, 1964), Vol. I. The π-angle degeneracy, $\theta_1' = \theta_2'$, $\theta_1 = \theta_2$, due to the symmetry of the $|\Psi\rangle$, is irrelevant within the argument expressing the contradiction: the degeneracy could be lifted by slight changes introduced in the optical circuit of the apparatus. Note that our work *does not* consist of a single-outcome experiment: as pointed out by Hardy, it falls half-way between two extremes represented by experiments testing the Bell's-type proof and the GHZ proof, respectively: in the first case it would only be necessary to have a statistical violation of quantum mechanics while in the latter the predictions of that theory can be violated by single events [7].

[19] The fact that the experimental results do not give exactly zero for those probabilities that are predicted to be zero does not impair the present test of the contradiction. Indeed the statistical analysis for these slight deviations from zero reveals that the contradiction still goes through as long as the following inequality holds: $P_{12}(\theta_1, \theta_2) > P_{12}(\theta_1', \theta_2') + P_{12}(\theta_1, \theta_2') + P_{12}(\theta_1', \theta_2)$. N. D. Mermin, in *Fundamental Problems in Quantum Theory*, edited by D. M. Greenberger and A. Zeilinger (Annals of the New York Academy of Sciences, New York, 1995). A Bell inequality theory involving mixed states has been given by R. Horodecki, P. Horodecki, and M. Horodecki, Phys. Lett. A **200**, 340 (1995); **210**, 223 (1996). A study on mixed states by the present π-entanglement method is now in progress in our laboratory.